DYNAMIC NETWORKS

(Dynamic <u>of</u> networks)

DYNAMIC NETWORKS

- Most real world networks are dynamic
  - Facebook friendship
    - People joining/leaving
    - Friend/Unfriend
  - Twitter mention network
    - Each mention has a timestamp
    - Aggregated every day/month/year => still dynamic
  - World Wide Web
  - Urban network
  - …

DYNAMIC NETWORKS

- Most real world networks are dynamic
  - Nodes can appear/disappear
  - Edges can appear/disappear
  - Nature of relations can change
- How to represent those changes?
- How to manipulate dynamic networks?

DYNAMIC NETWORKS

Semantic level

Relations	Interactions

Long term

-Friend
-Colleague
-Family relation
-…

Short term ?

-Collaborators in the same project
-Same team in a game
-Attendees of the same meeting
-…

Instantaneous

-e-mail
-Text message
-Co-authoring
…

With duration

-Phone call
-Discussion in real life
-Participate in a same meeting

DYNAMIC NETWORKS

Semantic level

Relations

Interactions

Representation level

Interval graphs

DN=(V,E,T,DV)
DV:VxTxT
E:VxVxTxT

Graph series

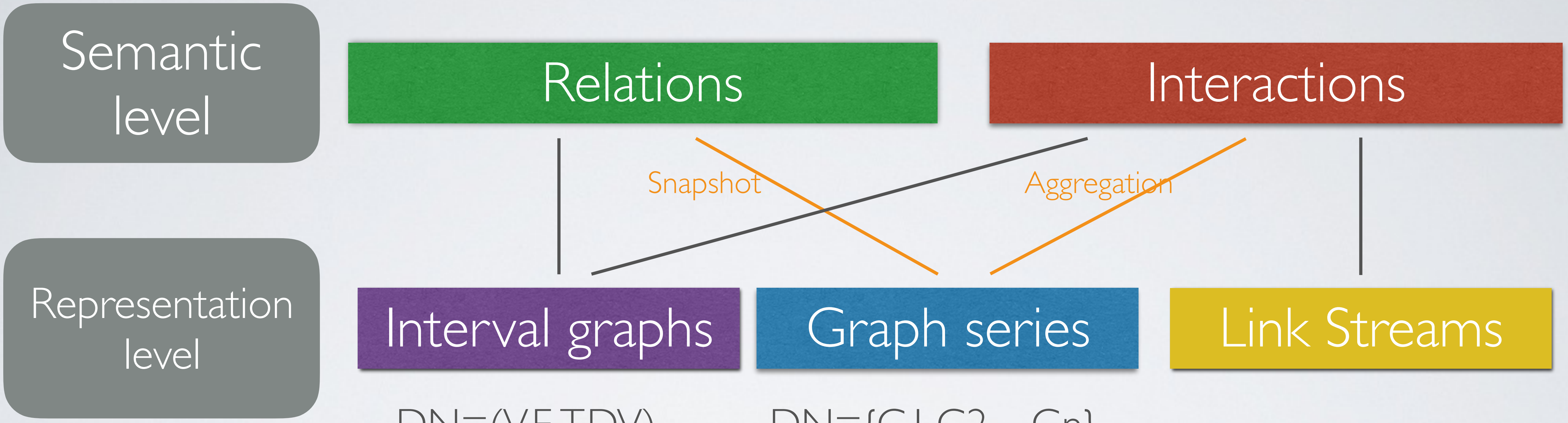
DN=[G1,G2…Gn]
Gi=(V,E)
E :VxV

(Or 3D tensor)

Link Streams

DN=(V,E,T)
E:VxVxT

DYNAMIC NETWORKS

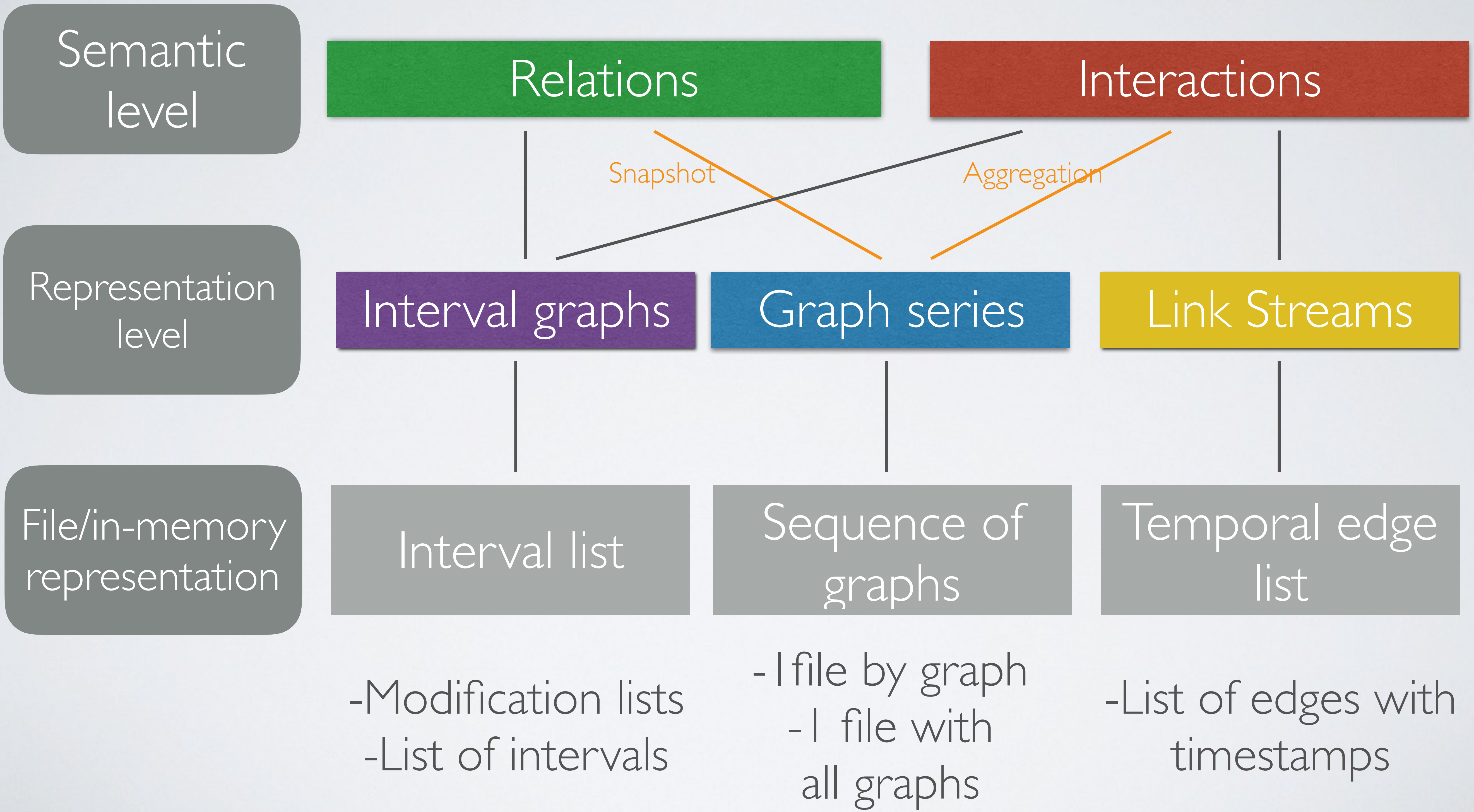


DN=(V,E,T,DV)
DV:VxTxT
E:VxVxTxT

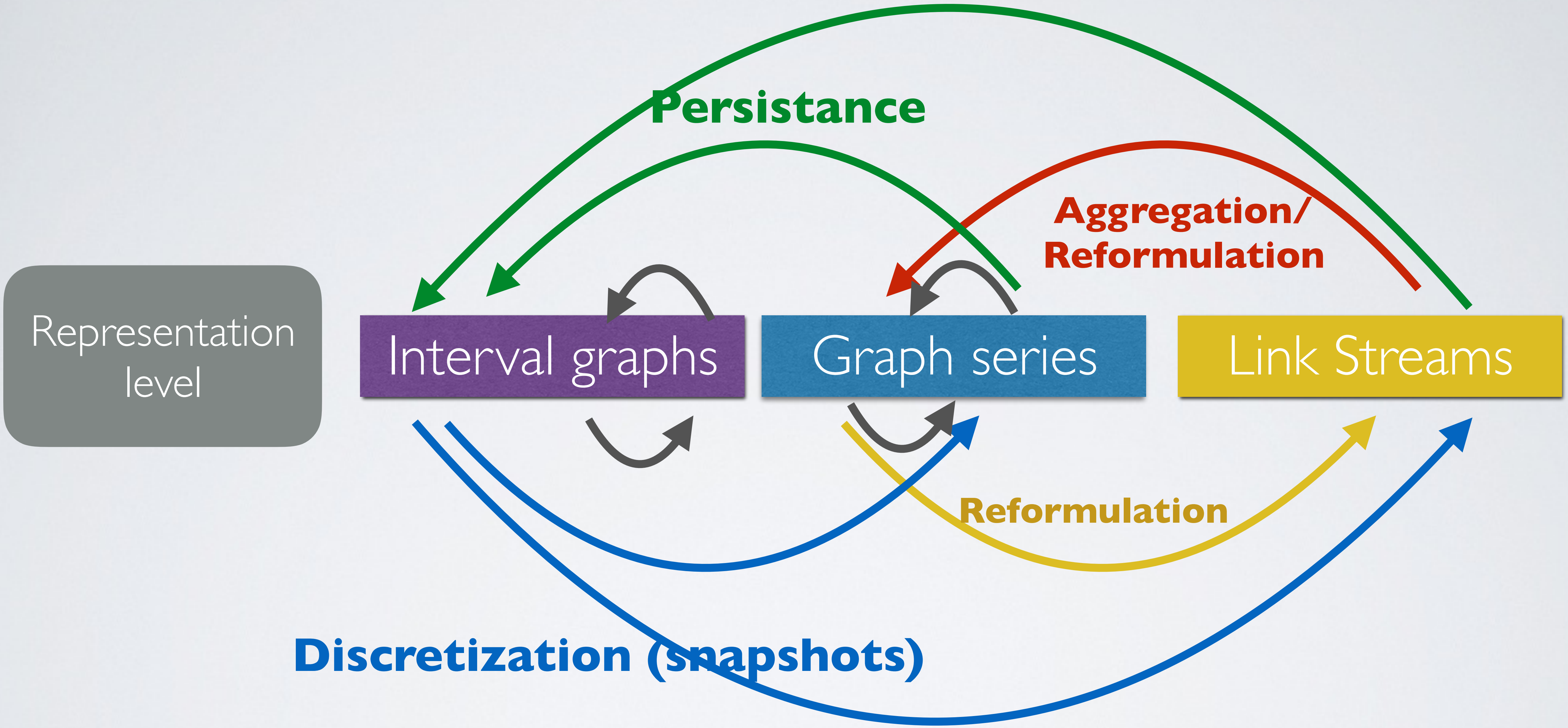
DN={G1,G2…Gn}
Gi=(V,E)
E :VxV

DN=(V,E,T)
E:VxVxT

DYNAMIC NETWORKS



DYNAMIC NETWORKS



DYNAMIC NETWORKS

- Exemple in practice: Sociopattern dataset
  - Every 20s, list of individuals at distance ≈ 1,5m
  - Dataset : **sequence of graphs** or **temporal edge list**

```
1353304100	1148	1644
1353304100	1613	1672
1353304100	656	682
1353304100	1632	1671

1353304120	1492	1613
1353304120	656	682
1353304120	1632	1671

1353304140	1148	1644

1353304160	656	682
1353304160	1108	1601
1353304160	1632	1671
1353304160	626	698
```

Types of network evolution

According to

1) Observation frequency
2) Network nature

Relations

The graph is more and more stable, until most observations are completely similar to previous/later ones (frequency faster than change rate)

Static Network → **Higher Observation Frequency**

Exhaustive / continuous time

The graph is less and less stable, until each observation is a graph in itself, thus completely different from previous/later ones (frequency faster than observed events rate)

Interactions

ANALYZING DYNAMIC NETWORKS

DISTINGUISHING:

-UNSTABLE SNAPSHOTS

-STABLE NETWORKS

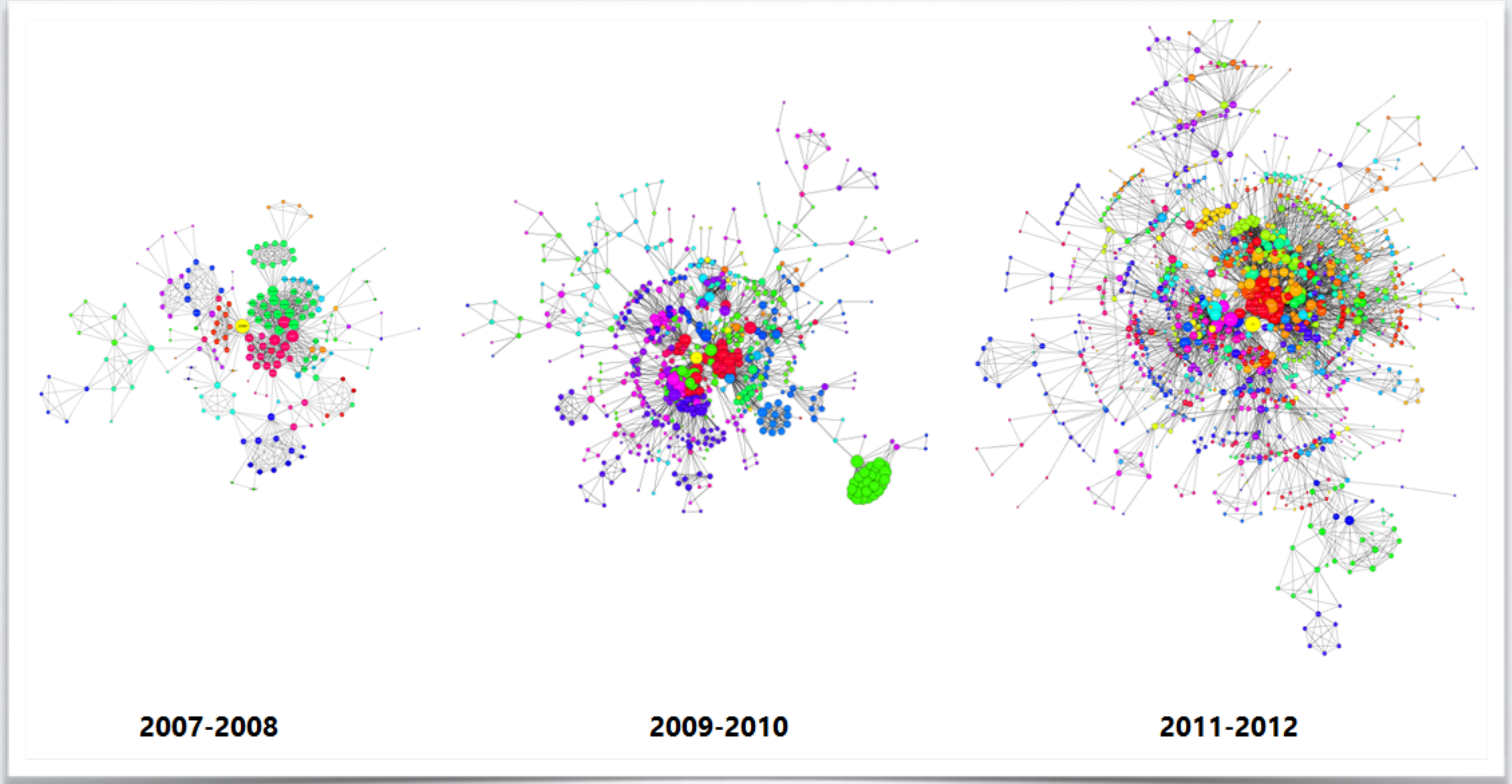
-(UNSTABLE) TEMPORAL NETWORKS

(WITH OR WITHOUT DURATION)

UNSTABLE SNAPSHOTS

UNSTABLE SNAPSHOTS

- The evolution is represented as a series of *a few* snapshots.
- Many changes between snapshots
  - Cannot be visualized as a "movie"



UNSTABLE SNAPSHOTS

- Each snapshot can be studied as a static graph
- The evolution of the properties can be studied "manually"
- "Node X had low centrality in snapshot t and high centrality in snapshot t+n"

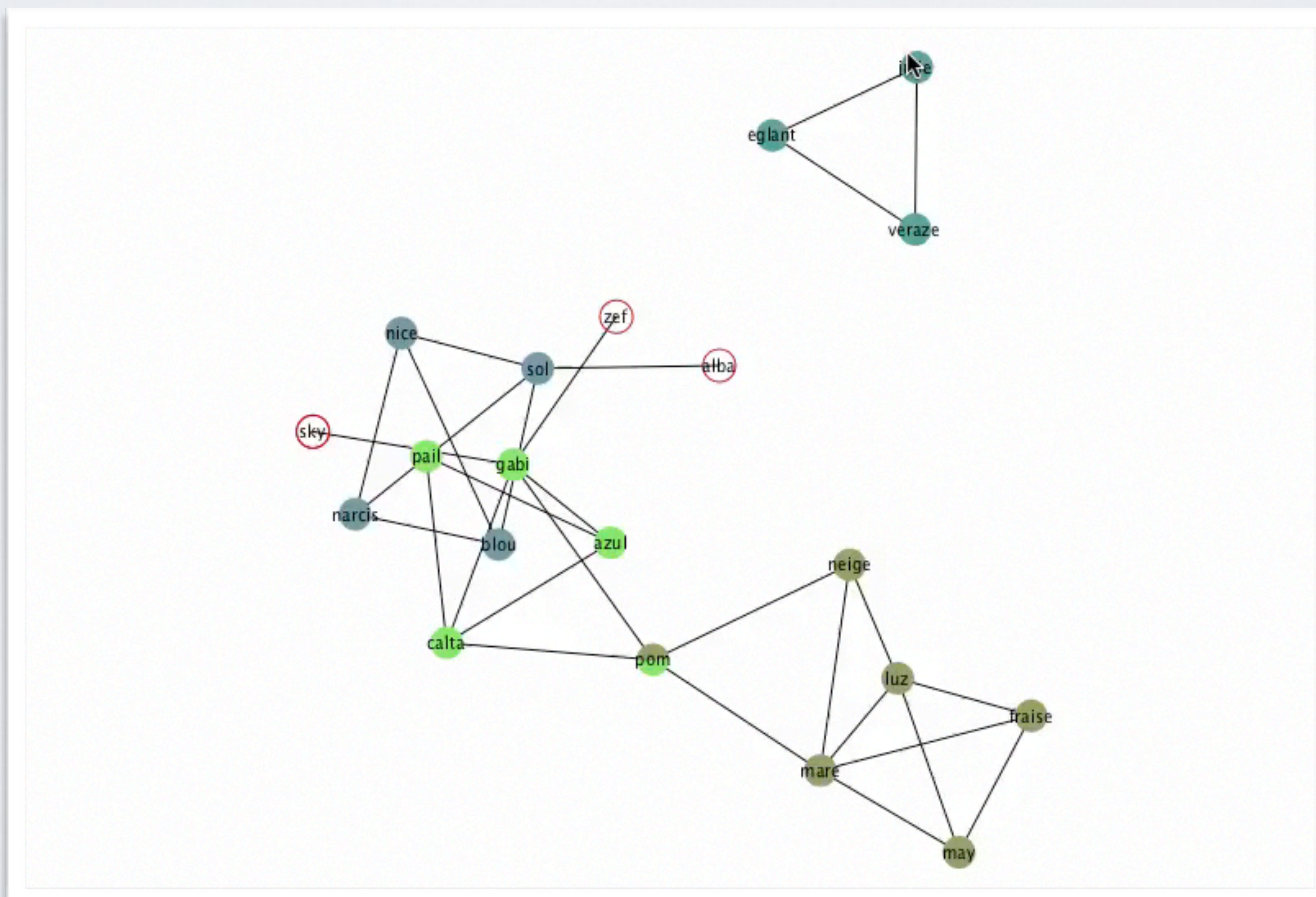
STABLE NETWORKS

STABLE NETWORK

- Edges change (relatively) slowly
- The network is well defined at any t
  - Temporal network: nodes/edges described by (long lasting) intervals
  - Enough snapshots to track nodes
- A static analysis at every (relevant) t gives a dynamic vision
- No formal distinction with previous case (higher observation frequency)

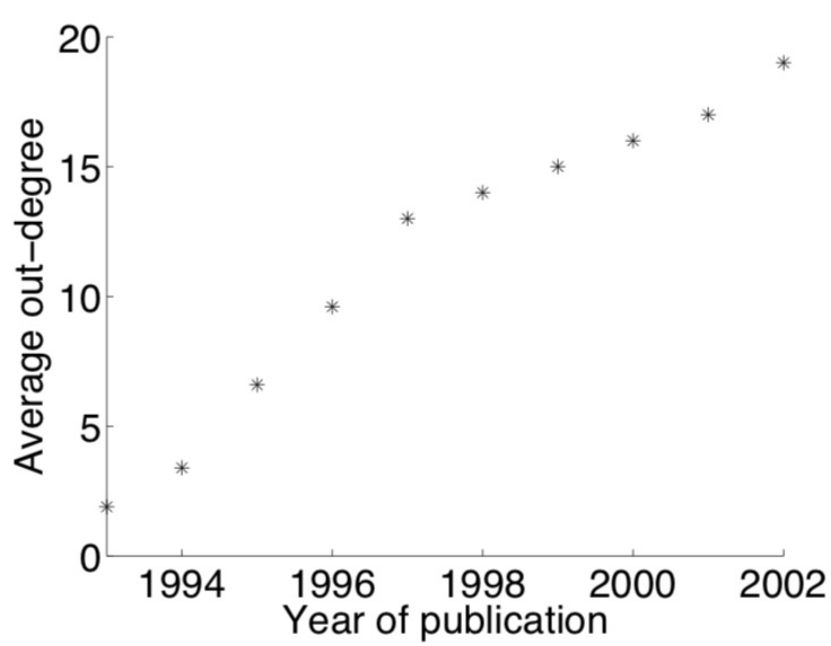
STABLE NETWORK

- Visualization
  - Problem of stability of node positions

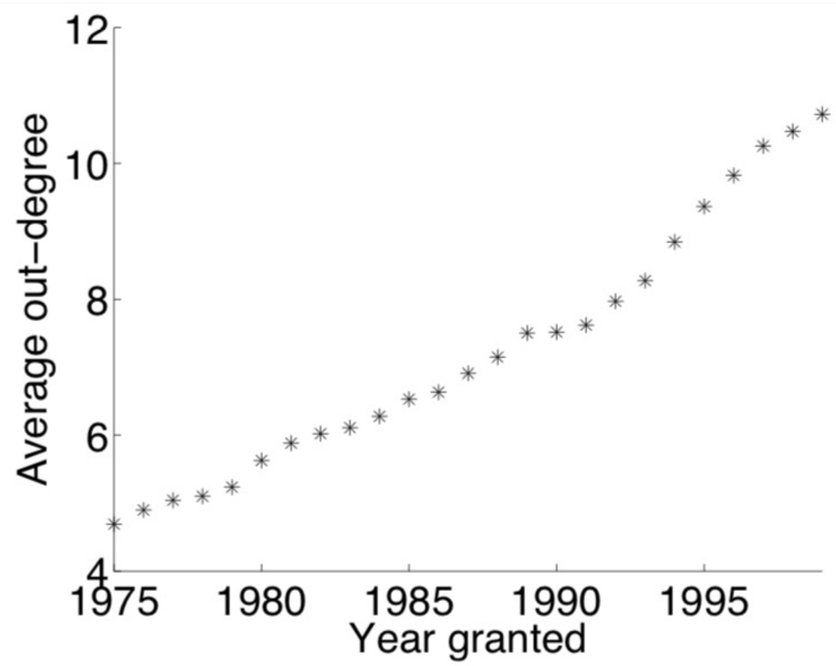


STABLE NETWORK

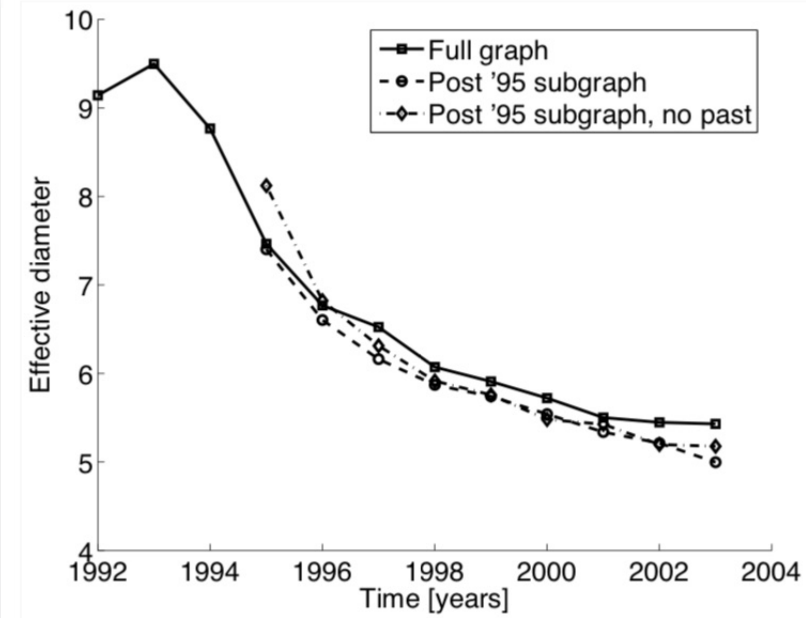
- Global graph properties



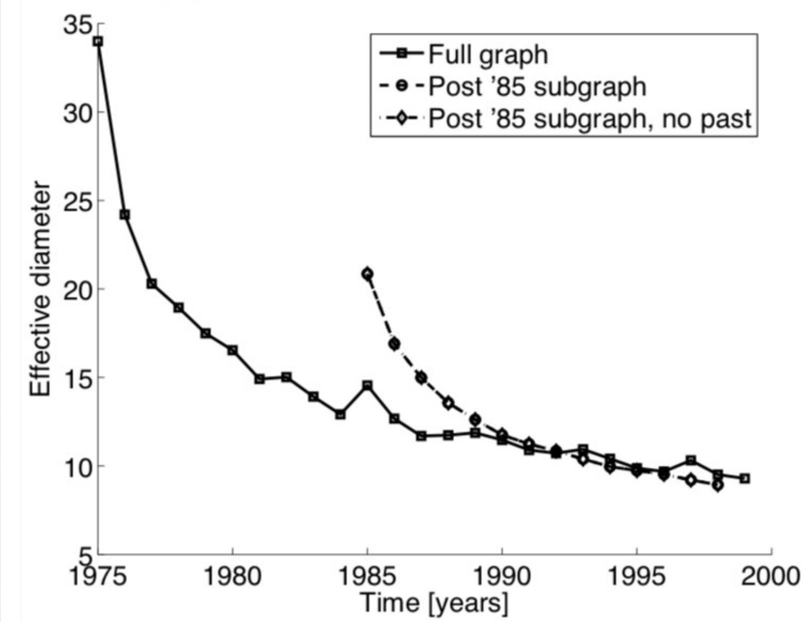
(a) arXiv



(b) Patents



(a) arXiv citation graph

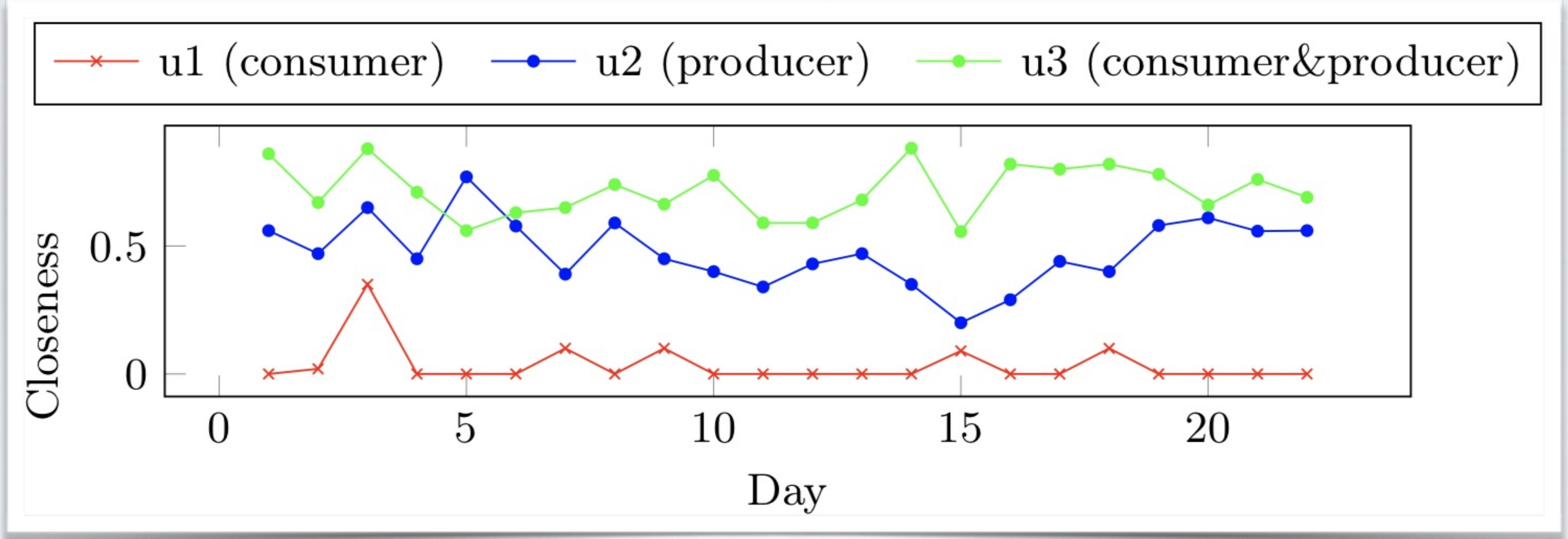


(c) Patents citation graph

Leskovec, Jure, Jon Kleinberg, and Christos Faloutsos. "Graph evolution: Densification and shrinking diameters." *ACM Transactions on Knowledge Discovery from Data (TKDD)* 1.1 (2007): 2.

STABLE NETWORK

- Centralities



TIME SERIES ANALYSIS

- TS analysis is a large field of research
- Time series: evolution of a value over time
  - Stock market, temperatures…
- "Killer app":
  - Detection of periodic patterns
  - Detection of anomalies
  - Identification of global trends
  - Evaluation of auto-correlation
  - Prediction of future values
- e.g. ARIMA (Autoregressive integrated moving average)

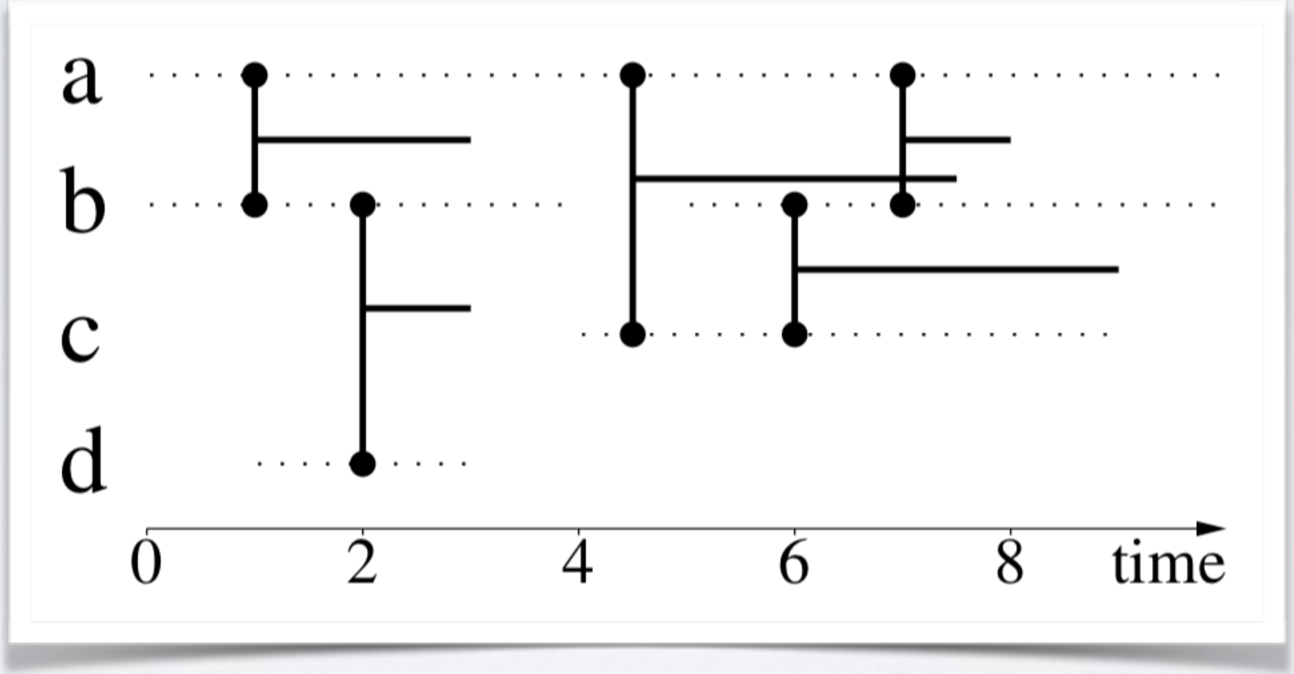
https://en.wikipedia.org/wiki/Autoregressive_integrated_moving_average

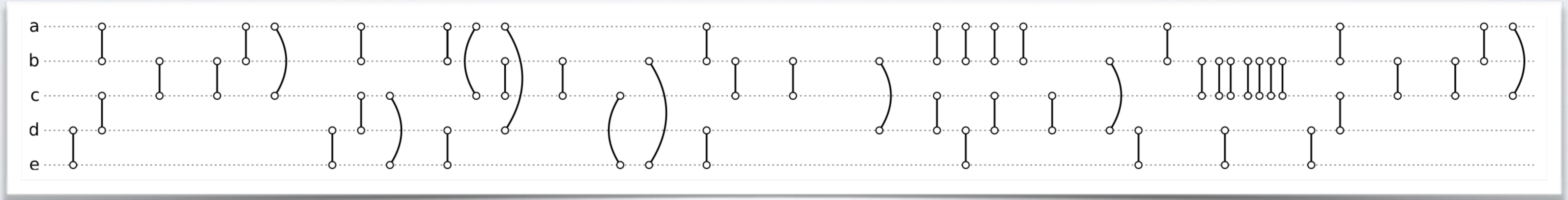
UNSTABLE TEMPORAL NETWORKS

UNSTABLE TEMPORAL NETWORK

- The network at a given t is not meaningful
- How to analyze such a network?

UNSTABLE TEMPORAL NETWORK





UNSTABLE TEMPORAL NETWORK

- Until recently, network was transformed using aggregation/ sliding windows
  - Information loss
  - How to chose a proper aggregation window size?
- Tools developed to deal with such networks

UNSTABLE TEMPORAL NETWORK

- [Holme 2012]: mostly about paths, walks, distances… (later class, diffusion on networks.)

Holme, Petter, and Jari Saramäki. "Temporal networks." *Physics reports* 519.3 (2012): 97-125.

- [Latapy 2018]: Other things (centralities, …)

Latapy, M., Viard, T., & Magnien, C. (2018). Stream graphs and link streams for the modeling of interactions over time. *Social Network Analysis and Mining*, *8*(1), 61.

- Idea: Generalize all graphs definitions to temporal networks
- => If all nodes and all edges always present, same values as for a static graph

CENTRALITIES
&
NETWORK PROPERTIES
IN STREAM GRAPHS

STREAM GRAPHS

$$\textbf{stream graph } S = (T, V, W, E)$$

T: Possible Time

V: vertices

W: Vertices presence time

E: Edges presence time

INDICES IN STREAM GRAPHS

Number of nodes:

$$\frac{\text{Total presence of nodes}}{\text{Total dataset duration}}$$

$$n = \sum_{v \in V} n_v = \frac{|W|}{|T|}$$

(not an integer value…)

e.g.: 2 if 4 nodes
half the time

INDICES IN STREAM GRAPHS

Number of edges:

$$\frac{\text{Total presence of edges}}{\text{Total dataset duration}}$$

$$m = \sum_{uv \in V \otimes V} m_{uv} = \frac{|E|}{|T|}$$

(not an integer value…)

e.g.: 1 if 1 edge all the time

INDICES IN STREAM GRAPHS

Neighborhood of a node

$$N(v) = \{(t,u), (t,uv) \in E\}$$

Degree of a node

$$d(v) = \frac{|N(v)|}{|T|} = \sum_{u \in V} \frac{|T_{uv}|}{|T|}$$

INDICES IN STREAM GRAPHS

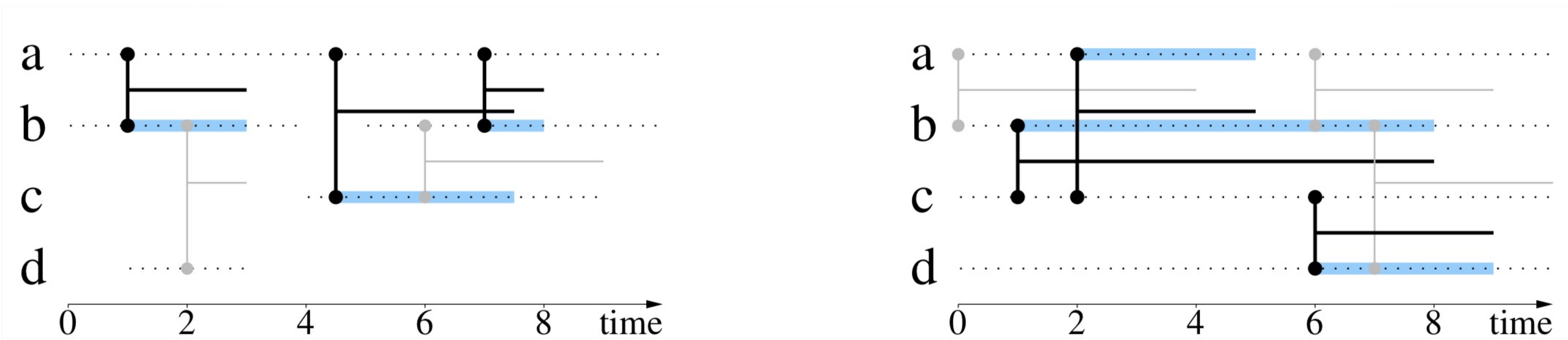


Figure 5: **Two examples of neighborhoods and degrees of nodes.** We display in black the links involving the node under concern, and in grey the other links. Left: $N(a) = ([1,3] \cup [7,8]) \times \{b\} \cup [4.5, 7.5] \times \{c\}$ is in blue, leading to $d(a) = \frac{3}{10} + \frac{3}{10} = 0.6$. Right: $N(c) = [2,5] \times \{a\} \cup [1,8] \times \{b\} \cup [6,9] \times \{d\}$ is in blue, leading to $d(c) = \frac{13}{10} = 1.3$.

INDICES IN STREAM GRAPHS

Average node degree

$$d(V) = \frac{1}{n} \cdot \sum_{v \in V} n_v \cdot d(v) = \sum_{v \in V} \frac{|T_v|}{|W|} \cdot d(v)$$

INDICES IN STREAM GRAPHS

Clustering coefficient of a node

$$cc(v) = \delta(N(v)) = \frac{\sum_{uw \in V \otimes V} |T_{vu} \cap T_{vw} \cap T_{uw}|}{\sum_{uw \in V \otimes V} |T_{vu} \cap T_{vw}|}$$

Probability that if we take 2 random neighbors at a random time, they are linked

INDICES IN STREAM GRAPHS

$$\delta(S) = \frac{\sum_{uv \in V \otimes V} |T_{uv}|}{\sum_{uv \in V \otimes V} |T_u \cap T_v|} = \frac{\int_{t \in T} |E_t| \, \mathrm{d}t}{\int_{t \in T} |V_t \otimes V_t| \, \mathrm{d}t}$$

Density (of a stream graph): probability if we take a random pair of nodes at a random time that there is an edge between them

INDICES IN STREAM GRAPHS

$$\delta(S) = \frac{\sum_{uv \in V \otimes V} |T_{uv}|}{\sum_{uv \in V \otimes V} |T_u \cap T_v|} = \frac{\int_{t \in T} |E_t| \, \mathrm{d}t}{\int_{t \in T} |V_t \otimes V_t| \, \mathrm{d}t}$$

Total edge presence

e.g.: 10 if 2 edges present over 5 periods

INDICES IN STREAM GRAPHS

$$\delta(S) = \frac{\sum_{uv \in V \otimes V} |T_{uv}|}{\sum_{uv \in V \otimes V} |T_u \cap T_v|} = \frac{\int_{t \in T} |E_t| \, \mathrm{d}t}{\int_{t \in T} |V_t \otimes V_t| \, \mathrm{d}t}$$

Total **overlapping time** between each pair of nodes

=>An edge is possible

INDICES IN STREAM GRAPHS

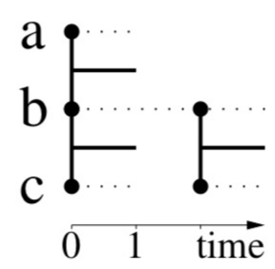


Figure 2: **Two stream graphs with $n = 2$ nodes, $m = 1$ link, but with different densities:** Left: $\delta = 0.75$. Right: $\delta = 1$.

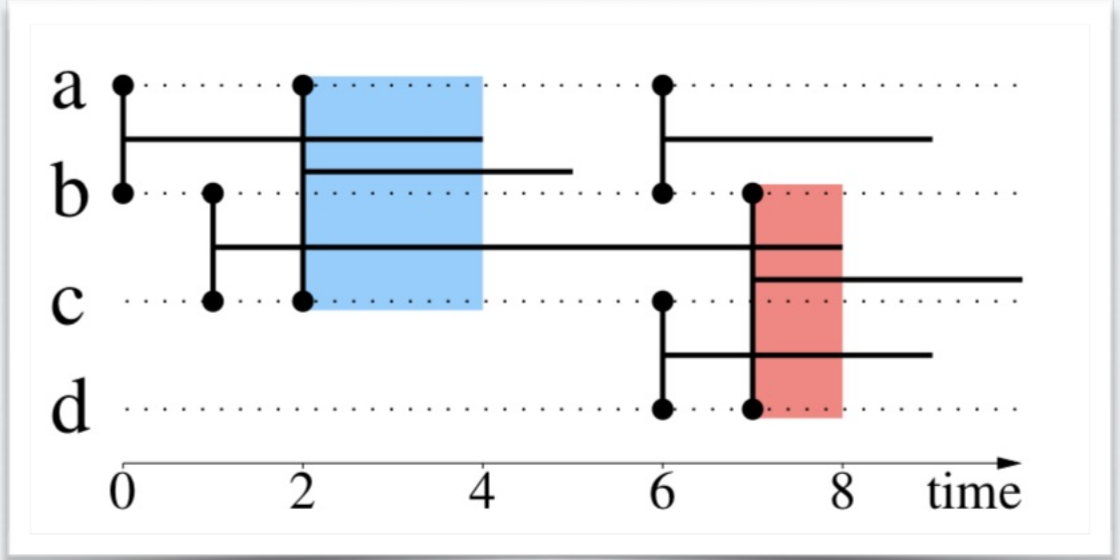
INDICES IN STREAM GRAPHS

- Note that we can define particular cases of density:
  - Density for a pair of nodes
  - Density for a node

$$\delta(uv) = \frac{|T_{uv}|}{|T_u \cap T_v|}, \quad \delta(v) = \frac{\sum_{u \in V, u \neq v} |T_{uv}|}{\sum_{u \in V, u \neq v} |T_u \cap T_v|}$$

INDICES IN STREAM GRAPHS

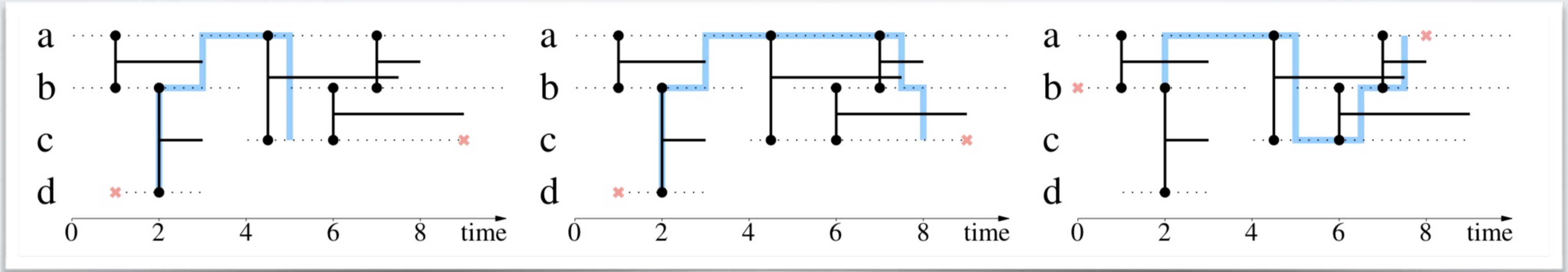
A clique of graph G is a cluster C of G of density 1. In other words, all pairs of nodes involved in C are linked together in G. A clique C is maximal if there is no other clique C' such that $C \subset C'$.



PATHS AND DISTANCES IN STREAM GRAPHS

PATHS

- A path in a stream graphs
  - Starts at a node and a date
  - Ends at a node and a date
  - Has a length (number of hops)
  - Has a duration (duration from leaving node to reaching node)



Path(d,1)(c9)
Length:3
Duration: 3

?

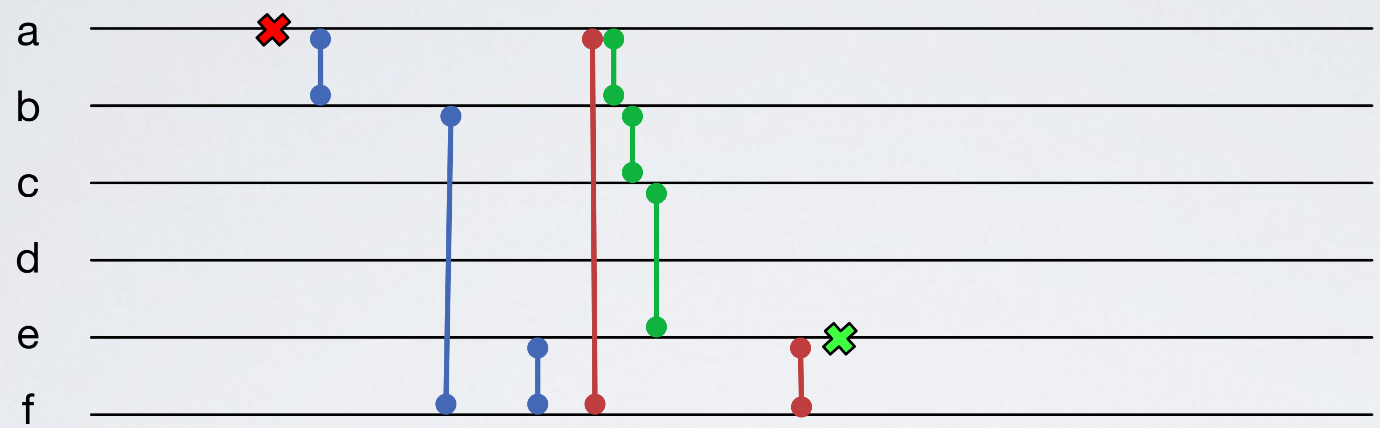
?

SHORTEST PATHS

- Several types of shortest paths in Stream graphs:
  - Shortest path: minimal length
  - Fastest path: minimal duration
  - Foremost path: first to reach

  - Fastest shortest paths
    - Minimum duration among minimal length
  - Shortest fastest paths
    - Minimal length among minimal duration

SHORTEST PATHS

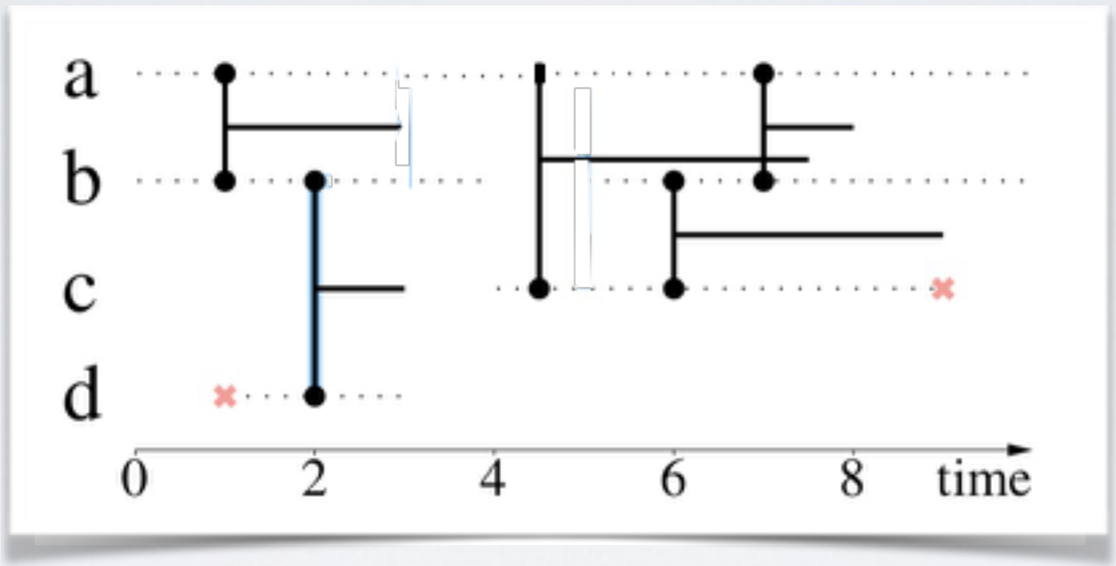


Blue: Foremost

Green: Fastest

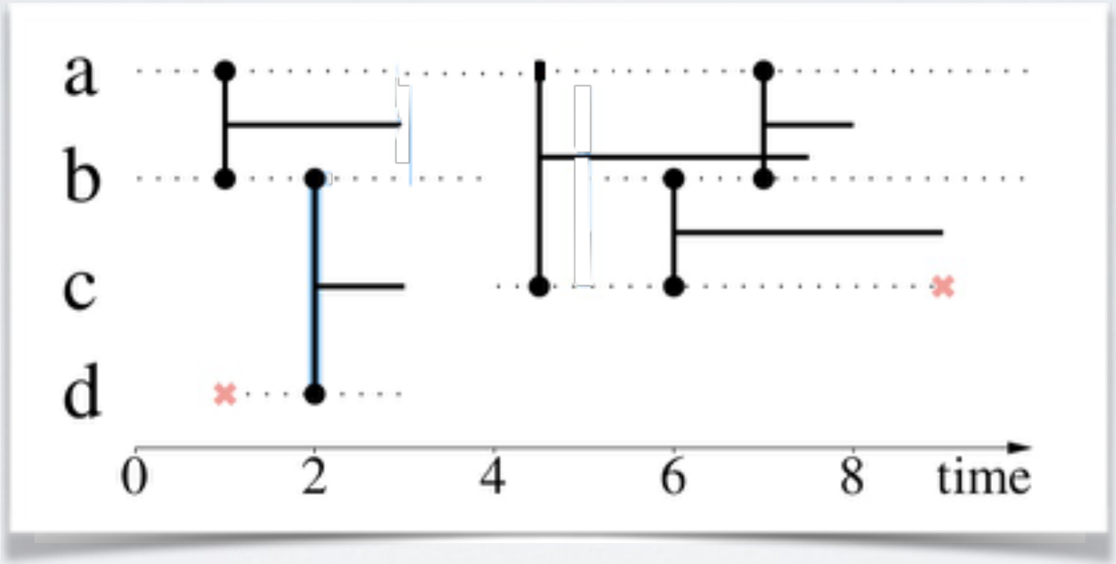
Red: Shortest

SHORTEST PATHS



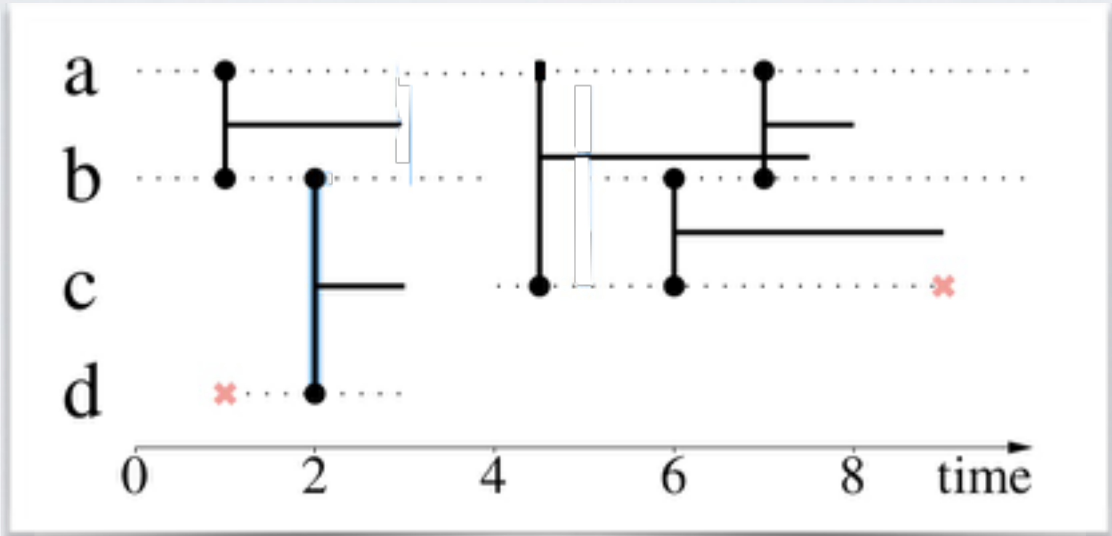
Shortest paths from (1, d) to (9, c) ?

SHORTEST PATHS



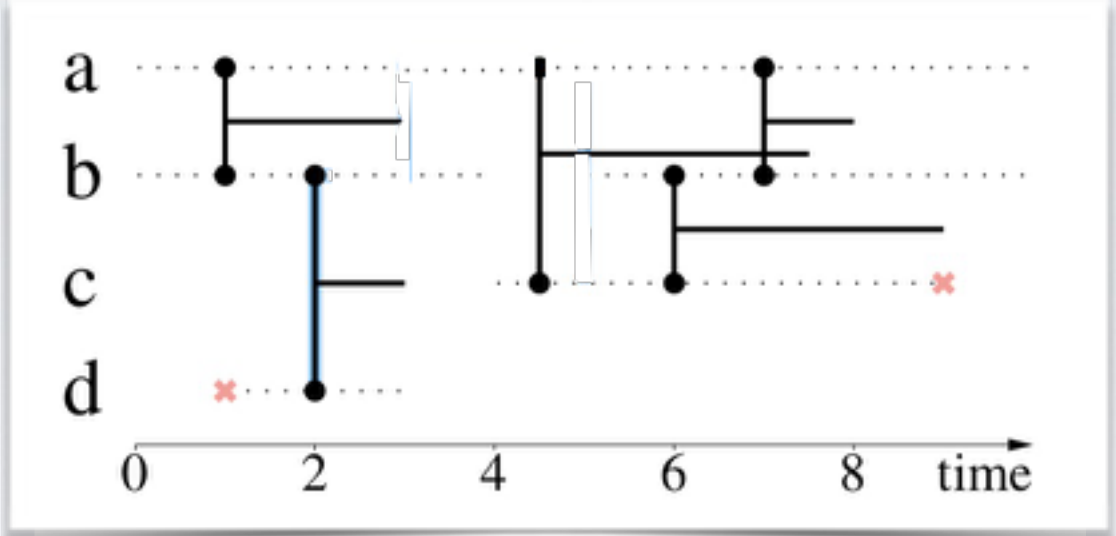
Shortest paths from (1, d) to (9, c) ?
=>e.g. (2.5,d,b)(3,b,a)(7,a,c)

SHORTEST PATHS



Fastest paths from (1, d) to (9, c) ?

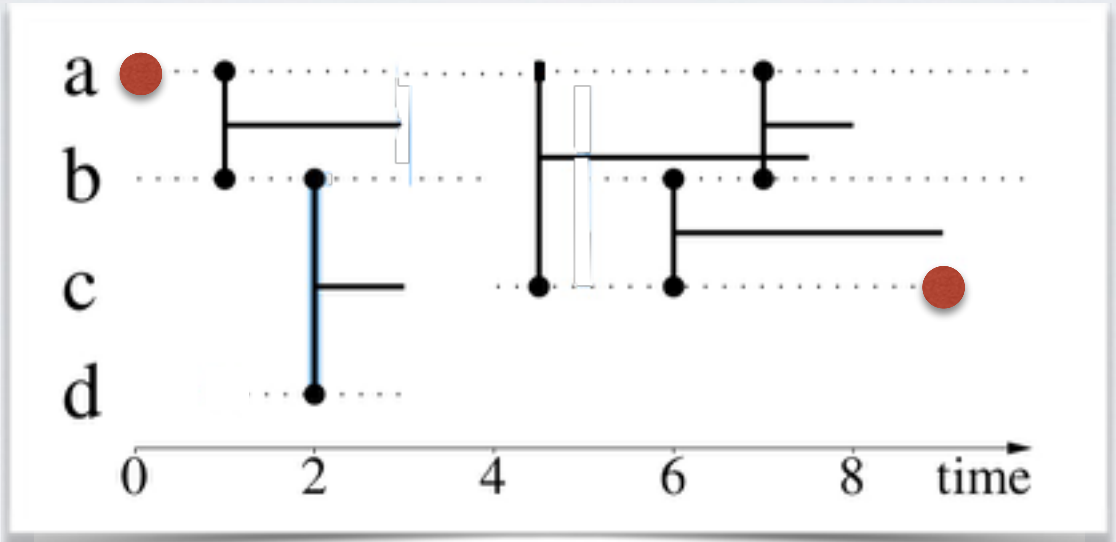
SHORTEST PATHS



Fastest paths from (1, d) to (9, c) ?

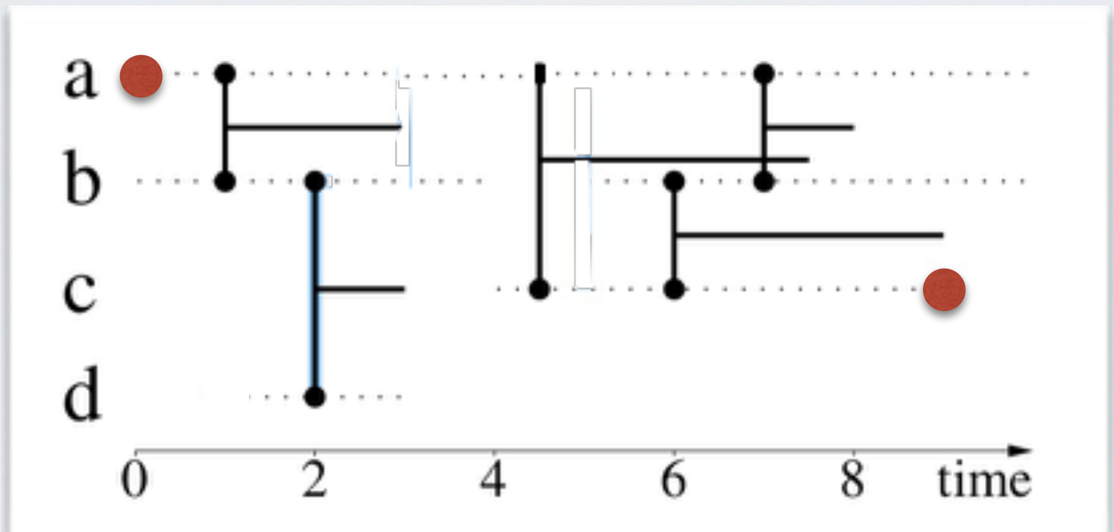
(3,d,b),(3,b,a),(4.5,a,c)

SHORTEST PATHS



Foremost paths from (0, a) to (9, c) ?

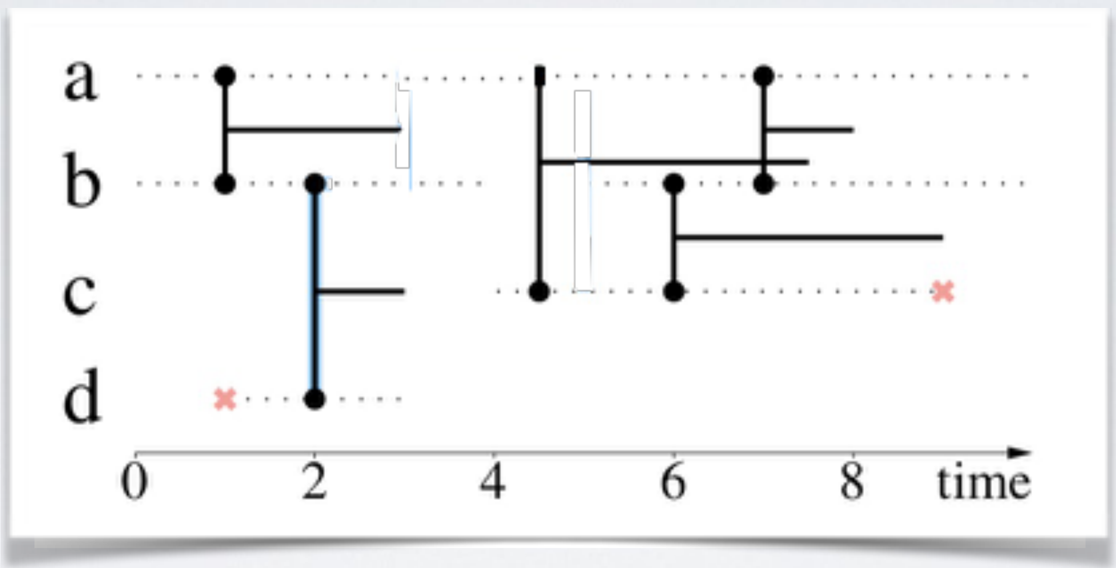
SHORTEST PATHS



Foremost paths from (0, a) to (9, c) ?

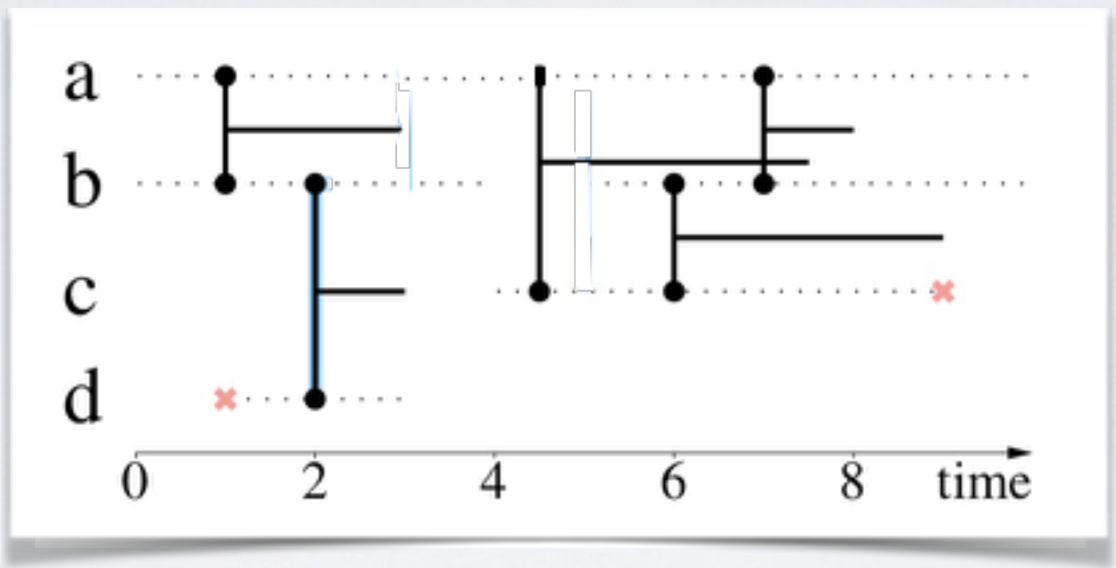
…(4.5,a,c)

SHORTEST PATHS



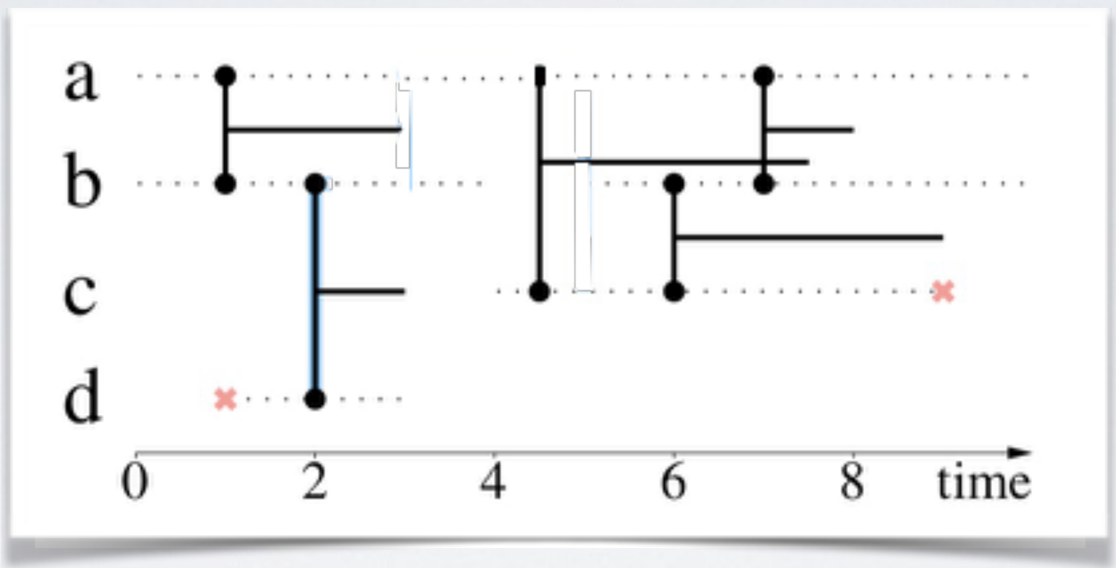
Fastest shortest path from (1, d) to (9, c) ?

SHORTEST PATHS



Fastest shortest path from (1, d) to (9, c) ?

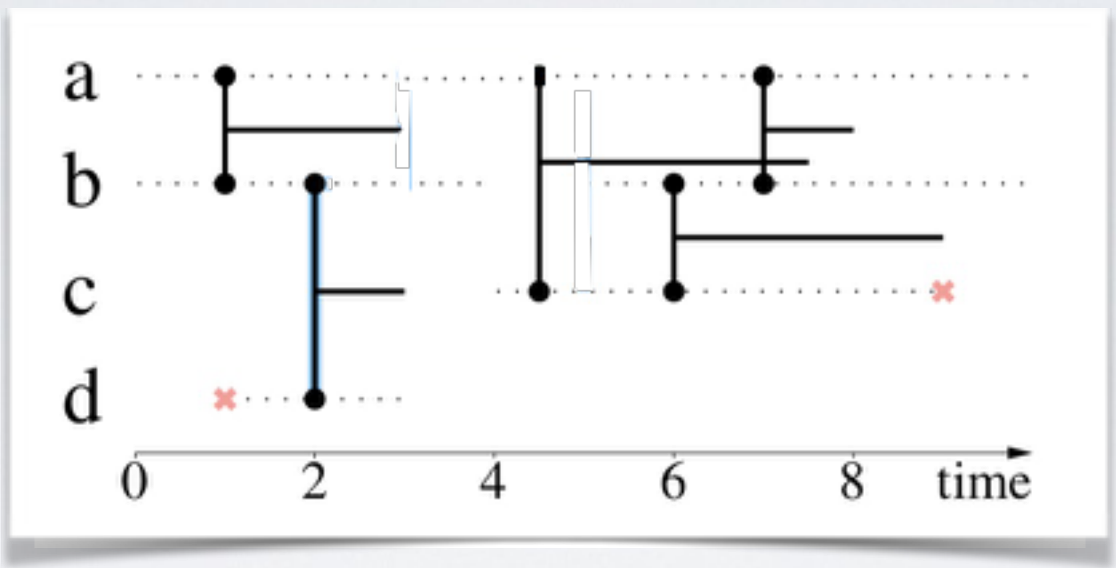
SHORTEST PATHS



Fastest Shortest path from (1, d) to (9, c) ?

(3, d, b), (3, b, a), (4.5, a, c)

SHORTEST PATHS

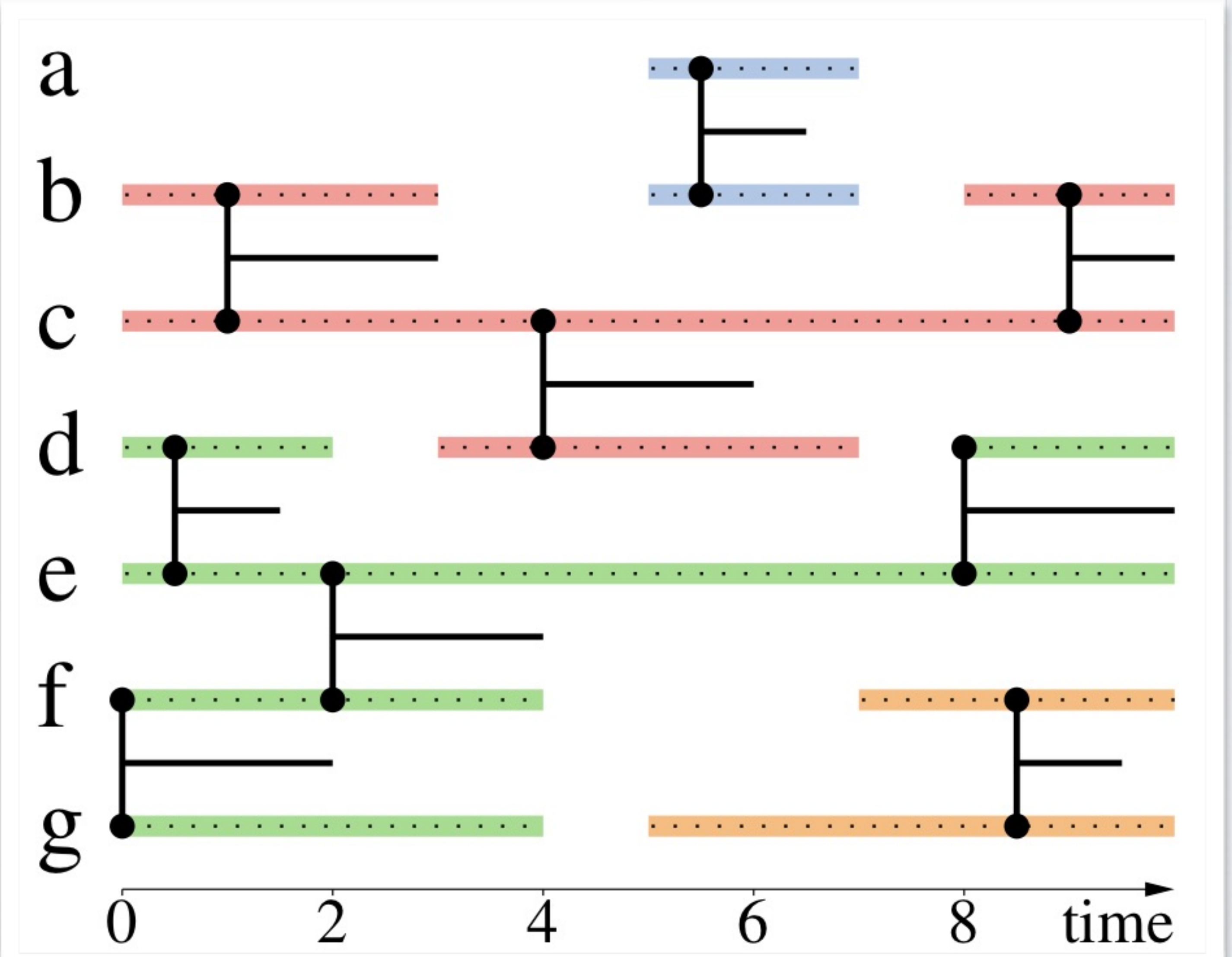


Shortest Fastest path from (1, d) to (9, c) ?

OTHER DEFINITIONS ON STREAM GRAPHS

CONNECTED COMPONENTS

- Weakly connected component:
  - There is at least a non-temporally respecting path



CLOSENESS - BETWEENNESS

$$\mathcal{C}_t(v) = \sum_{u \in V} \int_{\substack{s \in T \\ (s,u) \neq (t,v)}} \frac{1}{c_t(v,(s,u))} \, \mathrm{d}s$$

Shortest path in Static graphs is replaced by a *cost function,* any notion of distance (typically, time to reach)

$$\mathcal{B}(t,v) = \sum_{u \in V, w \in V} \int_{i \in T_u, j \in T_w} \frac{\sigma((i,u),(j,w),(t,v))}{\sigma((i,u),(j,w))} \, \mathrm{d}i \, \mathrm{d}j$$

Proportion of all the shortest fastest paths between all possible (time,node) pairs that go through (t,v)

Latapy, M., Viard, T., & Magnien, C. (2018). Stream graphs and link streams for the modeling of interactions over time. *Social Network Analysis and Mining*, *8*(1), 61.

RANDOM MODELS FOR DYNAMIC NETWORKS

RANDOM MODELS

- In many cases, in network analysis, useful to compare a network to a randomized version of it
  - Clustering coefficient, assortativity, modularity, …
- In a static graph, 2 main choices:
  - Keep only the number of edges (ER model)
  - Keep the number of edges and the degree of nodes (Configuration model)
- In dynamic networks, it is more complex…

RANDOM MODELS

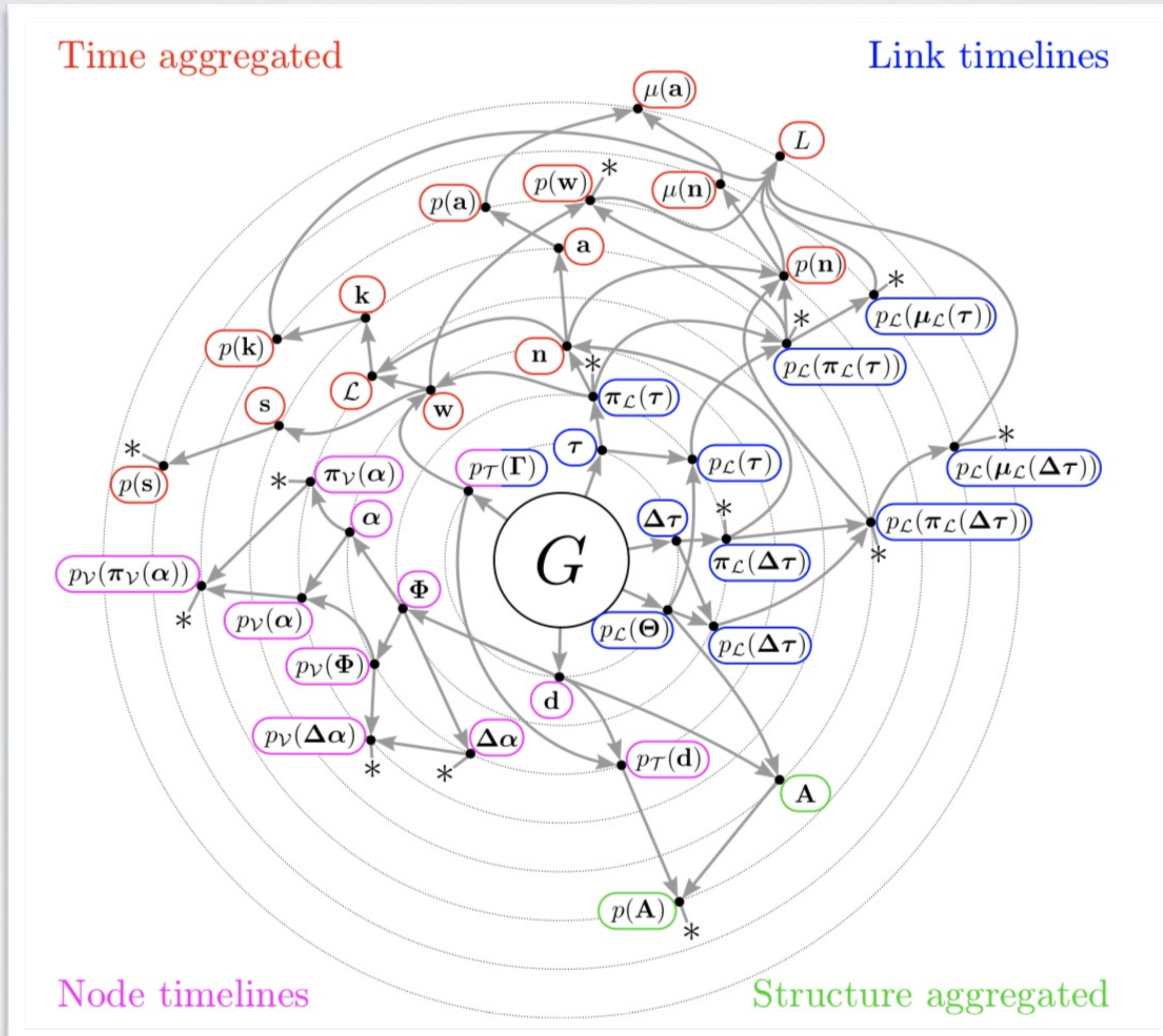
- [Gauvin 2018]

Gauvin, Laetitia, et al. "Randomized reference models for temporal networks." *arXiv preprint arXiv:1806.04032* (2018).

- Four families of shuffling:
  - Snapshot shuffling
    - =>Keep the order of snapshots, randomize network inside snapshot
  - Sequence Shuffling
    - =>Keep each snapshot identical, but switch randomly their order
  - Link Shuffling
    - => Randomize aggregated graph, keep activation times.
    - e.g., pick two node pairs activation time (u1,v1: t0, t1,…), (u2,v2: w0, w1, …) ad switch their activation time.
  - Timeline shuffling
    - => Randomize nodes/edges activation time, conserve the aggregated graph.
    - e.g. pick two edge observations (u1,v1,t1), (u2,v2,t2), switch t1 and t2

- Shufflings can be combined…

RANDOM MODELS



Gauvin, Laetitia, et al. "Randomized reference models for temporal networks." *arXiv preprint arXiv:1806.04032* (2018).

ADM network

with

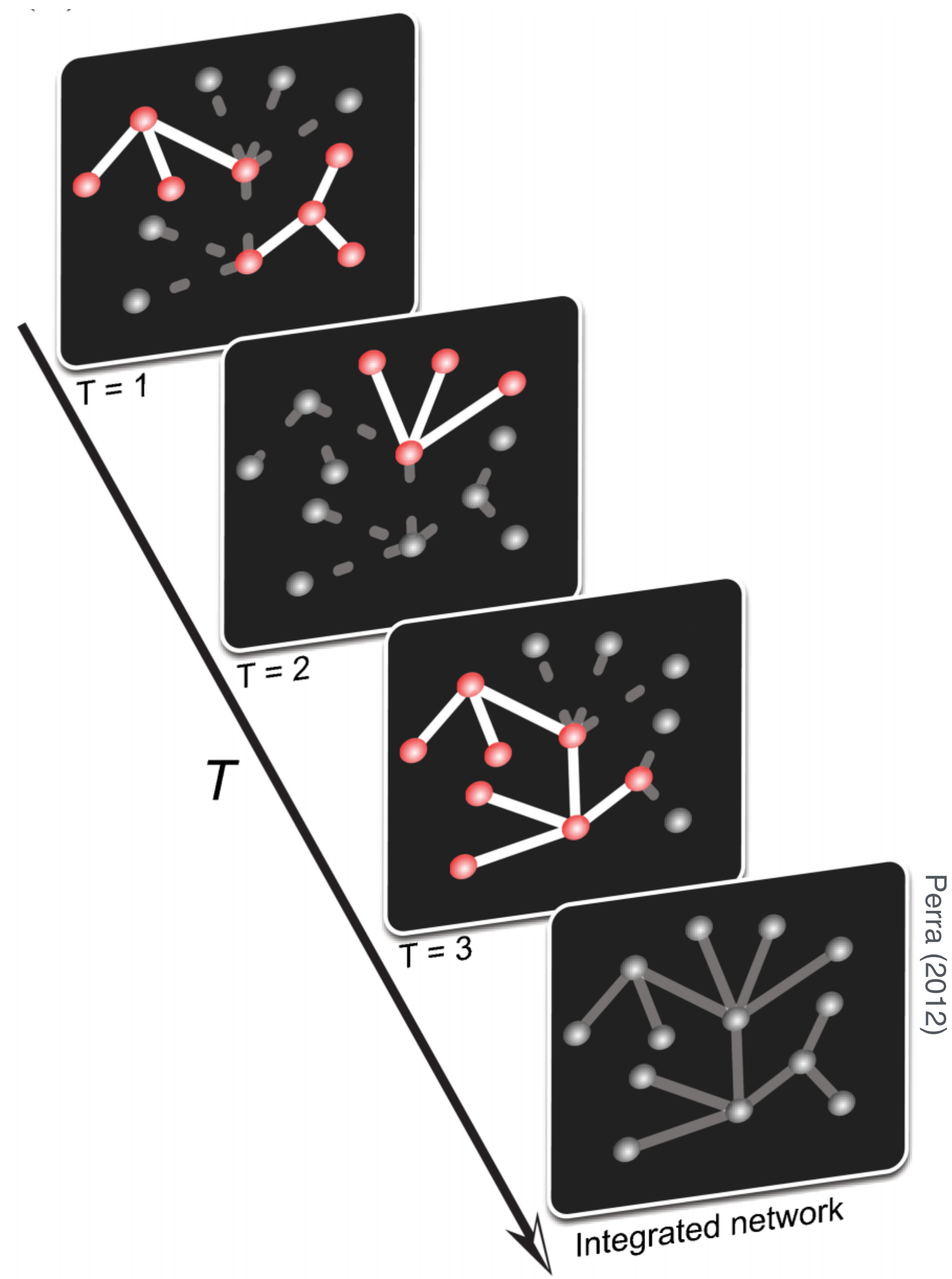
Social mechanisms

Activity driven model of time varying networks

Agent based model of temporal interactions

N. Perra, et.al., *Sci. Rep.* **2**, 469 (2012)

- It is only a general framework where additional mechanisms can be added
- It allows for understanding microscopic correlations shaping the emerging static structure
- It can be integrated in time to generate a static network structure
- It is capable of simulating dynamical processes co-evolving with the contact dynamics
- It takes a single assumption a priori: agents have different activity potentials



Activity driven model of time varying networks

Definition

- N disconnected nodes, with pre-assigned activity rates:

$$a_i = \eta x_i$$

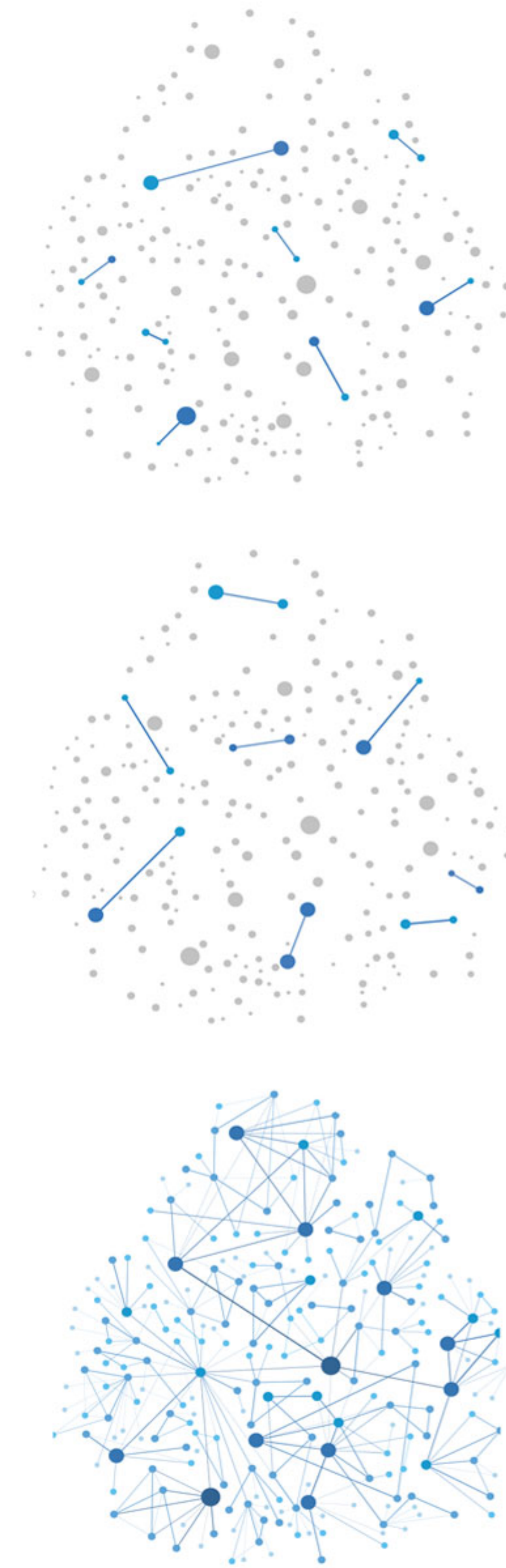
where

  - x_i is the activity potential of node i - sampled from an arbitrary distribution $F(x)$ and $x_i \in [\varepsilon, 1]$
  - η is a rescaling factor

- Each Δt time step start with N disconnected nodes:

  1. With probability $a_i \Delta t$ node i is activated and connect to *m* other nodes randomly
  2. With probability $1 - a_i \Delta t$ node i remains inactive (still can receive connections from other active nodes)

- In the end of each time step we delete each link and start the loop over again



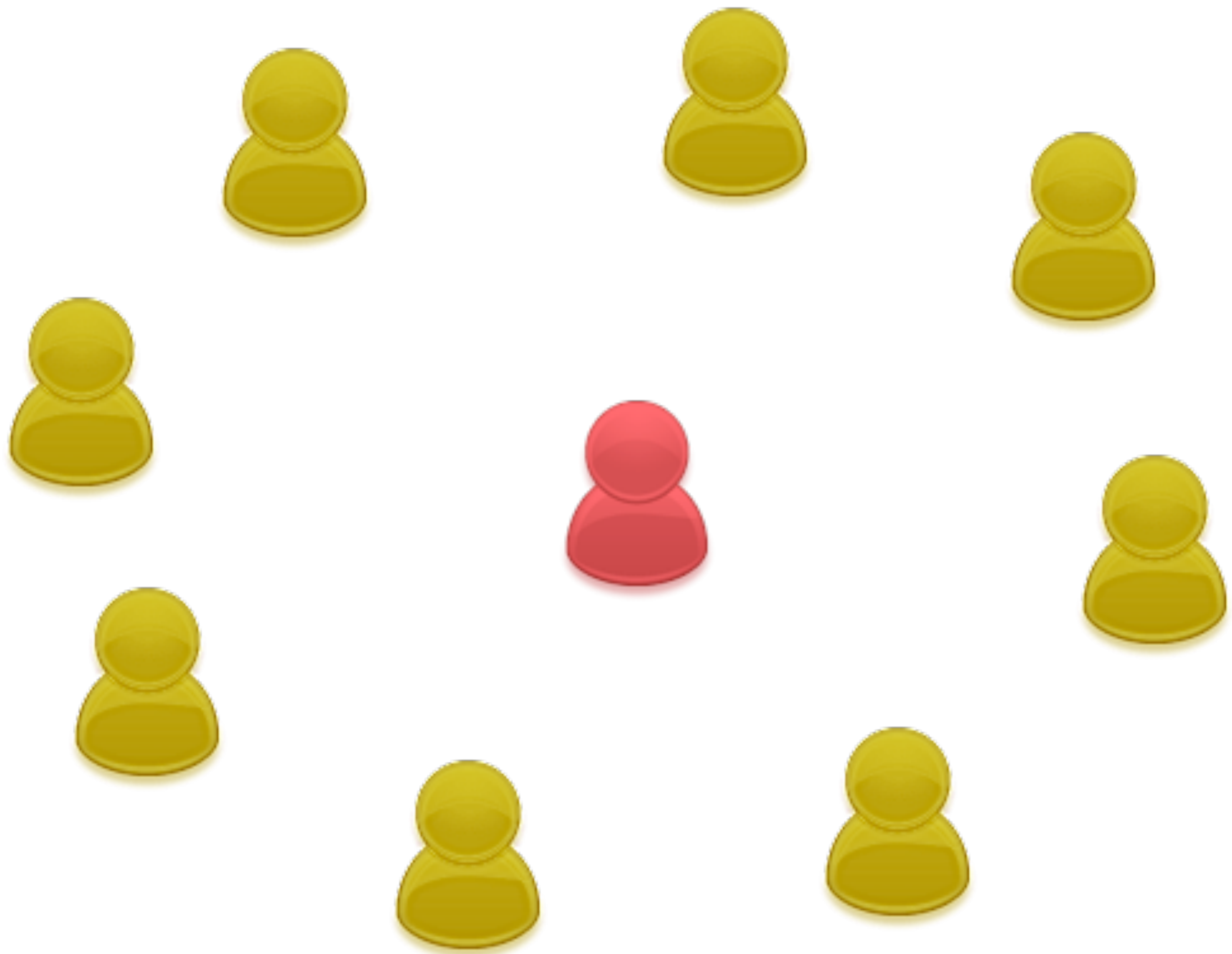
Activity driven model of time varying networks

Features

- The structure of the actual network at each Δt will be a random network
- The emerging degree distribution of the integrated network will follow the same scaling form as the pre-assigned activity distribution

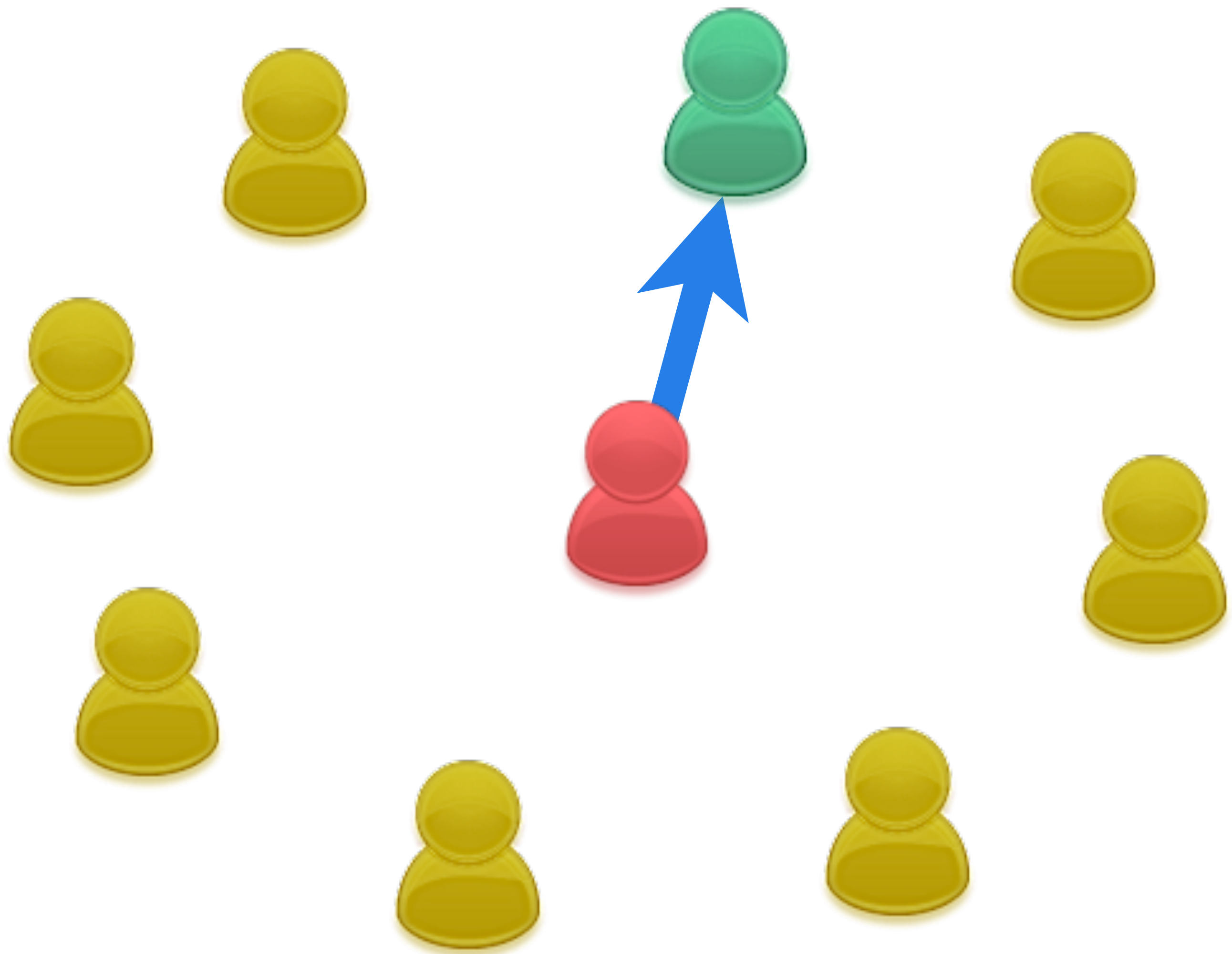
- Real node activity is different…

Egocentric network dynamics



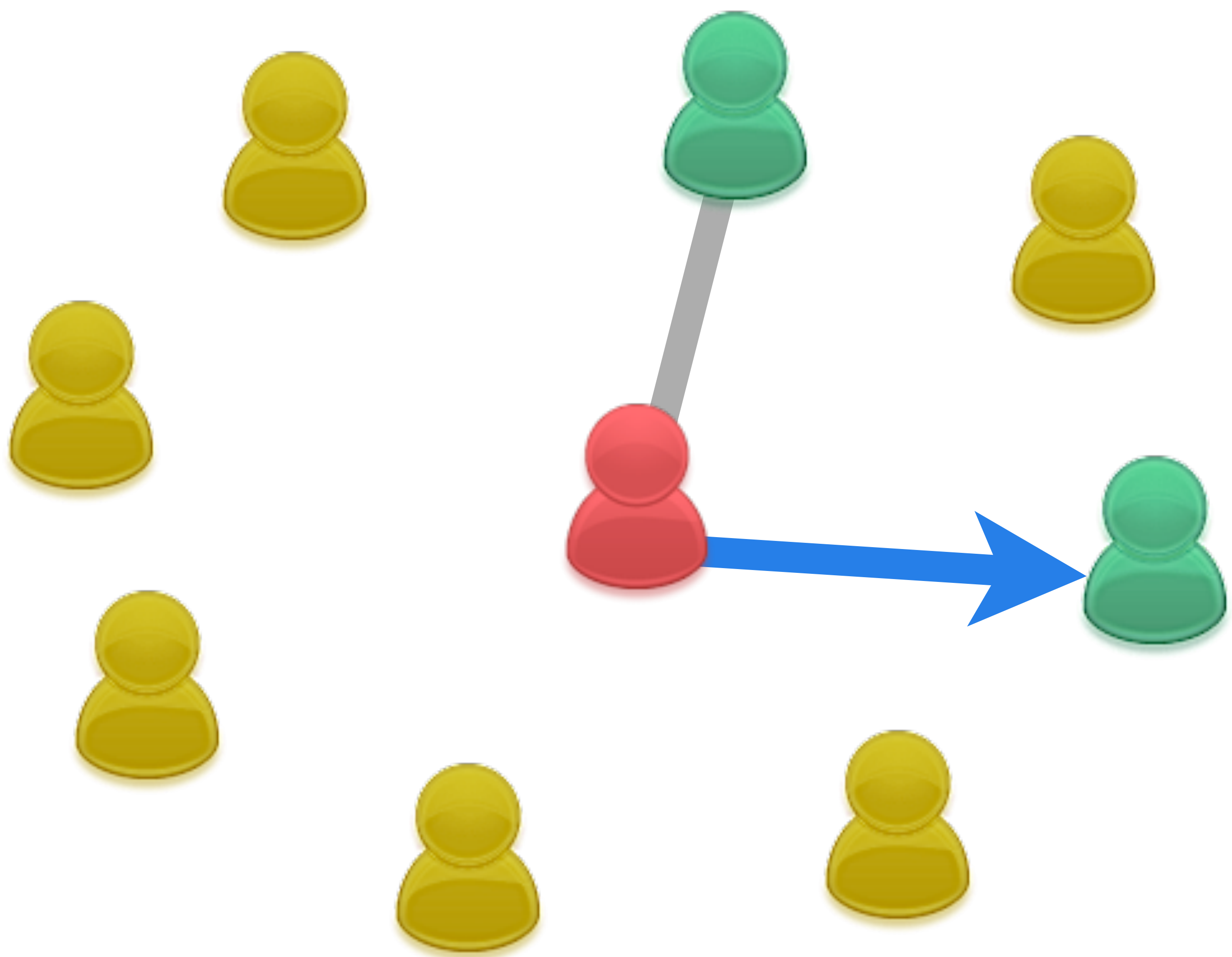
n=0

Egocentric network dynamics



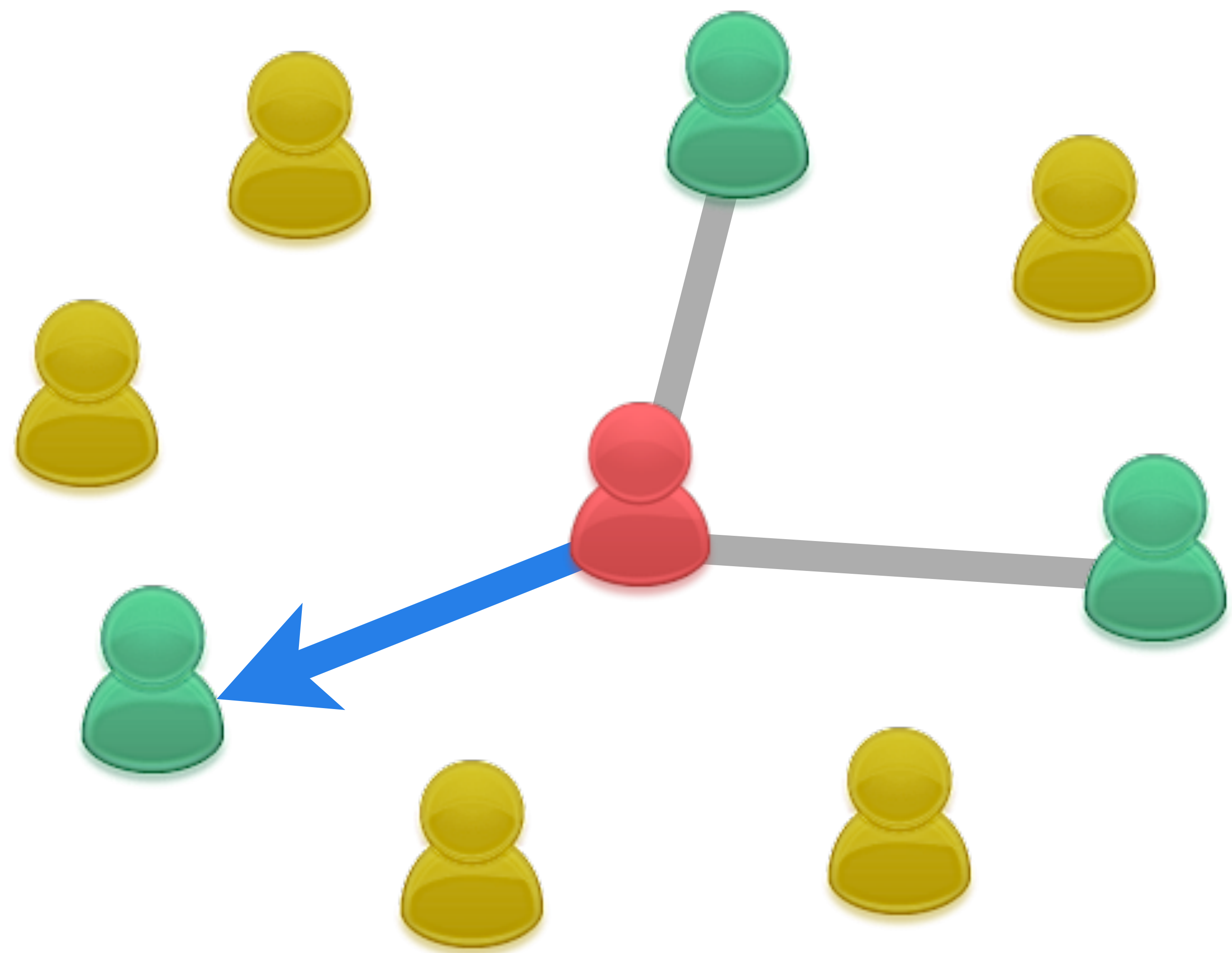
n=1

Egocentric network dynamics



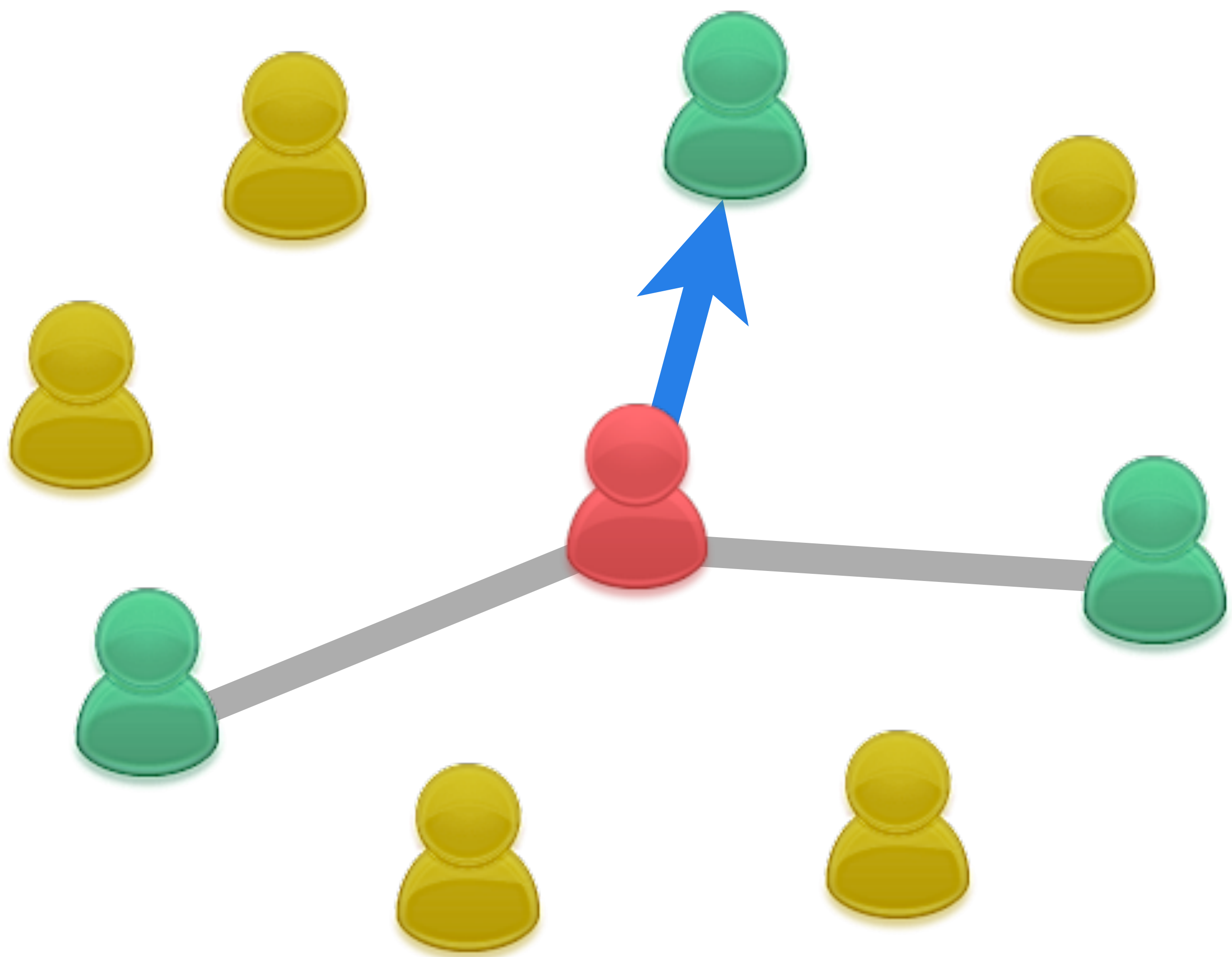
n=2

Egocentric network dynamics



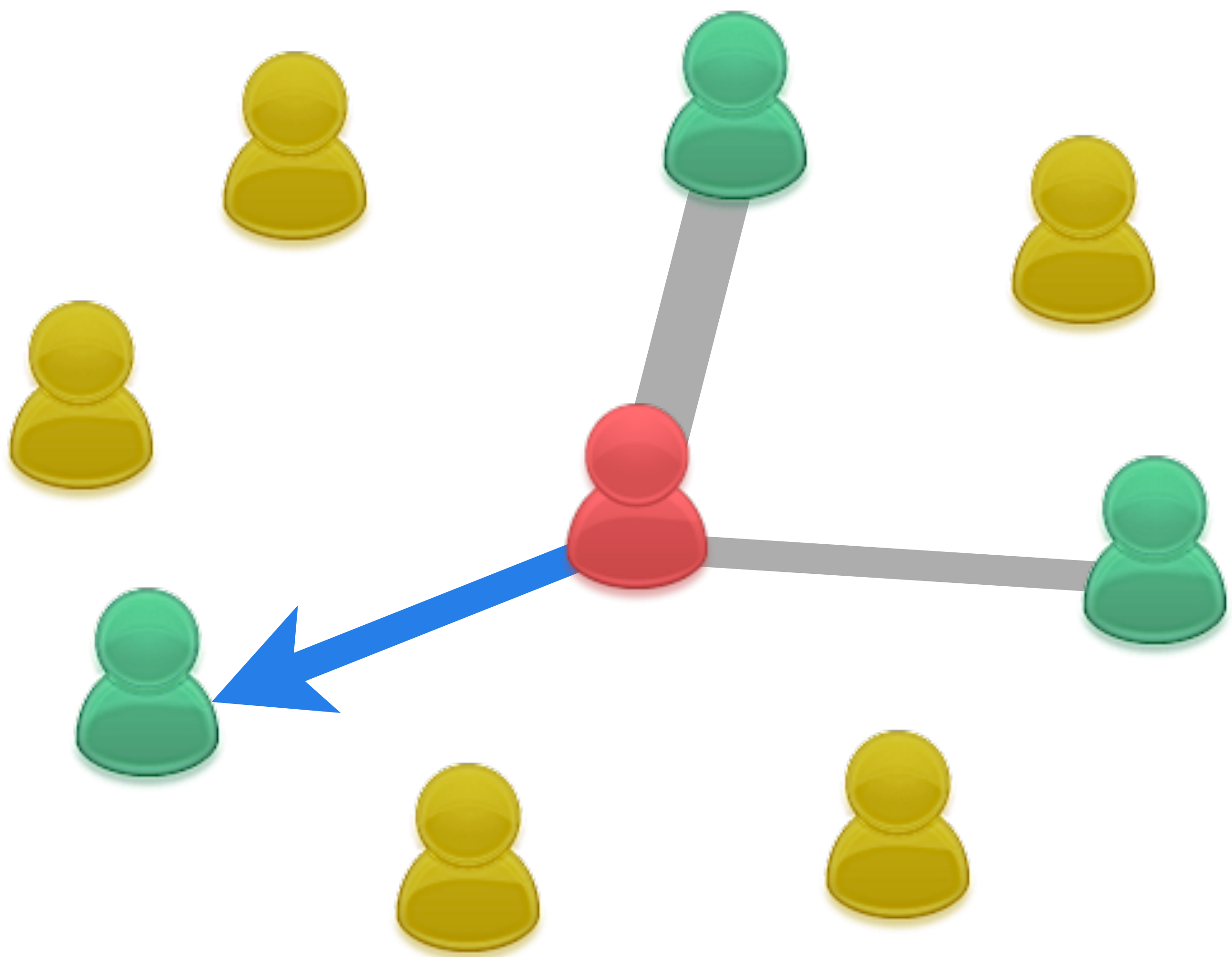
n=3

Egocentric network dynamics



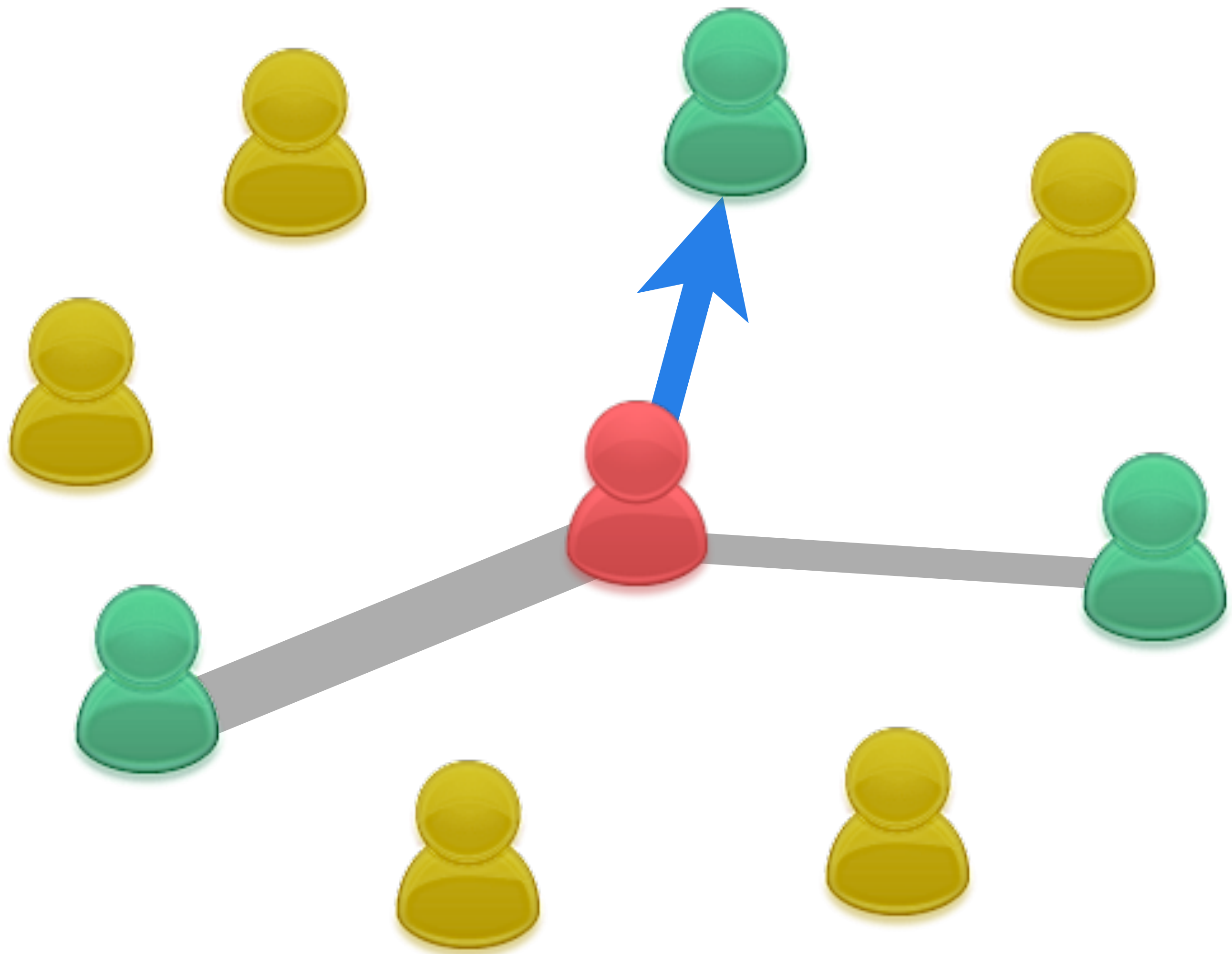
n=3

Egocentric network dynamics



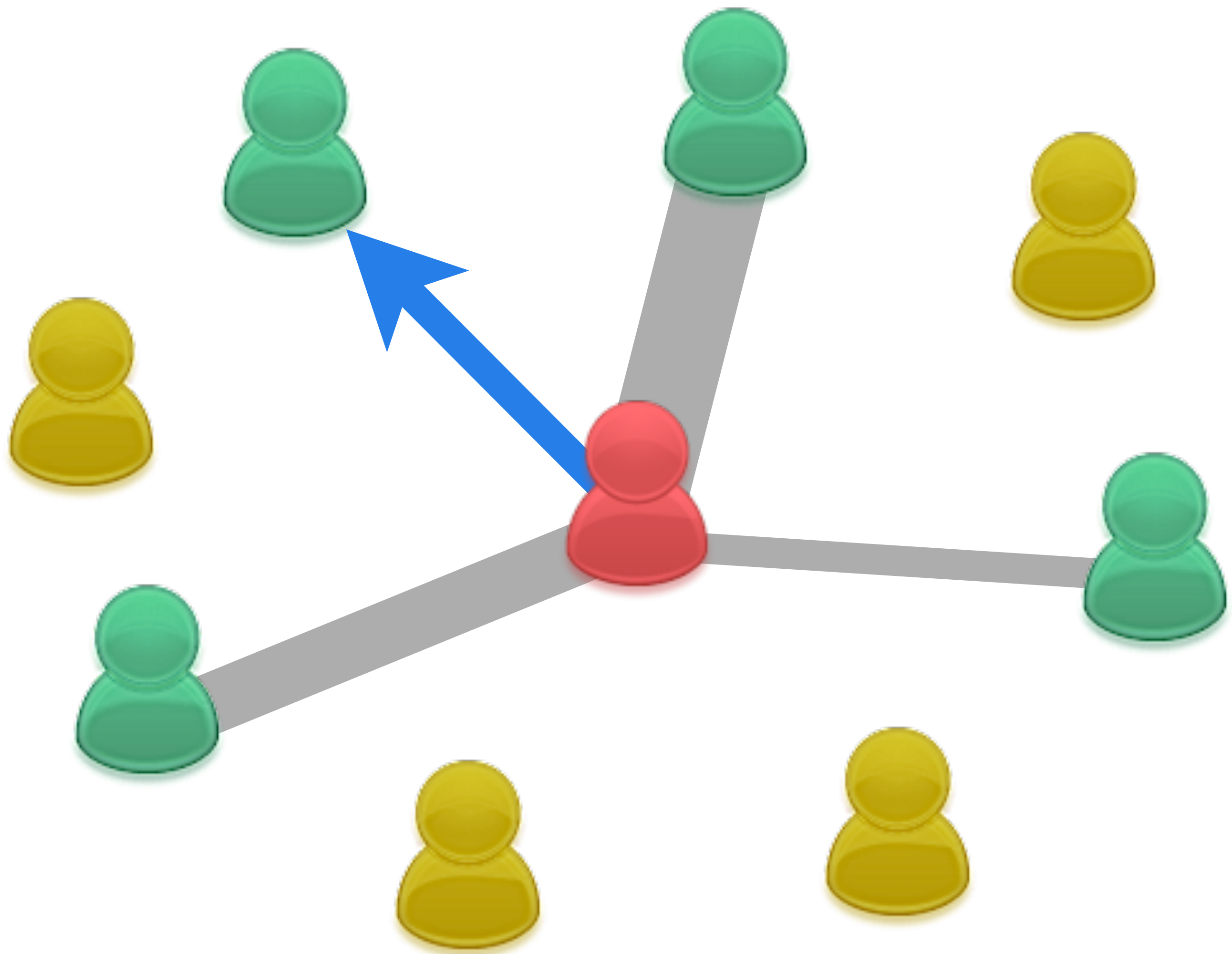
n=3

Egocentric network dynamics



n=3

Egocentric network dynamics



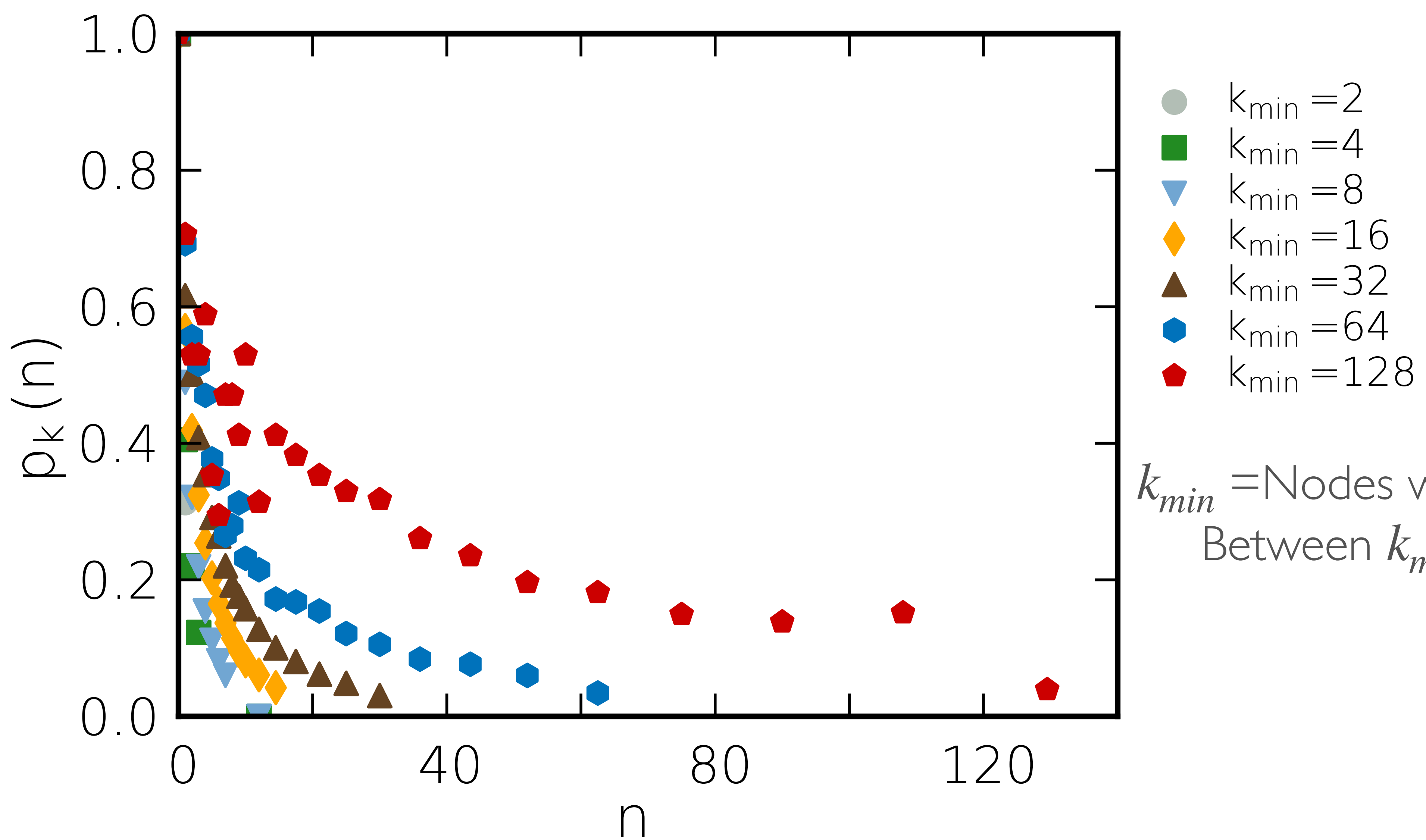
n=4

Egocentric network dynamics with memory

p(n) : probability that the next communication event of an agent with n social ties will occur via the establishment of a new (n + 1) social tie

Egocentric network dynamics with memory

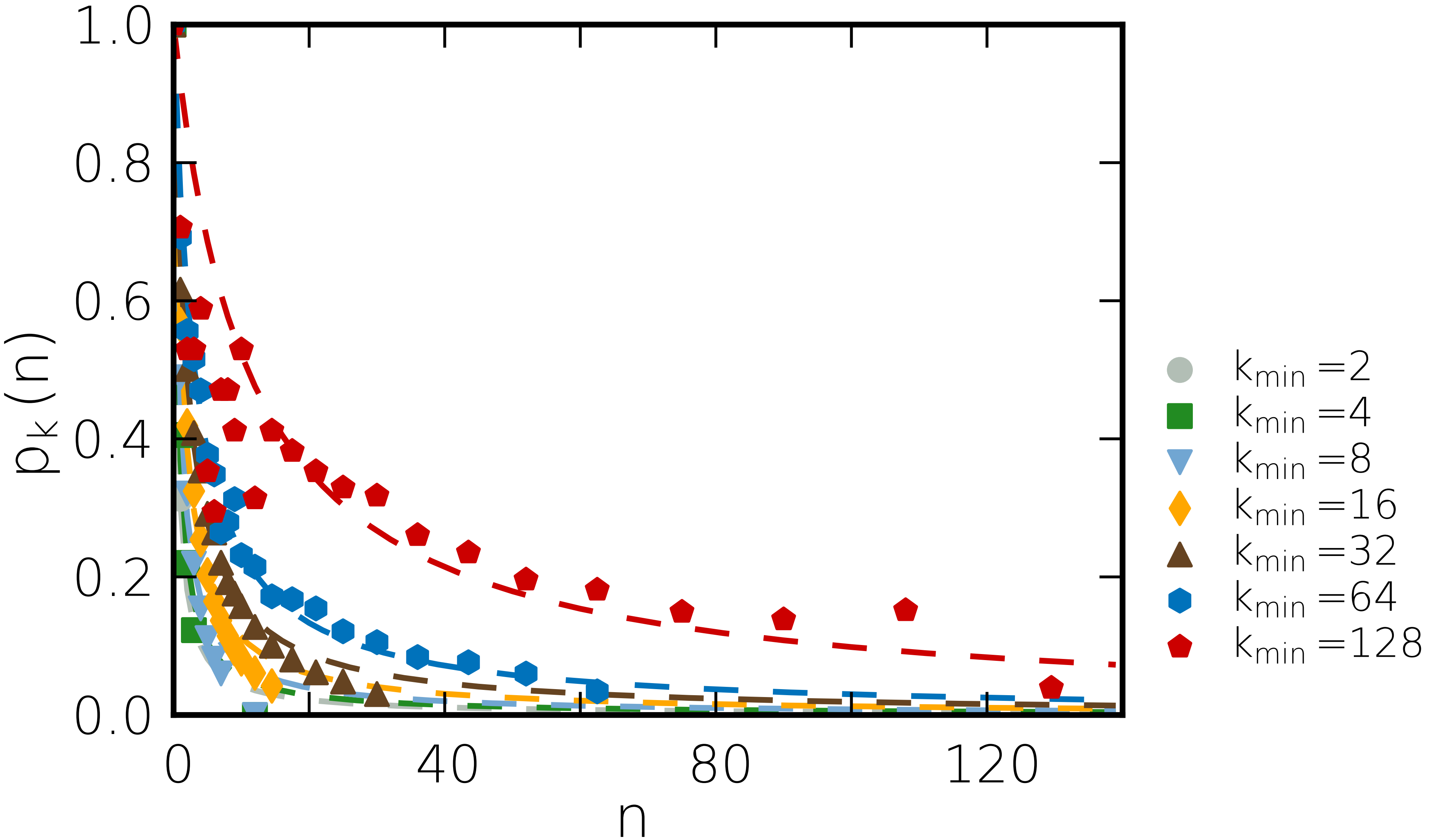
p(n) : probability that the next communication event of an agent with n social ties will occur via the establishment of a new (n + 1) social tie



k_{min} =Nodes with final degree Between k_{min} and $2k_{min}$

Egocentric network dynamics with memory

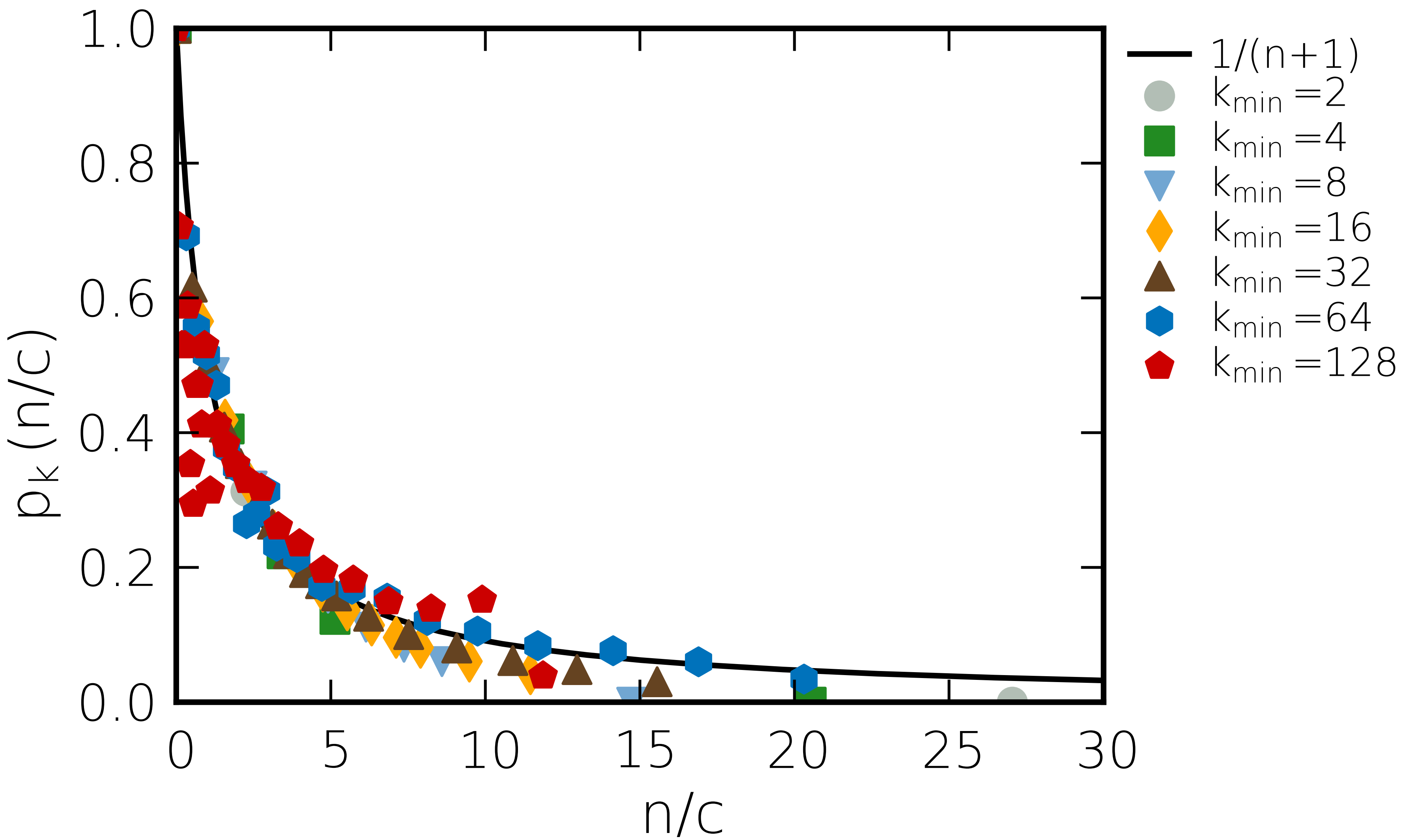
p(n) : probability that the next communication event of an agent with n social ties will occur via the establishment of a new (n + 1) social tie



$$p(n) = 1 - \frac{n}{n+c}$$

Egocentric network dynamics with memory

p(n) : probability that the next communication event of an agent with n social ties will occur via the establishment of a new (n + 1) social tie



$$p(n) = 1 - \frac{n}{n+c} \qquad p_k(n/c) = \frac{1}{n/c + c}$$

Social mechanisms

Activity driven network model

N. Perra, et.al., Sci. Rep. ***2*** *469 (2012)*

- N disconnected nodes with pre-assigned activity:

$$a_i = x_i \eta$$

where the activity potential is sampled from

$$F(x_i) \sim x_i^{-\nu} \text{ where } x_i \in [\epsilon, 1]$$

and η is a rescaling factor.

- In each iteration nodes become active with probability $a_i \Delta t$ and connect m nodes randomly.

$$\eta = 1 \quad \nu = 2.8 \quad \epsilon = 10^{-3}$$

$$m = 1 \quad \Delta t = 1 \quad c = 1$$

Memory & social reinforcement

M. Karsai, et.al., Sci. Rep. ***4*** *4001 (2014)*

- When a node is active it connects with probability

$$p(n) = c/(n + c)$$

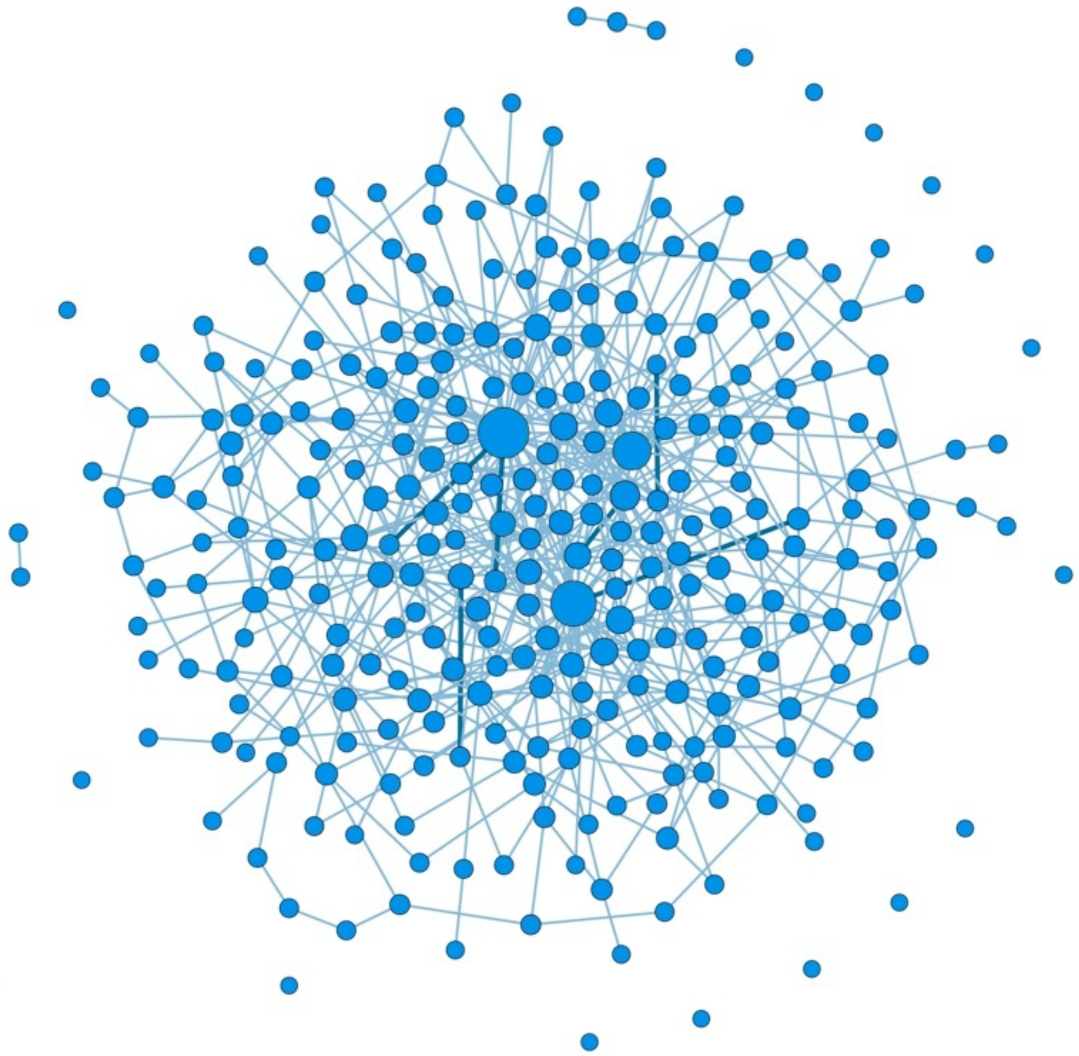
to a random node it has never connected before OR with probability

$$1 - p(n)$$

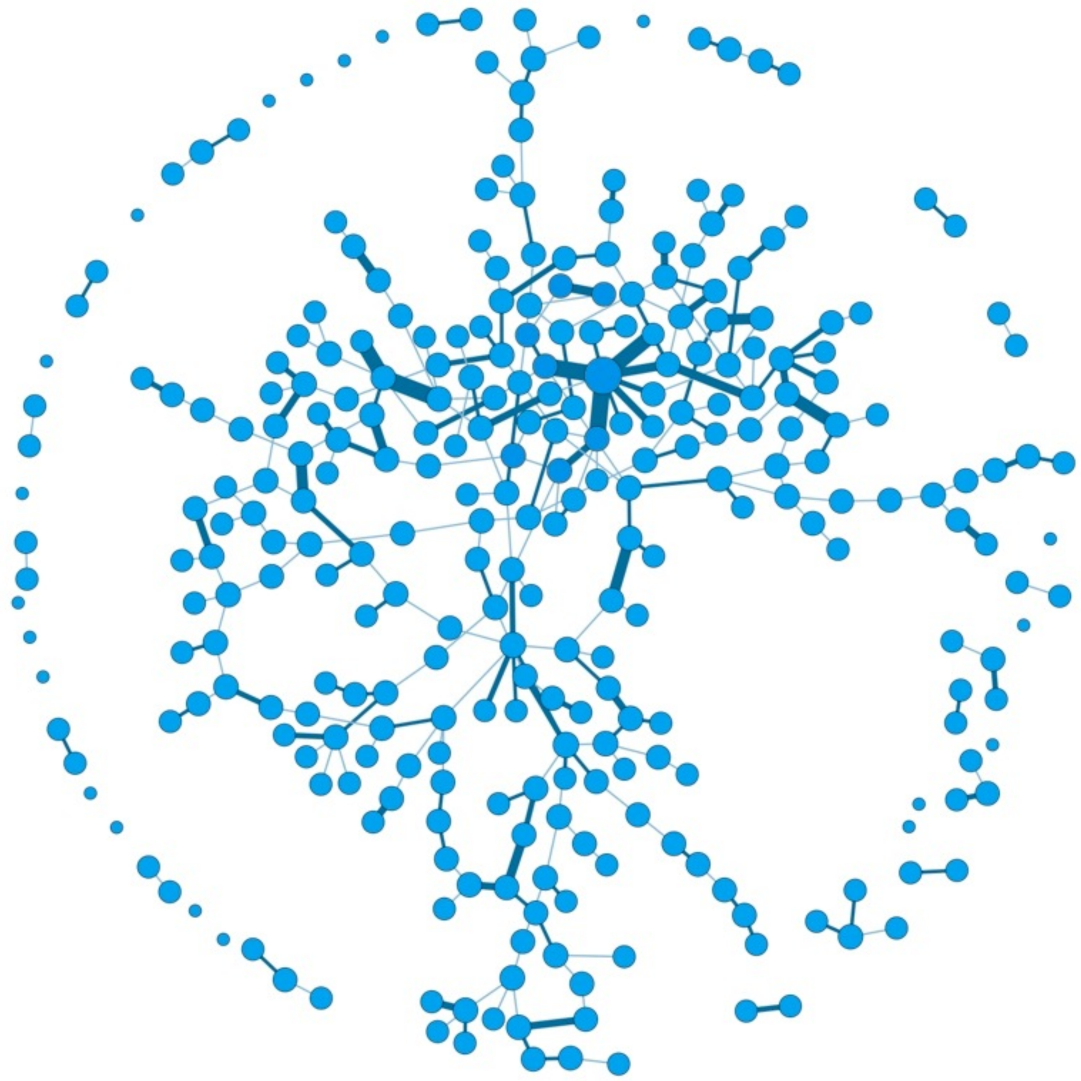
to one of the n node who it has connected earlier

- After each iteration links are deleted but *each node keeps remember to their previously connected egocentric network*
- A node can build a connection by initiating or receiving it

Activity driven network with memory



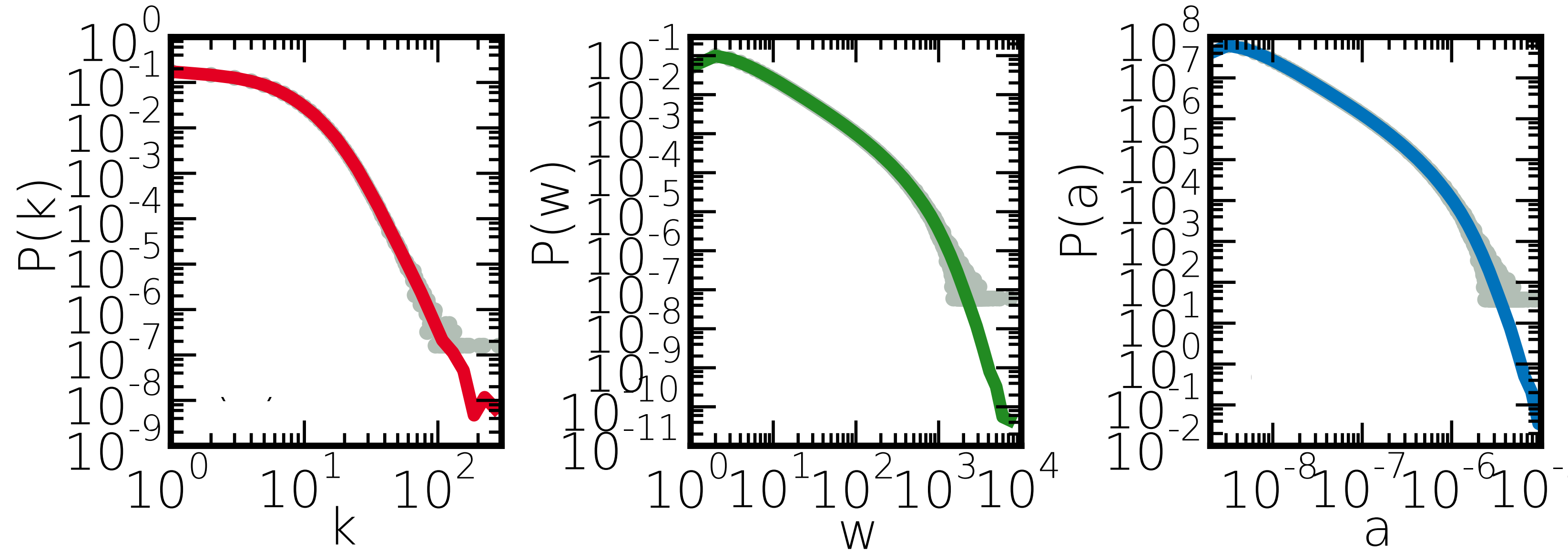
memoryless process



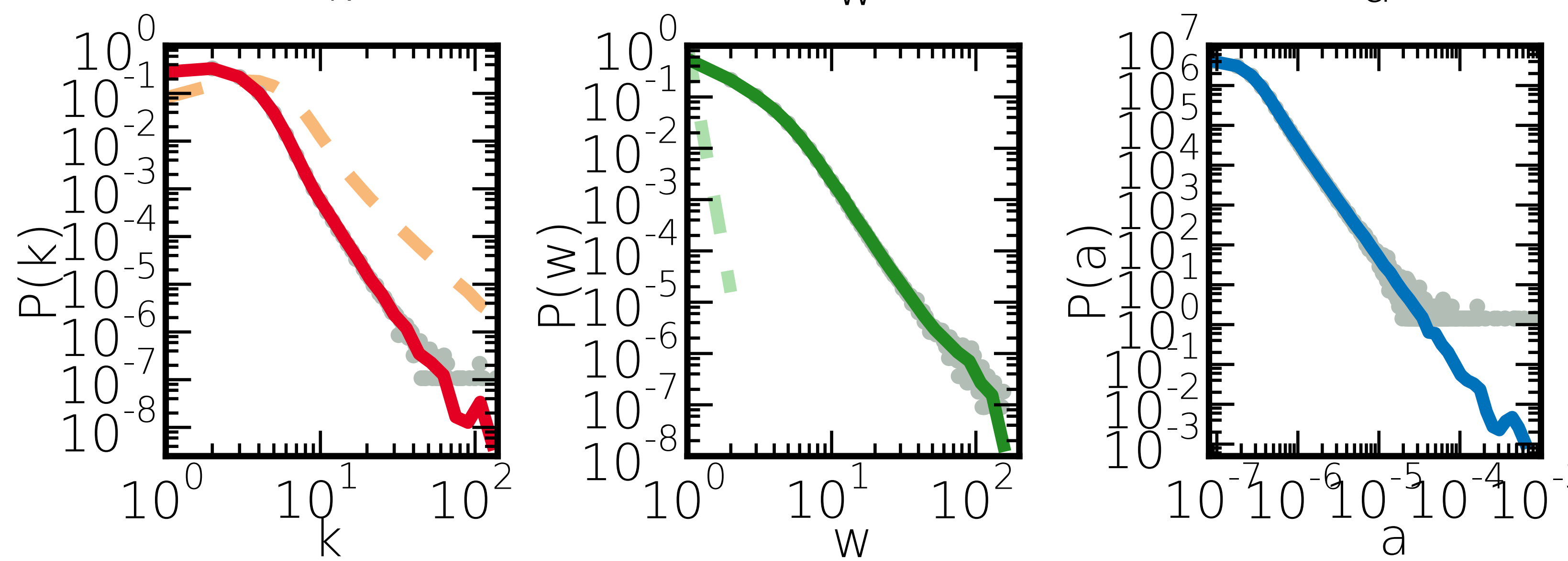
reinforced process

Activity driven network with memory

real network



model network



Evolution of the largest
connected component