CR07 - Complex Networks

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ENS Lyon - 17th January, 2019

- *The duration of the exam is 2h00.*
- **No documents are allowed (No book, no computer, no notes, no phones)**.
- **All phones must be switch off**.
- **Justify the proofs even it is simple ! Be concise !**

1 Graph representation and general measures [5 points]

The adjacency matrix is a useful graph representation for many analytical calculations. However, when we need to store a network in a computer, we can save computer memory by offering the list of links in a $L \times 2$ matrix, whose rows contain the starting and end point i and j of each link. Construct for the networks (a) and (b) in Fig. 1 :

(a) The corresponding adjacency matrices. [1p]

(b) The corresponding link lists. [1p]

(c) Determine the degree distribution and the average local clustering coefficient of the network shown in Fig.1a [1p]

(d) If you switch the labels of nodes 5 and 6 in Image Fig.1a, how does that move change the adjacency matrix ? And the link list ? [1p]

(e) In the (a) network, how many paths (with possible repetition of nodes and links) of length 3 exist starting from node 1 and ending at node 3 ? And in (b) ? [1p]

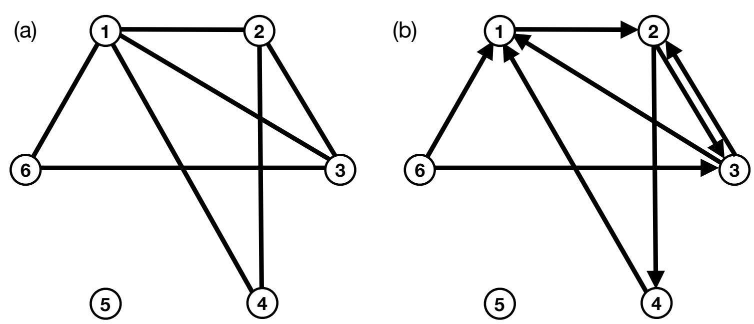

Figure 1 – (a) Undirected graph of 6 nodes and 7 links. (b) Directed graph of 6 nodes and 8 directed links.

2 Erdős-Rényi Networks [7 points]

(a) Provide the $G(n,p)$ definition of the Erdős and Rényi random graph $G(n,p)$ model. If n is a fixed large positive integer, and p is increasing from 0 to 1, the evolution of $G(n,p)$ passes through four clearly distinguishable phases. Describe briefly these four phases. [1p]

Consider an Erdős-Rényi Network with $N = 3,000$ nodes, connected to each other with probability $p = 10^{-3}$.

(b) What is the expected number of links, $\langle L \rangle$? [1p]

(c) In which regime is the network ? [1p]

(d) Calculate the probability p_c so that the network is at the critical point. [1p]

(e) Given the linking probability $p = 10^{-3}$, calculate the number of nodes N_{cr} so that the network has only one component. [1p]

(f) For the network in (e), calculate the average degree $\langle k_{\mathrm{cr}} \rangle$ and the average distance $\langle d \rangle$ between two randomly chosen nodes. [1p]

(g) Calculate the degree distribution p_k of this network (approximate with a Poisson degree distribution). [1p]

3 Spreading Processes [5 points]

Define the Susceptible-Infected-Susceptible model [1p] and calculate the characteristic time τ [2p] and the epidemic threshold λ_c [2p] of the SIS model for networks with exponential degree distribution. Assume that the networks are uncorrelated and infinite and the functional form of the degree distribution is $P(k) = ce^{-ck}$ with the corresponding $\langle k \rangle = 1/c$ first and $\langle k \rangle = 2/c^2$ second moments.

4 Community detection methods [5 points]

1. Define modularity and its features. [2p]
2. What is a dendrogram and introduce the general idea of hierarchical clustering ? [1p]
3. Introduce a modularity based community detection method. [2p]