(Dynamic of networks)

- Most real world networks are dynamic
 - Facebook friendship
 - People joining/leaving
 - Friend/Unfriend
 - Twitter mention network
 - Each mention has a timestamp
 - Aggregated every day/month/year => still dynamic
 - World Wide Web
 - Urban network

...

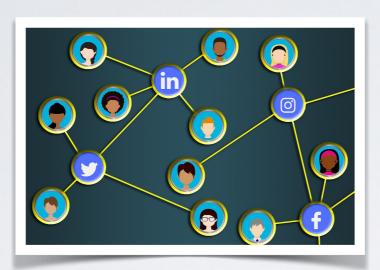
- Most real world networks are dynamic
 - Nodes can appear/disappear
 - Edges can appear/disappear
 - Nature of relations can change
- How to represent those changes?
- How to manipulate dynamic networks?

Dynamic Network Properties

Independently of the studied data, dynamic networks can have various properties:

- Edge presence can be punctual or with duration
- Node presence can be unspecified, punctual or continuous
- If time is continuous, it can be bounded on a period of analysis or ubounded
- If nodes have attributes, they can be constant or timedependent
- If edges have weights, they can be constant or timedependent

SEVERAL FORMALISMS

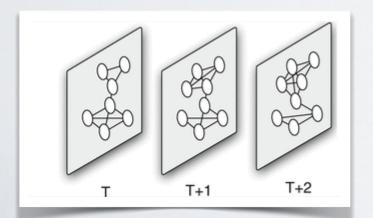


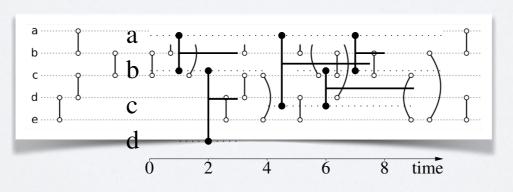


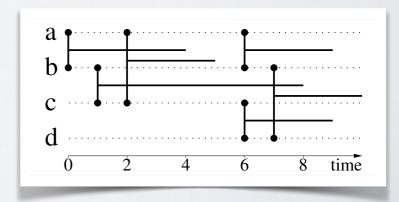












TEMPORAL NETWORK

Collected dataset, for instance in (t,u,v) format

Time (JV
--------	----

TITLE	u	V	
1353304100 1353304100 1353304100 1353304100) 1613) 656	1644 1672 682 1671	
1353304 20 1353304 20 1353304 20	656	1613 682 1671	
1353304140) 1148	1644	
1353304160 1353304160 1353304160 1353304160) 1108) 1632	1601 1671	

Examples:
-SocioPatterns
-Enron

-...

TEMPORAL NETWORK

Snapshots

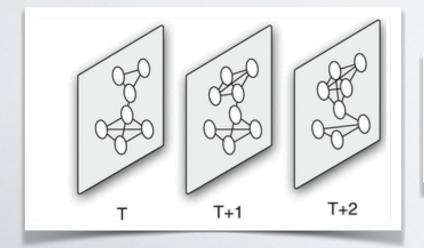
1353304100	1148 1644
1353304100	1613 1672
1353304100	656 682
1353304100	1632 1671
1353304120	1492 1613
1353304120	656 682
1353304120	1632 1671
1353304140	1148 1644
1353304160	656 682
1353304160	1108 1601
1353304160	1632 1671
1353304160	626 698

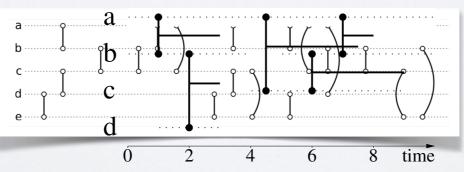
Link Stream

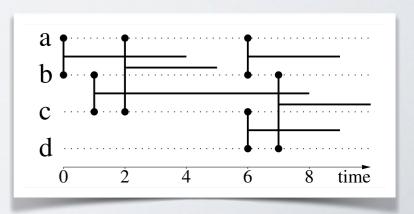
1353304100	1148	1644	
1353304100	1613	1672	
1353304100	656	682	
1353304100	1632	1671	
1353304120	1492	1613	
1353304120	656	682	
1353304120	1632	1671	
1353304140	1148	1644	
1353304160	656	682	
1353304160	1108	1601	
1353304160	1632	1671	
1353304160	626	698	

Interval Graph

1353304100 1353304100	1148 1644 1613 1672
1353304100	656 682
1353304100	1632 1671
1353304120	1492 1613
1353304120	656 682
1353304120	1632 1671
1353304140	1148 1644
1353304160	656 682
1353304160	1108 1601
1353304160	1632 1671
1353304160	626 698







Semantic level

Representation level

File/in-memory representation

Relations

Interval list

Snapshot

Interval graphs

Graph series

Aggregation

-Modification lists -List of intervals

Sequence of graphs

- I file by graph - I file with all graphs

Interactions

Link Streams

Temporal edge list

-List of edges with timestamps

Vocabulary

Many different names have been used to for networks changing with time, but there is no broad consensus in the literature on the meaning of those terms, unless they are used with an explicit reference to a paper defining those terms. Here is a list of the most popular:

- Dynamic Networks and Dynamic Graphs
- Longitudinal Networks
- Evolving Graphs
- Link Streams & Stream Graphs (Latapy, Viard, and Magnien 2018)
- Temporal Networks, Contact Sequences and Interval Graphs (Holme and Saramäki 2012)
- Time Varying Graphs (Casteigts et al. 2012)

ANALYZING DYNAMIC NETWORKS

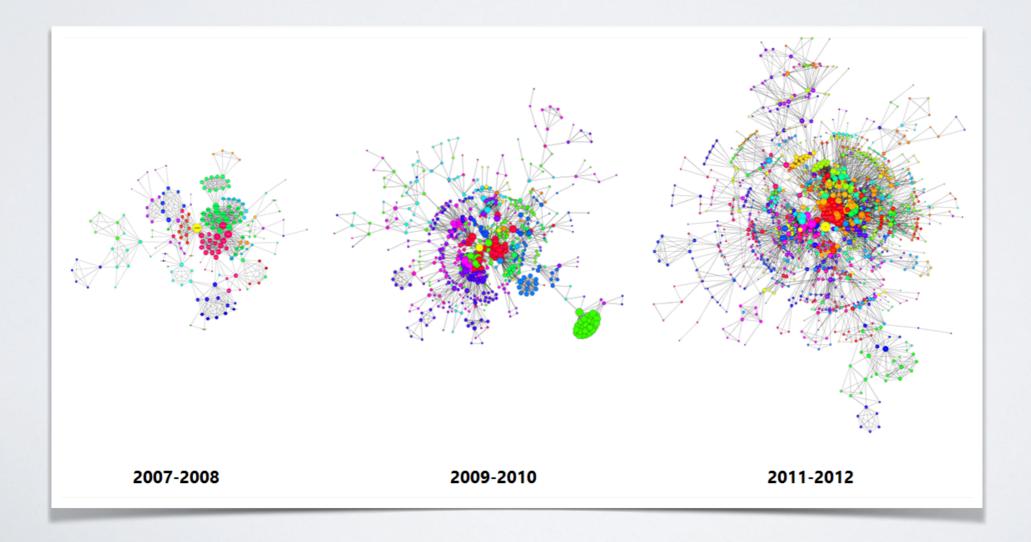
ANALYZING DYNAMIC NETWORKS

- Few snapshots
- Slowly Evolving Networks (SEN)
- Degenerate/Unstable temporal networks

FEW SNAPSHOTS

FEW SNAPSHOTS

- The evolution is represented as a series of a few snapshots.
- Many changes between snapshots
 - Cannot be visualized as a "movie"



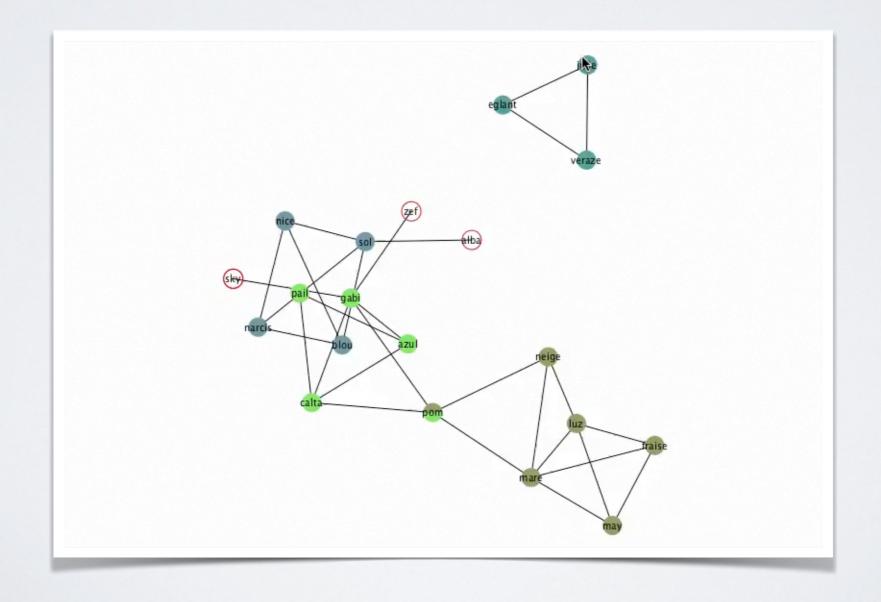
FEW SNAPSHOTS

- · Each snapshot can be studied as a static graph
- · The evolution of the properties can be studied "manually"
- "Node X had low centrality in snapshot t and high centrality in snapshot t+n"

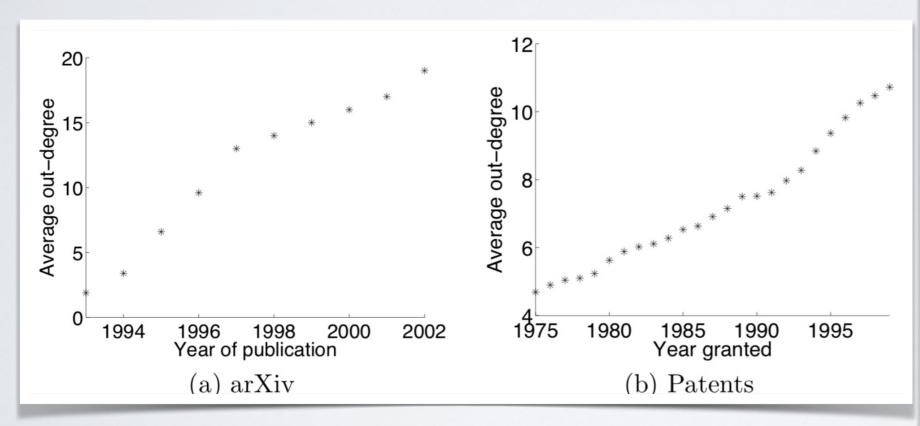
SLOWLY EVOLVING NETWORKS (SEN)

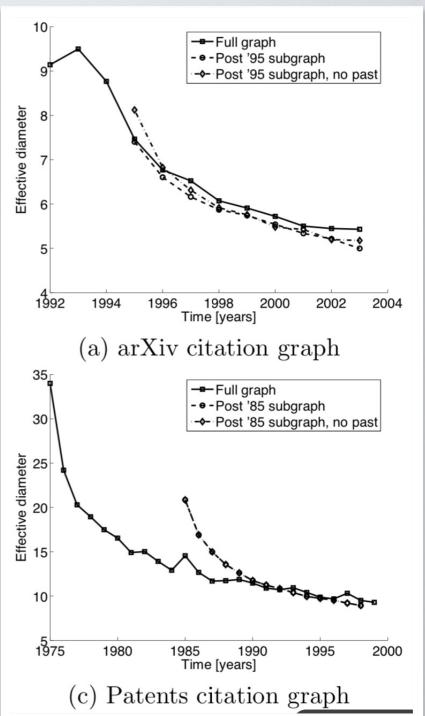
- Edges change (relatively) slowly
- The network is well defined at any t
 - Nodes/edges described by (long lasting) intervals
 - Enough snapshots to track nodes
- · A static analysis at every (relevant) t gives a dynamic vision
- No formal distinction with previous case (higher observation frequency)

- Visualization
 - Problem of stability of node positions



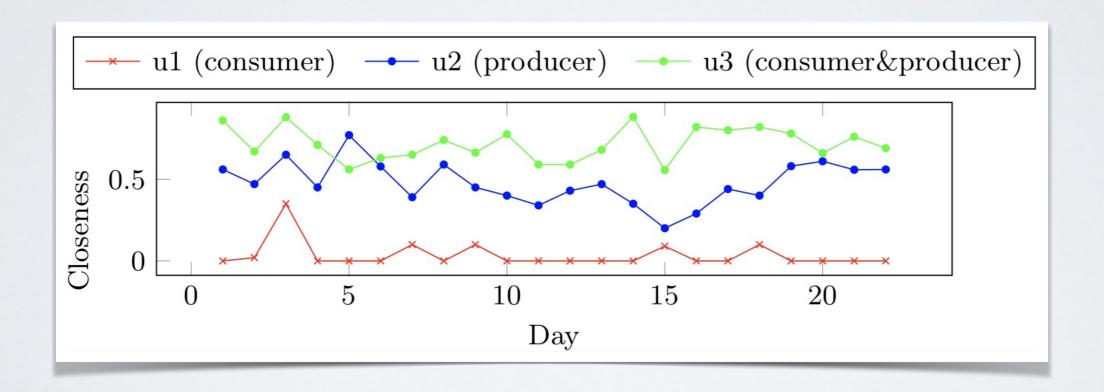
Global graph properties





Leskovec, Jure, Jon Kleinberg, and Christos Faloutsos. "Graph evolution: Densification and shrinking diameters." ACM Transactions on Knowledge Discovery from Data (TKDD) 1.1 (2007): 2.

Centralities



TIME SERIES ANALYSIS

- TS analysis is a large field of research
- Time series: evolution of a value over time
 - Stock market, temperatures...
- "Killer app":
 - Detection of periodic patterns
 - Detection of anomalies
 - Identification of global trends
 - Evaluation of auto-correlation
 - Prediction of future values
- e.g. ARIMA (Autoregressive integrated moving average)

https://en.wikipedia.org/wiki/Autoregressive_integrated_moving_average

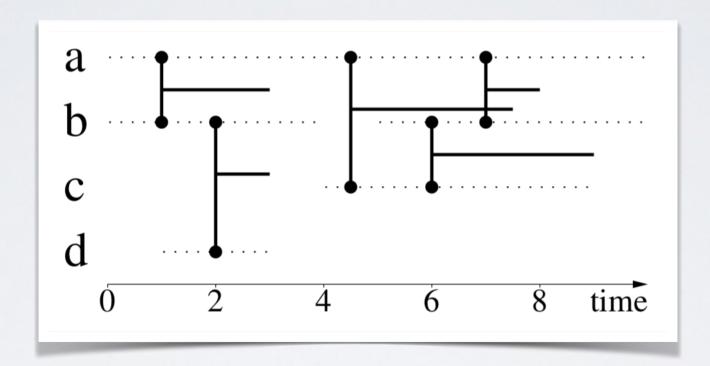
UNSTABLE/DEGENERATE TEMPORAL NETWORKS

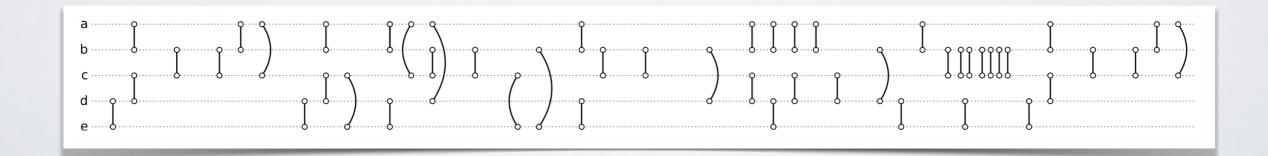
Matthieu Latapy, Tiphaine Viard, and Clémence Magnien. "Stream graphs and link streams for the modeling of interactions over time". In: *Social Network Analysis and Mining* 8.1 (2018), p. 61.

UNSTABLE TEMPORAL NETWORK

- The network at a given t is not meaningful
- How to analyze such a network?

UNSTABLE TEMPORAL NETWORK





UNSTABLE TEMPORAL NETWORK

- Common solution: transform into SEN using aggregation/ sliding windows
 - Information loss
 - How to chose a proper aggregation window size?
- New theoretical tools developed to deal with such networks
 - Link Streams & Stream Graphs (Latapy, Viard, and Magnien 2018)
 - Temporal Networks, Contact Sequences and Interval Graphs (Holme and Saramäki 2012)
 - Time Varying Graphs (Casteigts et al. 2012)

CENTRALITIES & NETWORK PROPERTIES IN STREAM GRAPHS

Stream Graph (SG)- Definition

Stream Graphs have been proposed in as a generic formalism – it can represent any type of dynamic networks, continuous, discrete, with or without duration, with the objective or redefining typical notions of graphs on dynamic networks, including degenerate ones.

Let's define a Stream Graph

$$S = (T, V, W, E)$$

T	Set of Possible times (Discrete or Time intervals)
V	Set of Nodes
W	Vertices presence time $V \times T$
E	Edges presence time $V \times V \times T$

^aLatapy, Viard, and Magnien 2018.

SG - Time-Entity designation

It is useful to work with Stream Graphs to introduce some new notions mixing entities (nodes, edges) and time:

 V_t

 E_t

 G_t

 v_t

 $(u,v)_t$

 T_u

 T_{uv}

Nodes At Time: set of nodes present at time t

Edges At Time: set of edges present at time t

Snapshot: Graph at time t, $G_t = (V_t, E_t)$

Node-time: v_t exists if node v is present at time t

Edge-time: $(u,v)_t$ exists if edge (u,v) is present at

time t

Times Of Node: the set of times during which \boldsymbol{u} is present

Times Of Edge: the set of times during which edge (u, v) is present

 N_u

Node presence: The fraction of the total time during which u is present in the network $\frac{|T_u|}{|T|}$

 L_{uv}

Edge presence: The fraction of the total time during which (u,v) is present in the network $\frac{|T_{uv}|}{|T|}$

SG - Redefining Graph notions

The general idea of redefining static network properties on Stream Graphs is that if the network stays unchanged along time, then properties computed on the stream graph should yield the same values as the same property computed on the aggregated graph.

SG - N & L

The number/quantity of nodes in a stream graph is defined as the total presence time of nodes divided by the dataset duration. In general, it isn't an integer.

More formally:

$$N = \sum_{v \in V} N_v = \frac{|W|}{|T|}$$

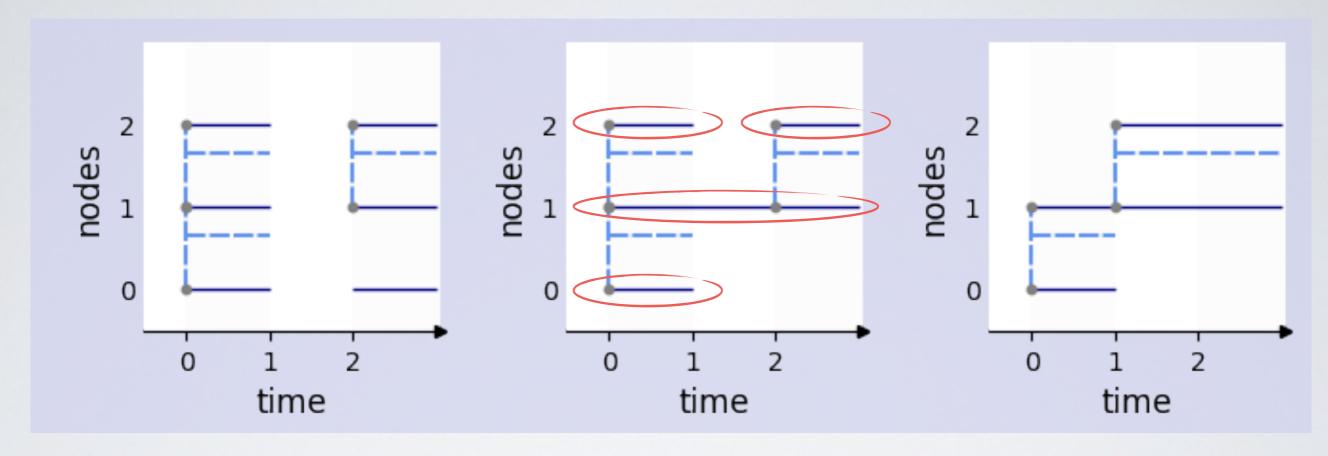
For instance, N=2 if there are 4 nodes present half the time, or Two streatwo nodes present all the time.

densities: Left: $\delta = 0$

In addition, $\delta(L)$ is eq

$$\frac{1}{|T|\cdot|V\otimes V|} \int_t |E_t| \, \mathrm{d}t = \frac{\int_t}{\int_t |V|}$$

Finally, if we consider of the corresponding grant of the corresponding gra



$$N=2$$

of nodes v and the same

induced by a subset V' subset

STREAM GRAPHS × V') \(\text{V'}\) \(\text{V'}\)

induced by a subset T' of

SG - L

$$V)\cap W, (T'\times V\otimes V)\cap K$$

The number of edges is defined as the optath respect to the divided by the total dataset duration. Is $([6,9],\{a,b,c\},[6,9])$ is the more formally:

$$L = \sum_{(u,v),u,v \in V} L_{uv} = \frac{|E|}{|T|}$$

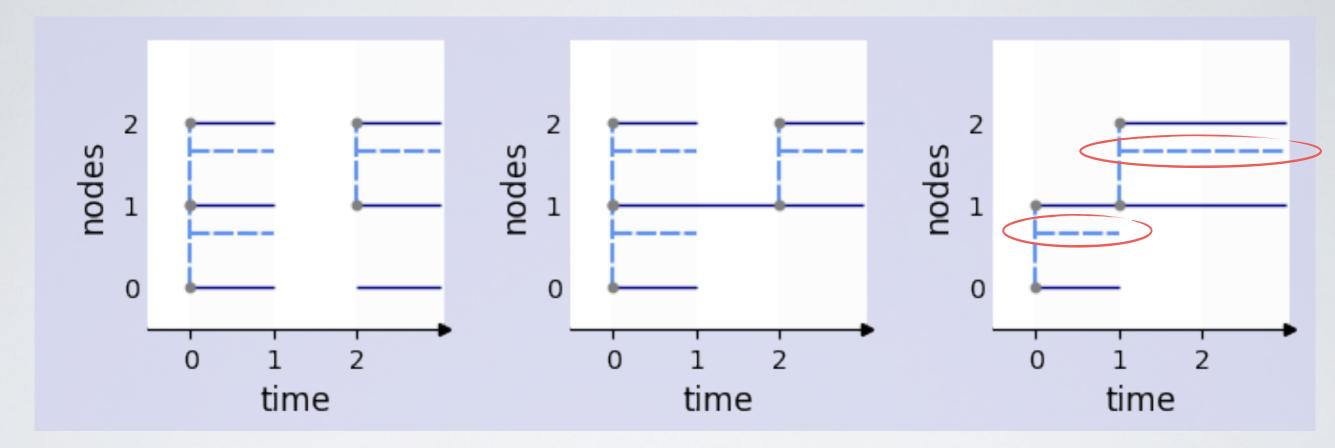
 $L = \sum_{(u,v),u,v \in V} L_{uv} = \frac{|E|}{|T|}$ For instance, L=2 if there are 4 edges possible between the content of the con two edges present all the time.

> A clique of graph G is a involved in C are linkedigter displa C' such that $C \subset C'$. induc

> > III alaca a alique

betwe deno

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$$L=1$$

betv deno

STREAM GRASHigues

SG - Edge domain - L_{max}

In Stream Graphs, several possible definitions of $L_{
m max}$ could exist ue of

- · Ignoring nodes duration: $2 \frac{11}{2} pairs of nodes involved$
- · Ignoring nodes co-presence is \max in there is $\sum_{t=1}^{t} \max_{t=1}^{t} \sum_{t=1}^{t} \sum_{t$
- Taking nodes co-presence into Weousay that a clique $L_{\max}^{3} = \sum_{(u,v),u,v \in V} |T_{u} \bigcap T_{v}|$ uniform). It is then

set) meaning that all pair

(resp. uniform). It is then fit straining that all pairs of the set of the se

The density in static networks can be understood as the fraction of existing edges among all possible edges,

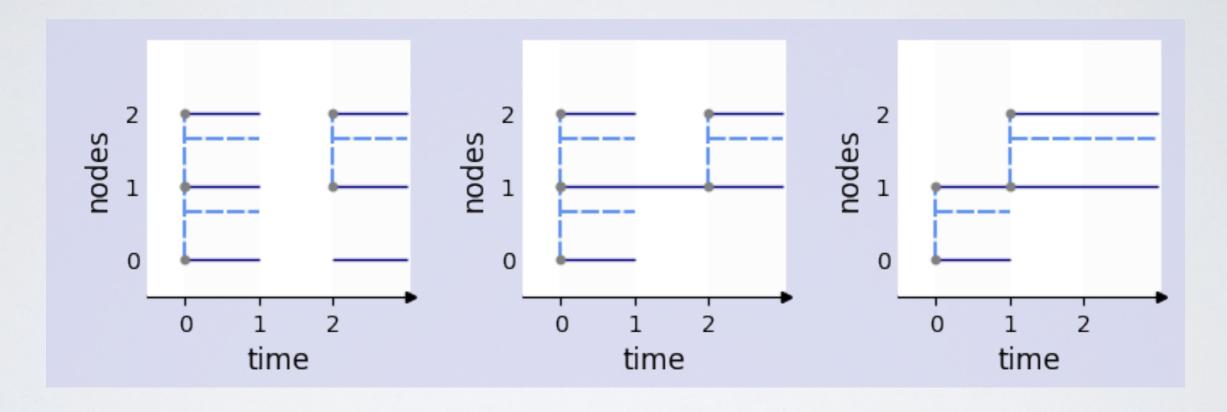
$$d = \frac{L}{L_{\text{max}}}.$$

In the following, we will using the samples of mapping pact cliques involving three and $[7,8] \times \{b,c,d\}$. Its oth $[2,5] \times \{a,c\}, [1,8] \times \{b,c\},$

For instance, in Figure 4

$$N=2$$

$$L=1$$



$$d = \frac{3}{6} = \frac{1}{2}$$

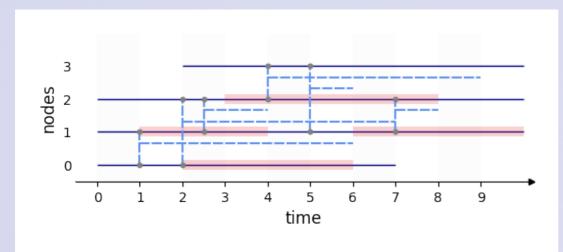
$$d = \frac{3}{4}$$

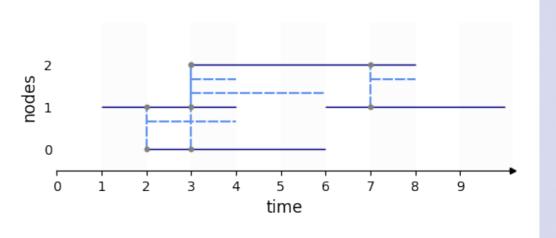
$$d = \frac{3}{3} = 1$$

STREAM GRAPHS

SG - Clusters & Substreams

In static networks, a cluster is a set of nodes, and we have defined an (induced) subgraph of this cluster as a graph composed of the nodes of the cluster and the edges existing between those nodes. In Stream Graphs, a clusters C is as subset of W, and the corresponding (induced) substream $S(C) = \{T, V, C, E(C)\}$, with $E(C) = \{(t, (u, v)) \in E, (t, u), (t, v) \in C\}$.



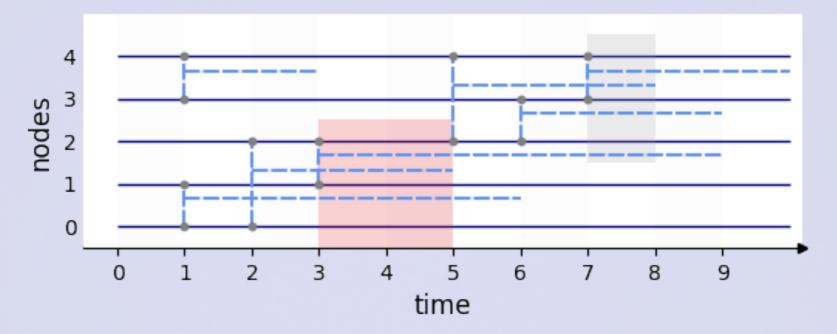


Example of subgraph (red, left) and induced substream (right).

STREAM GRAPHS

SG - Cliques

Having defined substreams and density, we can now naturally define a clique by analogy with static networks as a substream of density 1. A clique is said to be a **maximal clique** if it is not included in any other clique.



Red and Grey are the two maximal cliques of size three in this Stream Graph.

ques involving three nodes of the link stream L of Figure 1 (right): $[2,4] \times \{$ $[8] \times \{b, c, d\}$. Its other maximal compact cliques are $[0, 4] \times \{a, b\}$, $[6, 9] \times \{a, b\}$ $\{a,c\}, [1,8] \times \{b,c\}, [7,10] \times \{b,d\}, [6,9] \times \{c,d\}$ (involving two nodes each $\inf_{u=0}^{a} \frac{1}{u}$ in Figu SG - Neighborhood N(u)ct clique. Howeve ximal as it is included when eighborhood y(u) of node y is defined as the cluster communication. al compact clique posed of node-times such as an edge-time exists between it and a node-time of u, i.e., ntersectsanother The maximal content of the part of the pa $[0,4] \times \{a,b\}$ is not a maximal clique because it is for instance included in the $\{a,b\} \cup [6,9] \times \{c \text{ SG - Degree } k(u)\}$ $\sin \sin dt = \sin dt = \cos dt$ The degree $\sin dt = \cos dt = \cos dt$ as the quantity of node in $\sin dt = \cos dt = \cos dt$. Figure in S does not in general induce a clique in G(S): for instance, $[0,1] \times \{c,d\}$ is a clique for the example $k(u) \neq k(u) \neq d$ but $\{a,b,c,d\}$ is not a clique $[b,e] \times X$ is a compact dustaphove insteame for an **node** v in V, and the density $\frac{|T_v|}{|V_t|}$ and $\delta(t) = \frac{|E_t|}{|V_t \otimes V_t|}$. Example, the neighborhood of node 2 is highlighted in grey. 0, respectively, then we define $\delta(uv)$,

There is a unique not maximal eit

 $tin(t_i, u_i v_i) \in E, [\alpha, t_0] \times \{$ 8 time This sequence is similar neighborhoods and degrees of modes have to be node under concern, and in grey the other links. Left: if (α, u) -- $[4.5, 7.5] \times \{c\}$ is in blue, leading to $d(a) = \frac{3}{10} + \frac{3}{10} = \frac{3}{5} = \frac{3$ SG - Ego-network connected. It is a weakl node de gree of S as follows. The Ego network G_u of node u is defined as the substream induced by its neighborhood, i.e., $G_u = (T, V, N(u), E(igur))14$ for an illustrat SG - Clustering coefficient

The clustering coefficient C(u) of node u is defined as the density of the ego-network of \underline{u} view u_{j+1} then $P' = (u_0, v_0), \ldots, (u_{i-1}, v_{i-1}), (u_{j+1}, v_{j+1}), \ldots, (u_k, also is a path from <math>u$ to v. If one iteratively removes the cycles of P in this way, eventually obtains a G(up) e-path V(v) u to v.

The path P is a shortest path from u to v if there is no path in G of length lower t k. Then, k is called the distance between u and v and it is denoted by $\partial(u,v)$. If there no path between u and v then their distance is infinite. The diameter of G is the large finite distance between two nodes in V.

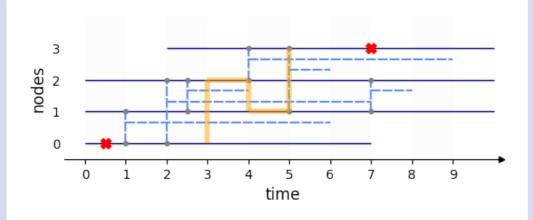
Figure 14: Weakly coefficients

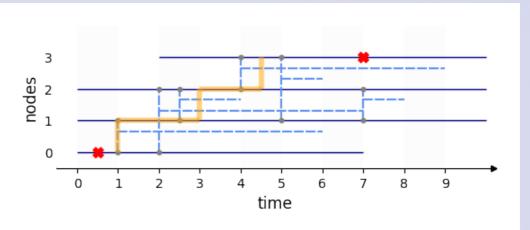
PATHS AND DISTANCES IN STREAM GRAPHS

PATHS

SG - Paths

In a Stream Graph S=(T,V,W,E), a **path** P from node-time x_{α} to node-time y_{ω} is a sequence $(t_0,x,v_0),(t_1,v_0,v_1),...,(t_k,v_k,y)$ of elements of $T\times V\times V$ such that $t_0\geq \alpha,t_k\leq \omega,((t_i,u_i,v_i))\in E$. We say that P starts at t_0 , arrives at t_k , has length k+1 and duration t_k-t_0 .





Examples of two paths from (node 0, t=0.5) to (node 3, t=1). The left one starts at 3, arrives at 5, has length 3 and duration 2. The right one starts at 1, arrives at 4.5, has length 3 and duration 3.5.

m and its corresponding grayshuahedmarpalehnissaveryche in restricted carr sponding path is a cycle in the graph aspects of a network using the two level reresentations introduced in Section II A 2 above. models as they all conserve the nodes \mathcal{V} , the tempor nnectedness and connected teamponents SG - Shortest -E)-Eyentshufflings furthermore conserve the multiset he =(V,E) is connected if for based up the line time representation with the feature Vis connected iength) is contributed for the static graph of a network but not the individual timelines, and time no other connected patricipation Thereof the the connected the continue of the connected the connect $tey form a partition^3 of V siblication of the period of$ Furthermore, one can reproduct has a present the second of stance: Eventsinks in G^{stat}). In practice they are implemented stream **Fastest** shortest paths which is introduced in the several more restricted class of minimal length -1 distributing the time times v_k of the hardest paths paths being the timest and the period v_k of elements of a to v_k of v_k of elements of a to v_k of v_k $E, [\alpha, t_0] \times \{u\} \subseteq W, [t_k$ resentations introduced in Section II.A.2 above t_k as x_0 ice is similar to a path frequency of the individual timeling the similar to a path frequency of the practice they are implemented by frily have $t_0 \geq \alpha$, $t_{i+1} \geq t_i$; tributing the tinstant are one insequence between the

me

We display links. Left:

 $\frac{3}{10} + \frac{3}{10} = 0.6$. $=\frac{13}{10}=1.3.$ topological aspects of a network using the two level representations introduced in Section IIA2 above. network's static topology, $G^{\text{scation}} = (1, \mathcal{L})$, as well as any constraints on the content of the individual timelines $\Theta_{(i,j)} \in \Theta$. In practice they are implemented by redistributing the (instantaneous) events in or between the

Based on the link-timeline representation (Def. II.6), we Before place I had in ging which End to Bring it hoost altiforgraph SG - Connected mixing the static graph indicated at the mediates and on think shielding prossible ich pandom Eigthe. Simelines entos et the

Various definitions for the company of the beautiful by redisposed for temporal networks the instantant of the condent of the c 2018) for details. One of the main person is the weakly connected as Ins (b) ncomponent, defined suchias two the deatimes belongite the same infiguration connected component if anthous limit (Frette) is Impathotic motion to the plemented by randomizing the links \mathcal{L} in the static graph and reother, *ignoring time*. distributing the timelines $\Theta_{(i,j)} \in \Theta$ on the new links without replacement. Based on the snapshot-sequence representation in the shuffings, on the other hand, constrain the

Hetwork's static topology, Generally, shufflings, which mire the order of the snapshots but not the shutsaust buy the conserve of the snapshots but the time porserventse discount of the snapshot state of the snapshot state of the snapshot state of the snapshot state of the snapshot satisfies the snapshot sa The shullings furthermore conserve the multiset of the representations (a) the representations (b) the representations (b) the representations (c) the representations (c) the representation (c) the represen $\ldots, (u_{i-1}, v_{i-1}), (u_{j+1}, v_{j+1}), \ldots, (u_k, v_k)$

ere is no path in G of length lower than and it is denoted by $\partial(u,v)$. If there is finite. The diameter of G is $\mathsf{Example}^t$ of a Stream g

noves the cycles of P in this way, one

P from $(\alpha, u) \in W$ to $(\omega, v) \in W$ is a EL, which conserves only the number of instantaneous elements of $T \times V \times V$ such that $u_0 = u$,

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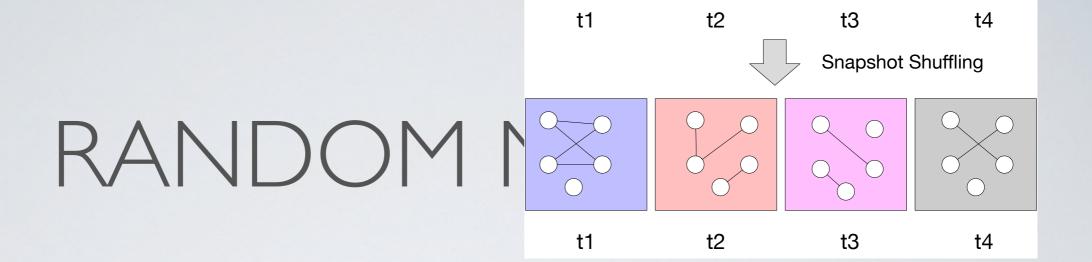
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RANDOM MODELS FOR DYNAMIC NETWORKS

Laetitia Gauvin et al. "Randomized reference models for temporal networks". In: SIAM Review 64.4 (Nov. 2022)

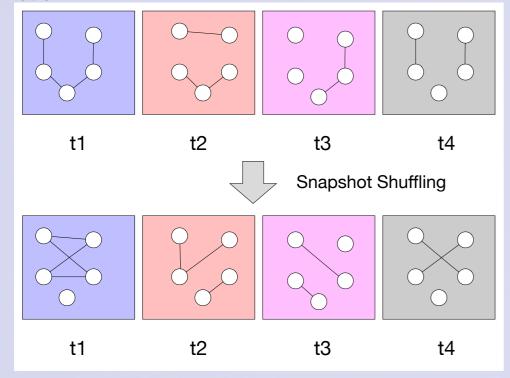
RANDOM MODELS

- In many cases, in network analysis, useful to compare a network to a randomized version of it
 - Clustering coefficient, assortativity, modularity, ...
- In a static graph, 2 main choices:
 - Keep only the number of edges (ER model)
 - Keep the number of edges and the degree of nodes (Configuration model)
- In dynamic networks, it is more complex...



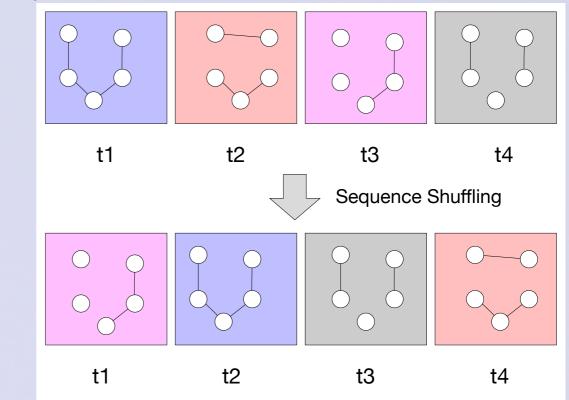
Snapshot Shuffling

Snapshot Shuffling keeps the order of snapshots, randomize edges inside snapshots. Any random model for static network can be used, such as ER random graphs or a degree preserving randomization.



Sequence Shuffling

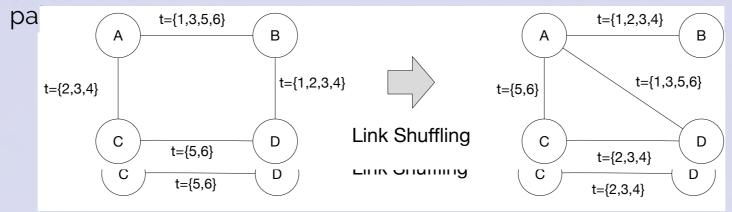
Sequence Shuffling keeps each snapshot identical, switch randomly their order.



RANDOM MODELS

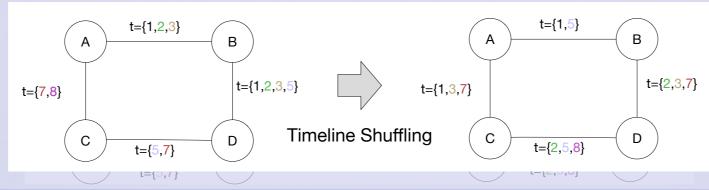
Link Shuffling

Link Shuffling keeps activation time per node pairs, randomize the aggregated graph. For instance, a simple way to achieve this is to pick two node pairs at random (connected or not) of the aggregated graph, and to exchange activation time of these node



Timeline Shuffling

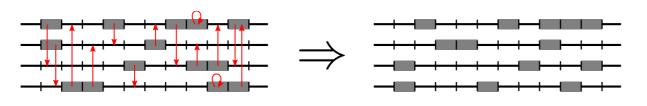
Timeline Shuffling keeps the aggregated graph, randomize edges activation time. For instance, a simple way to achieve this is to redistribute randomly activation period among all edges, e.g.:



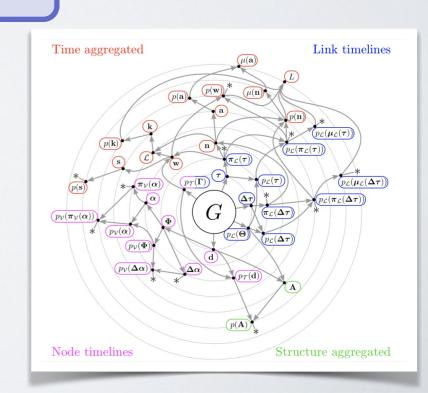
RANDOM MODELS

More constrained Shuffling

Variants of these shufflings with additional constraints have been proposed, for instance the **Local timeline shuffling**, randomizing events time edge by edge, or the **Weight constrained timeline shuffling**, randomizing events while conserving the number of observations for each edge. See (Gauvin et al. 2018) for details.



Laetitia Gauvin et al. "Randomized reference models for temporal networks". In: *SIAM Review* 64.4 (Nov. 2022)



DYNAMIC COMMUNITY DETECTION

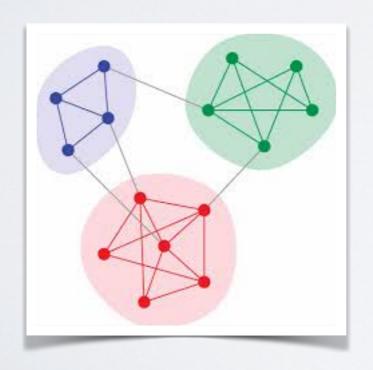
Rossetti, G., & Cazabet, R. (2018). Community discovery in dynamic networks: a survey. *ACM Computing Surveys* (CSUR), 51(2), 1-37.

Cazabet, R., Boudebza, S., & Rossetti, G. (2020). Evaluating community detection algorithms for progressively evolving graphs. *Journal Of Complex Networks*

COMMUNITY DETECTION

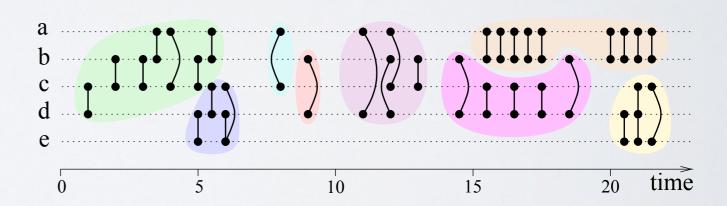
Static networks

Clusters: Sets of nodes



Dynamic Networks

Clusters: Sets of time-nodes, i.e., pairs (node, time)

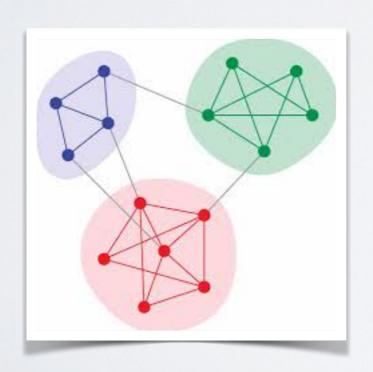


Gaumont, N., Viard, T., Fournier-S'Niehotta, R., Wang, Q., & Latapy, M. (2016). Analysis of the temporal and structural features of threads in a mailing-list. In *Complex Networks VII*

COMMUNITY DETECTION

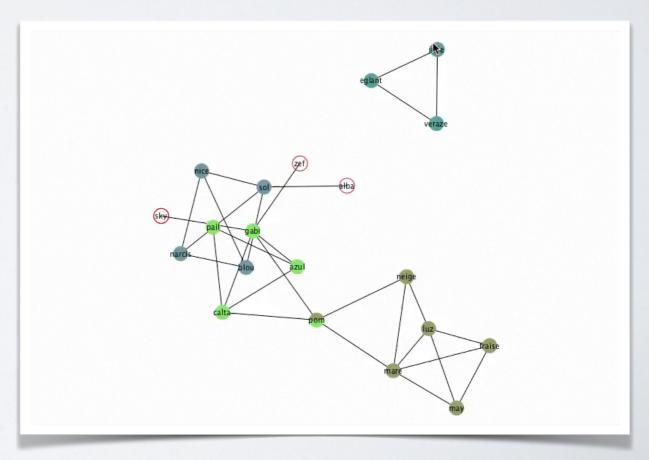
Static networks

Clusters: Sets of nodes



Dynamic Networks

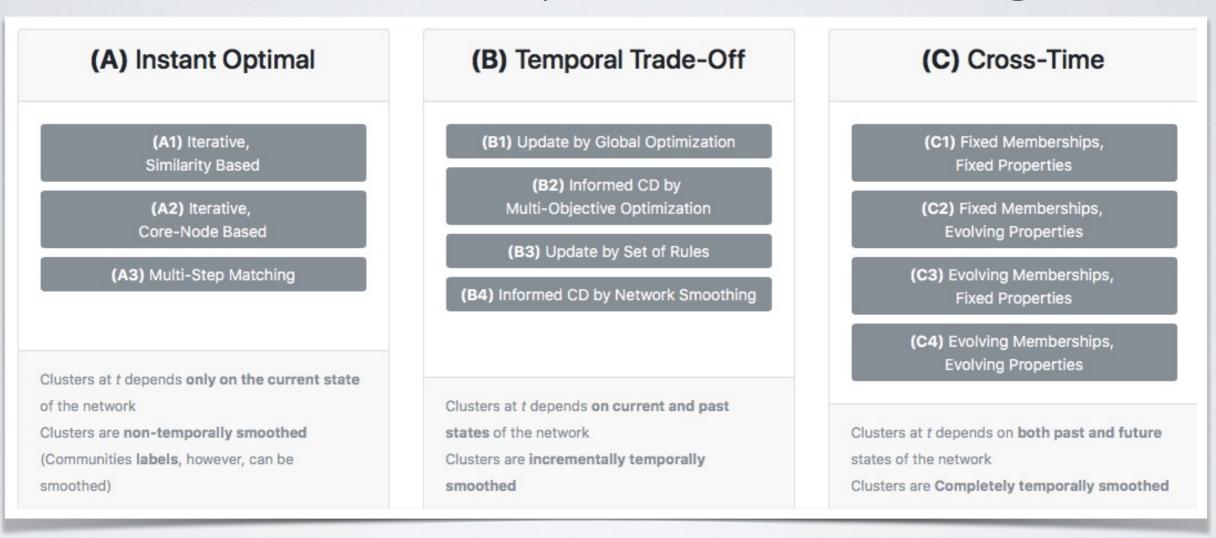
Clusters: Sets of time-nodes, i.e., pairs (node, time)



APPROACHES TO DCD

DYNAMIC COMMUNITIES?

More than 50 methods published, broad categories



Rossetti, G., & Cazabet, R. (2018). Community discovery in dynamic networks: a survey. *ACM Computing Surveys* (CSUR), 51(2), 1-37. 54

CATEGORIES

- Instant optimal:
 - Allows reusing static algorithms
 - No partition smoothing
 - Labels can be smoothed
 - Simple to parallelize

CATEGORIES

- Temporal trade-off
 - Cannot be parallelized (iterative)
 - => Best suited for real-time analysis / tasks
- Cross-Time
 - Requires to know the whole evolution in advance
 - > => Not suited for real-time analysis, potentially the best smoothed (a posteriori interpretation)

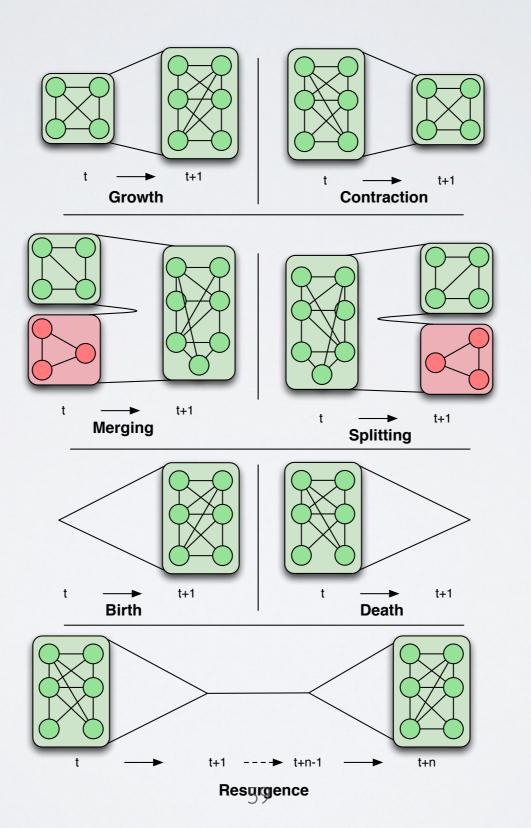
WHAT MAKES DCD INTERESTING

NARRATIVES?

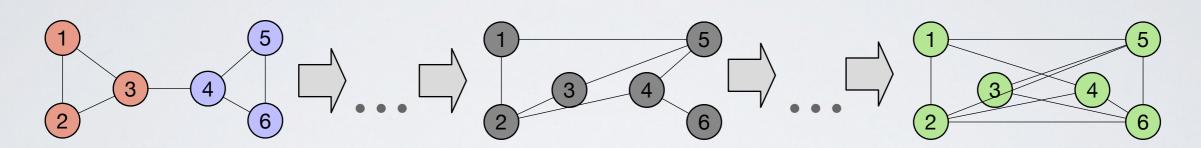
SMOOTHNESS / STABILITY

- No Smoothness: Partition at **t** should be the same as found by a static algorithm.
- Smoothness: Partition at **t** is a trade-off between "good" communities for the graph at **t** and similarity with partitions at different times

COMMUNITY EVENTS



PROGRESSIVE EVOLUTION



2 communities

??

I community

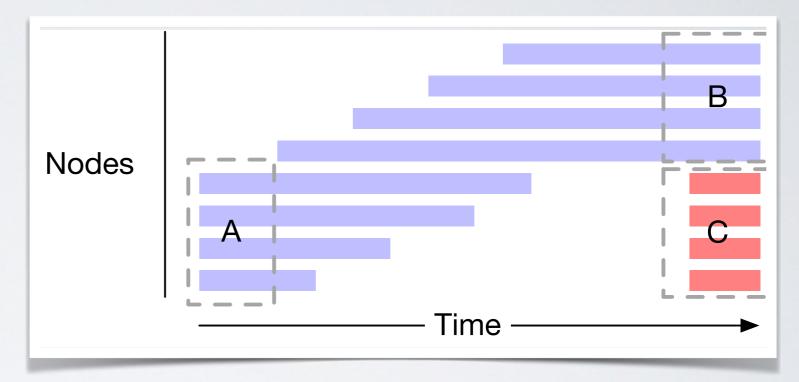
Intermediate state

How to track communities, giving a coherent dynamic structure?

IDENTITY PRESERVATION

Ship of Theseus [Plutarch., 75]





2 problems:

I)Find node clusters at each t 2)Assign labels between same communities at \neq t

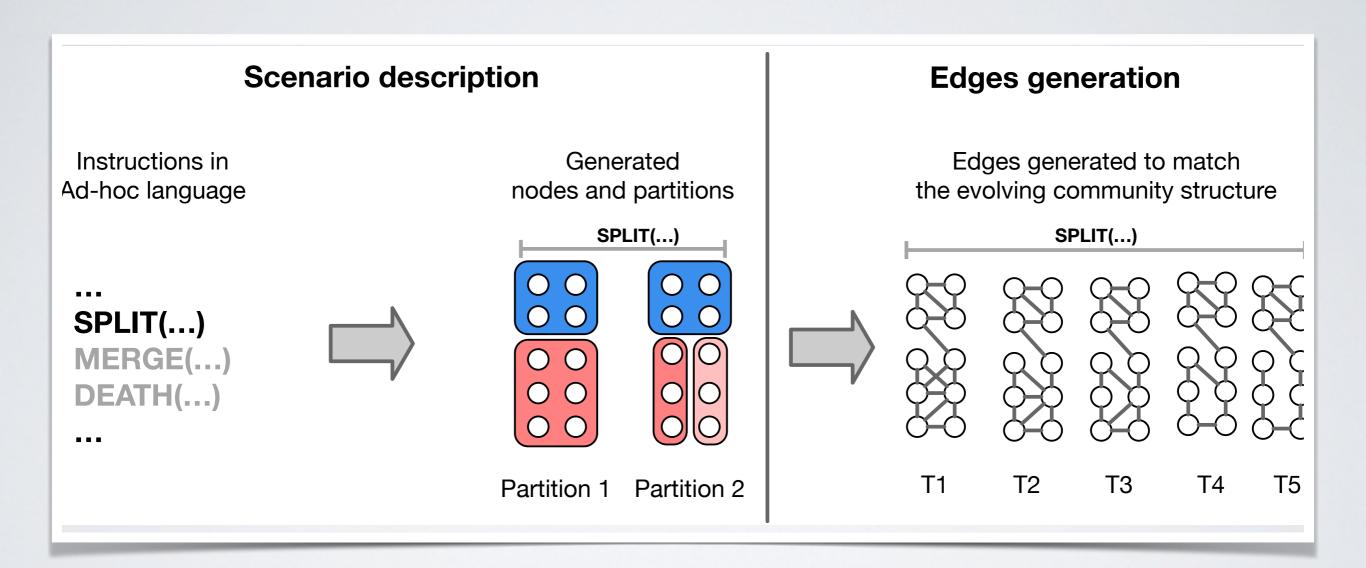
Cazabet, R., & Rossetti, G. (2019). Challenges in community discovery on temporal networks. In *Temporal Network Theory* (pp. 181-197). Springer, Cham.

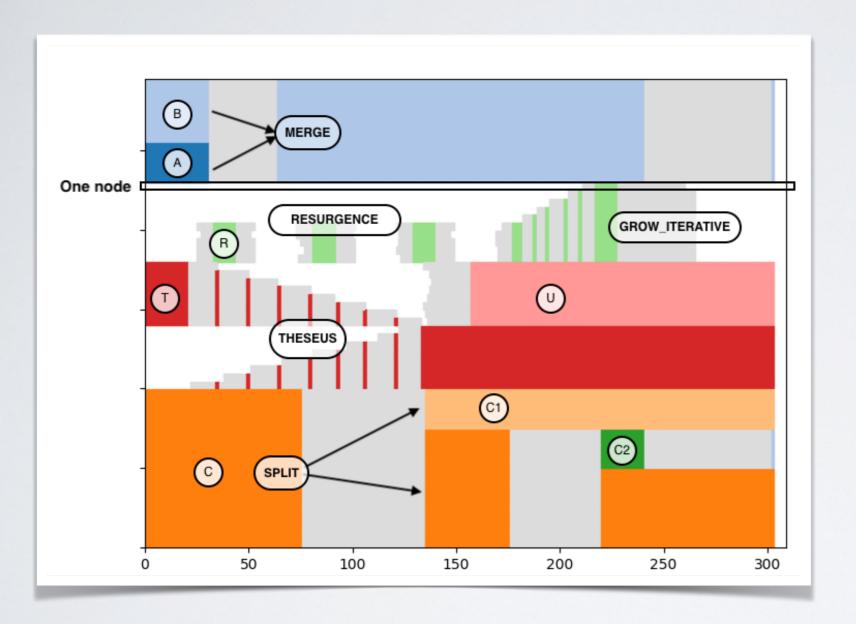
EMPIRICAL EVALUATION

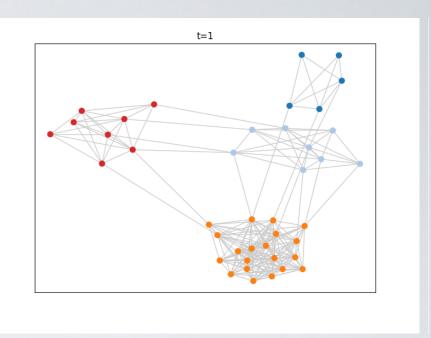
SETTING

- Choose methods based on the same definition of a static community: Modularity (most widespread), but different approaches to dynamics
- Generate dynamic networks with planted dynamic community structure

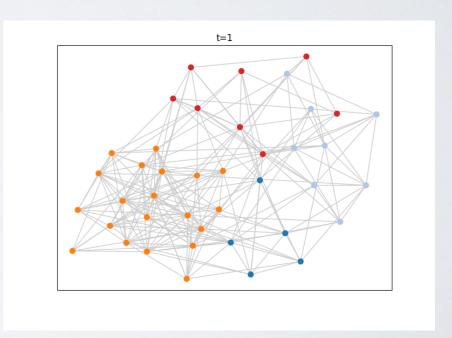
SETTING







(b) The static graph at time t=0, version sharp $(\alpha = 0.9, \beta = 0.05, \beta_r = 0.01)$



(c) The static graph at time t=0, version blurred

$$(\alpha = 0.8, \beta = 0.25, \beta_r = 0.01)$$

METHODS

Instant Optimal

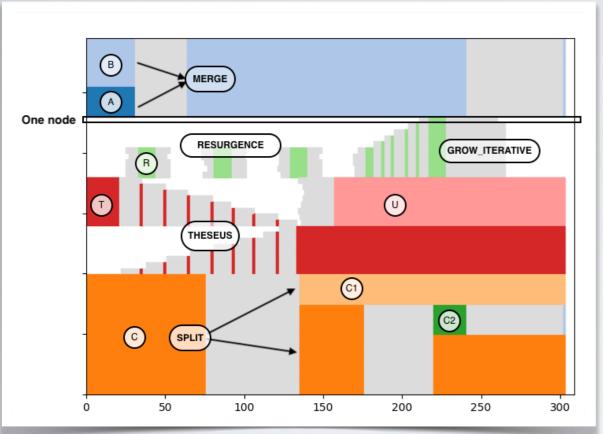
- No smoothing
 - Louvain at each step, match with Jaccard

Temporal trade-off

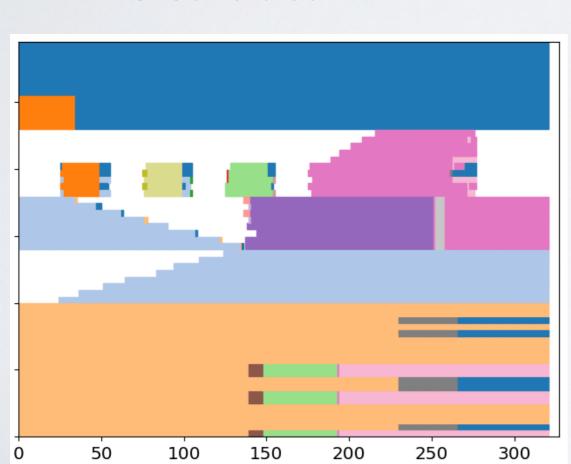
- Implicit Global
 - Louvain at each step in initialized by the previous partition (same local maximum), +Jaccard
- DYNAMO
 - Update partition only based on edge changes to keep modularity high
- Smoothed-graph
 - Each snapshot is modified to artificially raise the probability to obtain similar partition as previous step, then Louvain+Jaccard

Cross-Time

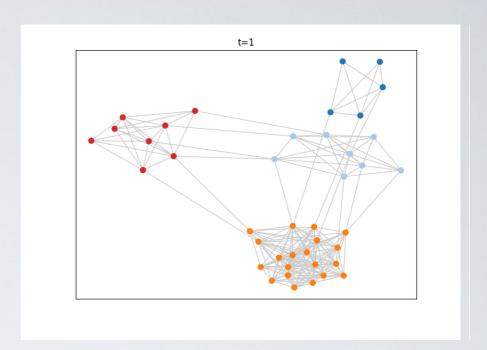
- Transversal Network
 - Create a single graph by adding edges between same nodes in successive snapshots (Mucha et al.), then (modified) Modularity optimization
- Label-Smoothing
 - Create a "Community Survival graphs": nodes are static communities (Louvain), edges weighted by Jaccard Similarity. Apply Louvain on it.



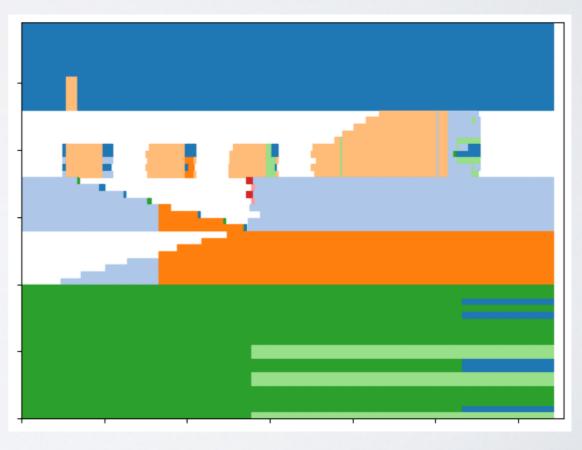
Ground truth



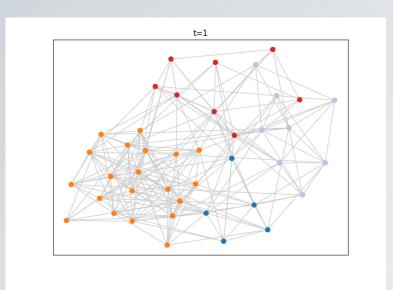
(a) No-Smoothing



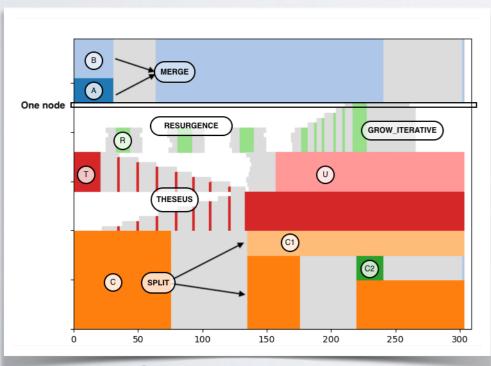
(b) The static graph at time t=0, version sharp $(\alpha = 0.9, \beta = 0.05, \beta_r = 0.01)$



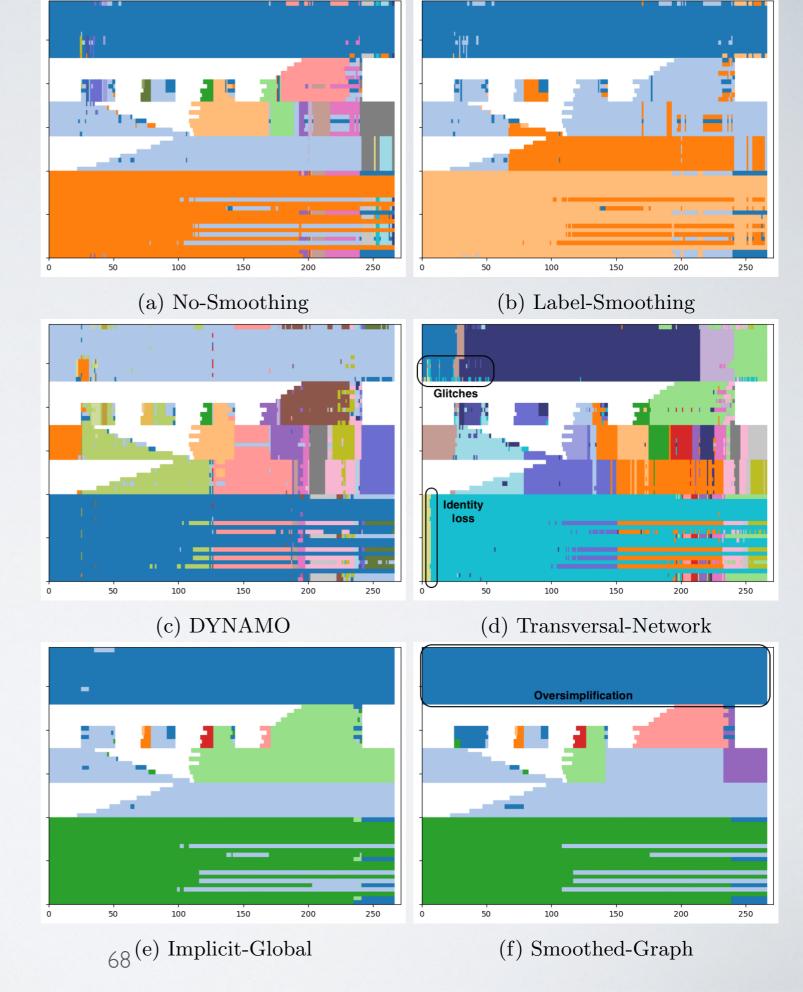
(b) Label-Smoothing



(c) The static graph at time t=0, version blurred ($\alpha = 0.8, \beta = 0.25, \beta_r = 0.01$)



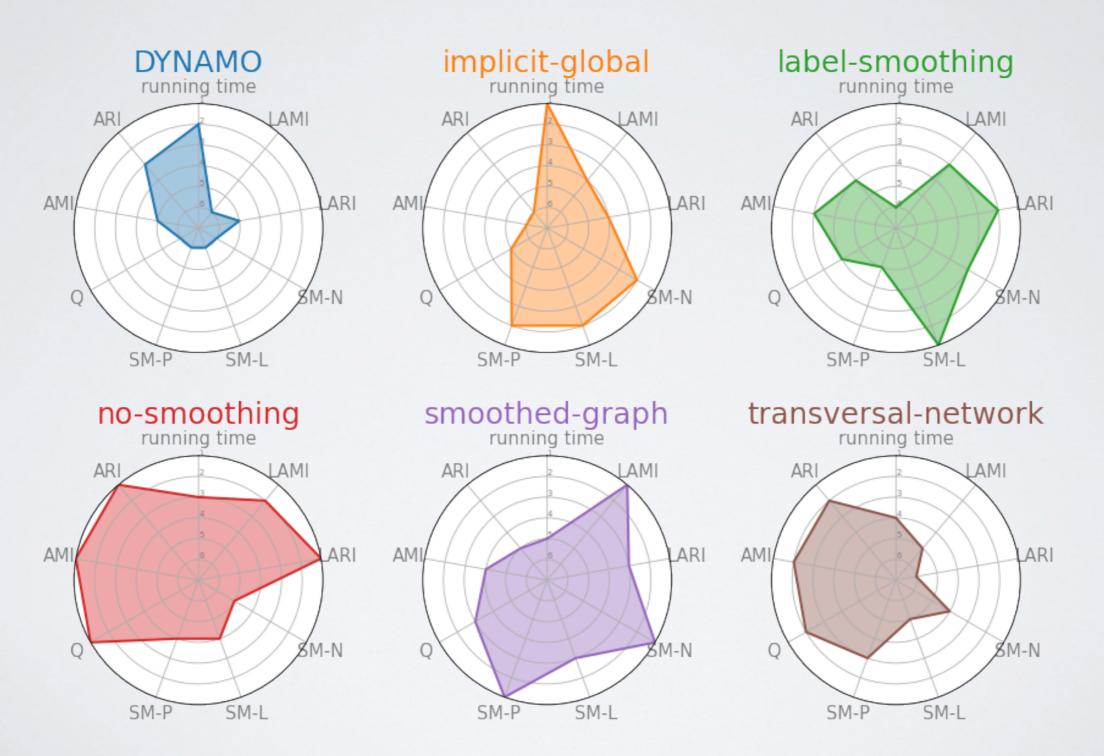
Ground truth



MEASURING DC QUALITY?

- Evaluation at each step (No smoothness)
 - Average Mutual Information (similarity at each step)
 - Average Modularity
- Evaluation of Smoothness
 - ▶ SM-Partitions: Average Mutual Information between successive partitions
 - Label independent, insensitive to glitches, Identity loss
 - ▶ SM-Nodes: Inverse of number of affiliation change
 - Sensitive to glitches
 - SM-Labels: Inverse of Shannon entropy of nodes labels
 - Sensitive to Identity Loss
- Longitudinal Score
 - Modified mutual information of time-node (u,t)

MEASURING DC QUALITY?



TO SUM UP ON DYNAMIC GRAPHS

TO SUM UP

- · Currently, most practitioners still use the snapshot approaches
 - No widespread framework
 - No widespread coding libraries (pathpy, tnetwork, tacoma=>limited usage)
 - Datasets still relatively limited
- · But considered an important topic to work on
 - Dynamic is everywhere
 - Dynamic changes many things in many cases