(Dynamic of networks)

- Most real world networks are dynamic
  - Facebook friendship
    - People joining/leaving
    - Friend/Unfriend
  - Twitter mention network
    - Each mention has a timestamp
    - Aggregated every day/month/year => still dynamic
  - World Wide Web
  - Urban network

**...** 

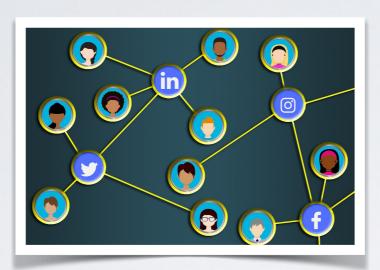
- Most real world networks are dynamic
  - Nodes can appear/disappear
  - Edges can appear/disappear
  - Nature of relations can change
- How to represent those changes?
- How to manipulate dynamic networks?

### **Dynamic Network Properties**

Independently of the studied data, dynamic networks can have various properties:

- Edge presence can be punctual or with duration
- Node presence can be unspecified, punctual or continuous
- If time is continuous, it can be bounded on a period of analysis or ubounded
- If nodes have attributes, they can be constant or timedependent
- If edges have weights, they can be constant or timedependent

### SEVERAL FORMALISMS

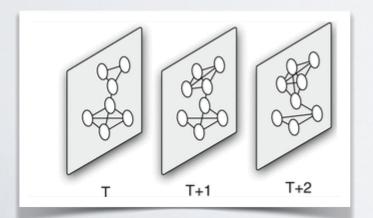


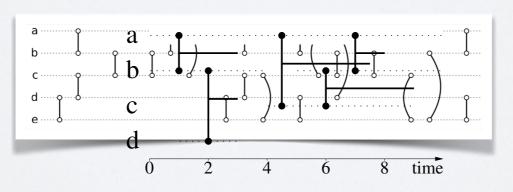


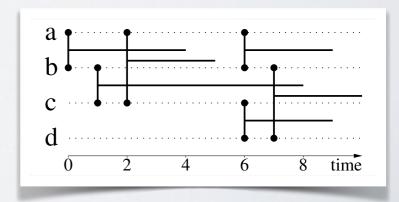












### TEMPORAL NETWORK

Collected dataset, for instance in (t,u,v) format

Time (	JV
--------	----

TITLE	u	V	
1353304100 1353304100 1353304100 1353304100	) 1613 ) 656	1644 1672 682 1671	
1353304 20   1353304 20   1353304 20	656	1613 682 1671	
1353304140	) 1148	1644	
1353304160 1353304160 1353304160 1353304160	) 1108 ) 1632	1601 1671	

Examples:
-SocioPatterns
-Enron

-...

### TEMPORAL NETWORK

### Snapshots

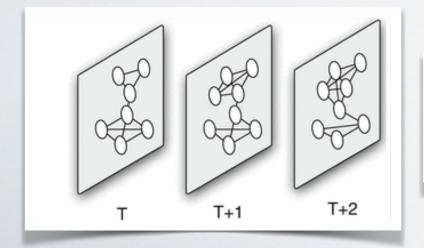
1353304100	1148 1644
1353304100	1613 1672
1353304100	656 682
1353304100	1632 1671
1353304120	1492 1613
1353304120	656 682
1353304120	1632 1671
1353304140	1148 1644
1353304160	656 682
1353304160	1108 1601
1353304160	1632 1671
1353304160	626 698

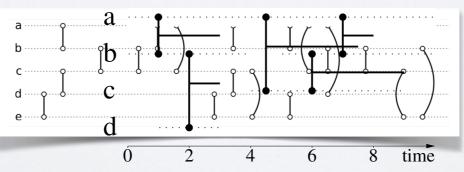
### Link Stream

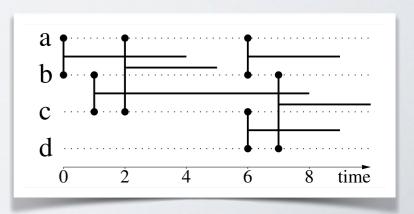
1353304100	1148	1644	
1353304100	1613	1672	
1353304100	656	682	
1353304100	1632	1671	
1353304120	1492	1613	
1353304120	656	682	
1353304120	1632	1671	
1353304140	1148	1644	
1353304160	656	682	
1353304160	1108	1601	
1353304160	1632	1671	
1353304160	626	698	

### Interval Graph

1353304100 1353304100	1148 1644 1613 1672
1353304100	656 682
1353304100	1632 1671
1353304120	1492 1613
1353304120	656 682
1353304120	1632 1671
1353304140	1148 1644
1353304160	656 682
1353304160	1108 1601
1353304160	1632 1671
1353304160	626 698







#### Vocabulary

Many different names have been used to for networks changing with time, but there is no broad consensus in the literature on the meaning of those terms, unless they are used with an explicit reference to a paper defining those terms. Here is a list of the most popular:

- Dynamic Networks and Dynamic Graphs
- Longitudinal Networks
- Evolving Graphs
- Link Streams & Stream Graphs (Latapy, Viard, and Magnien 2018)
- Temporal Networks, Contact Sequences and Interval Graphs (Holme and Saramäki 2012)
- Time Varying Graphs (Casteigts et al. 2012)

## ANALYZING DYNAMIC NETWORKS

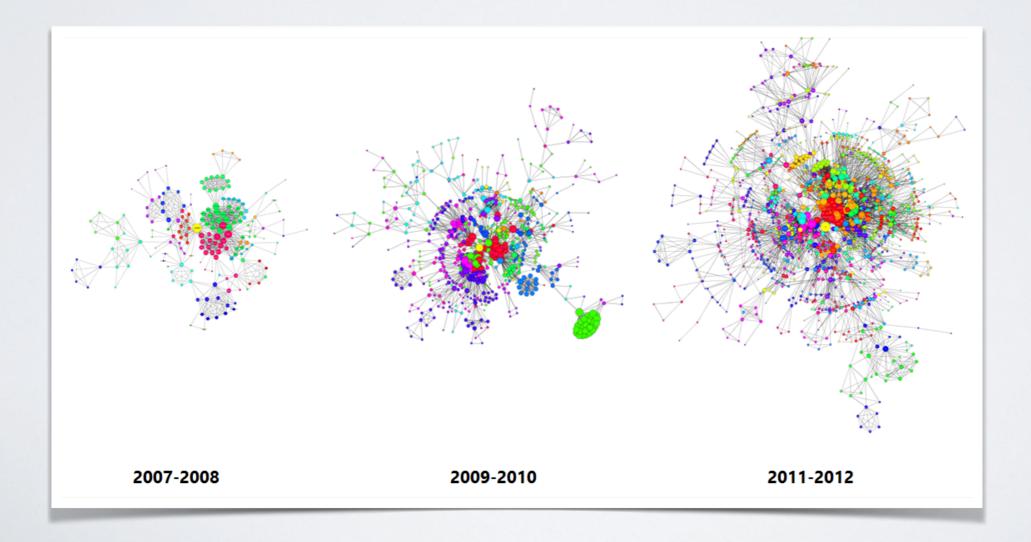
### ANALYZING DYNAMIC NETWORKS

- Few snapshots
- Slowly Evolving Networks (SEN)
- Degenerate/Unstable temporal networks

### FEW SNAPSHOTS

### FEW SNAPSHOTS

- The evolution is represented as a series of a few snapshots.
- Many changes between snapshots
  - Cannot be visualized as a "movie"



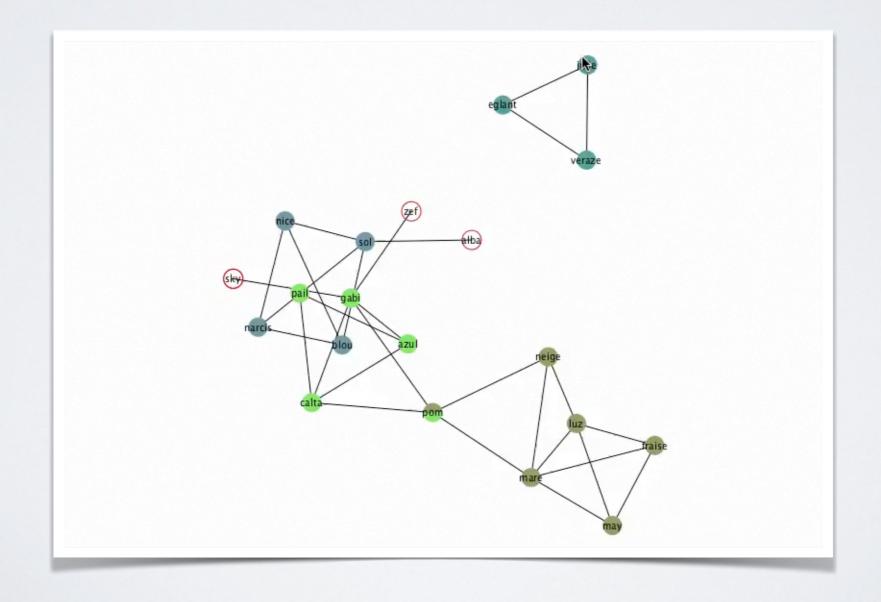
### FEW SNAPSHOTS

- · Each snapshot can be studied as a static graph
- · The evolution of the properties can be studied "manually"
- "Node X had low centrality in snapshot t and high centrality in snapshot t+n"

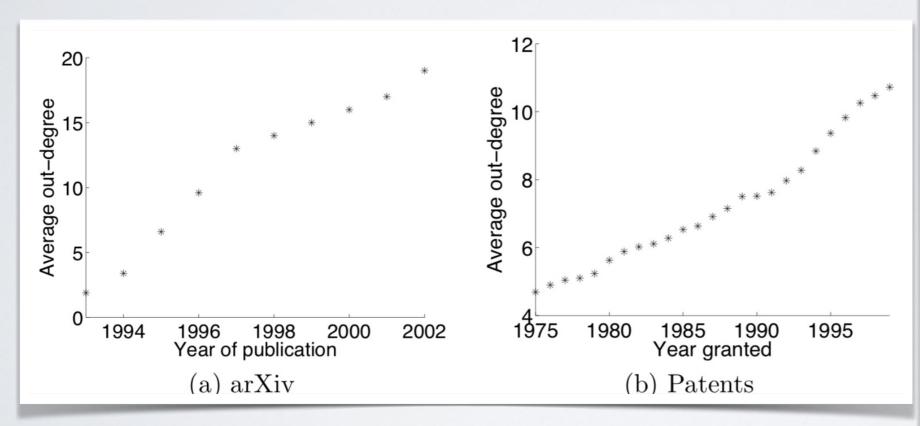
### SLOWLY EVOLVING NETWORKS (SEN)

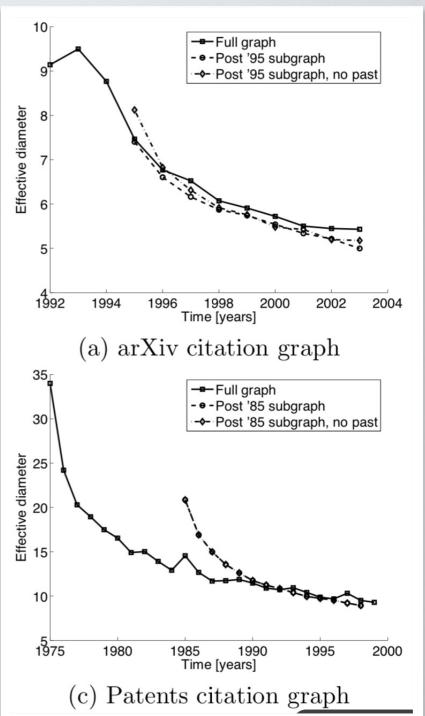
- Edges change (relatively) slowly
- The network is well defined at any t
  - Nodes/edges described by (long lasting) intervals
  - Enough snapshots to track nodes
- · A static analysis at every (relevant) t gives a dynamic vision
- No formal distinction with previous case (higher observation frequency)

- Visualization
  - Problem of stability of node positions



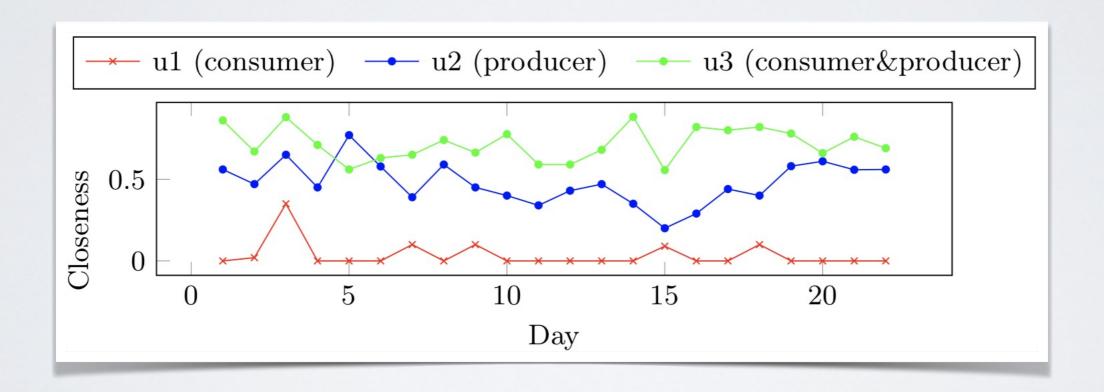
Global graph properties





Leskovec, Jure, Jon Kleinberg, and Christos Faloutsos. "Graph evolution: Densification and shrinking diameters." ACM Transactions on Knowledge Discovery from Data (TKDD) 1.1 (2007): 2.

#### Centralities



### TIME SERIES ANALYSIS

- TS analysis is a large field of research
- Time series: evolution of a value over time
  - Stock market, temperatures...
- "Killer app":
  - Detection of periodic patterns
  - Detection of anomalies
  - Identification of global trends
  - Evaluation of auto-correlation
  - Prediction of future values
- e.g. ARIMA (Autoregressive integrated moving average)

https://en.wikipedia.org/wiki/Autoregressive\_integrated\_moving\_average

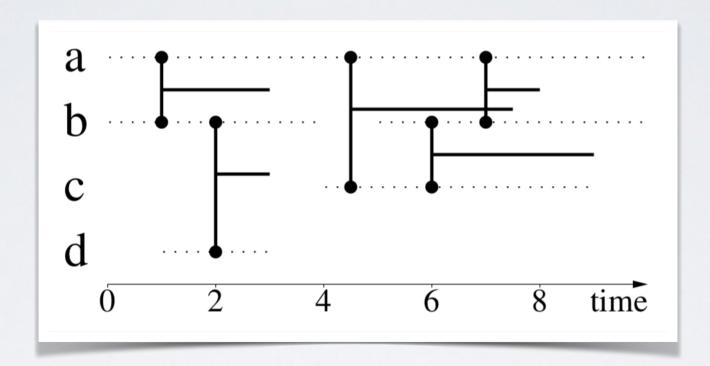
### UNSTABLE/DEGENERATE TEMPORAL NETWORKS

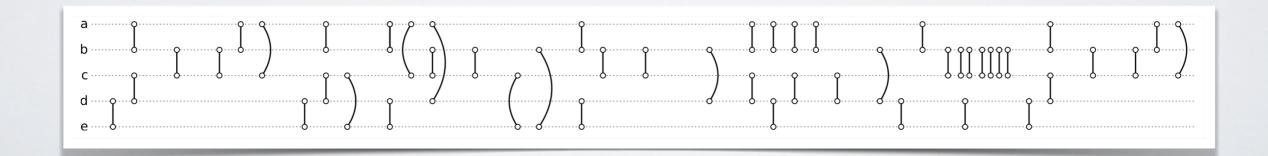
Matthieu Latapy, Tiphaine Viard, and Clémence Magnien. "Stream graphs and link streams for the modeling of interactions over time". In: *Social Network Analysis and Mining* 8.1 (2018), p. 61.

### UNSTABLE TEMPORAL NETWORK

- The network at a given t is not meaningful
- How to analyze such a network?

## UNSTABLE TEMPORAL NETWORK





### UNSTABLE TEMPORAL NETWORK

- Common solution: transform into SEN using aggregation/ sliding windows
  - Information loss
  - How to chose a proper aggregation window size?
- New theoretical tools developed to deal with such networks
  - Link Streams & Stream Graphs (Latapy, Viard, and Magnien 2018)
  - Temporal Networks, Contact Sequences and Interval Graphs (Holme and Saramäki 2012)
  - Time Varying Graphs (Casteigts et al. 2012)

# CENTRALITIES & NETWORK PROPERTIES IN STREAM GRAPHS

#### Stream Graph (SG)- Definition

Stream Graphs have been proposed in as a generic formalism – it can represent any type of dynamic networks, continuous, discrete, with or without duration, with the objective or redefining typical notions of graphs on dynamic networks, including degenerate ones.

Let's define a Stream Graph

$$S = (T, V, W, E)$$

T	<b>Set of Possible times</b> (Discrete or Time intervals)
V	Set of Nodes
W	Vertices presence time $V \times T$
E	Edges presence time $V \times V \times T$

<sup>&</sup>lt;sup>a</sup>Latapy, Viard, and Magnien 2018.

### **SG - Time-Entity designation**

Stream Graphs introduce some new notions mixing entities (nodes, edges) and time:

 $V_t$ 

 $E_t$ 

 $G_t$ 

 $v_t$ 

 $(u,v)_t$ 

 $T_u$ 

 $T_{uv}$ 

**Nodes At Time**: set of nodes present at time t

**Edges At Time**: set of edges present at time t

**Snapshot**: Graph at time t,  $G_t = (V_t, E_t)$ 

**Node-time**:  $v_t$  exist if node v is present at time t

**Edge-time**:  $(u,v)_t$  exist if edge (u,v) is present at

time t, 0 otherwise

**Times Of Node**: the set of times during which u is present

**Times Of Edge**: the set of times during which edge (u, v) is present

 $N_u$ 

**Node presence**: The fraction of the total time during which u is present in the network  $\frac{|T_u|}{|T|}$ 

 $L_{uv}$ 

**Edge presence**: The fraction of the total time during which (u,v) is present in the network  $\frac{|T_{uv}|}{|T|}$ 

### **SG - Redefining Graph notions**

The general idea of redefining static network properties on Stream Graphs is that if the network stays unchanged along time, then properties computed on the stream graph should yield the same values as the same property computed on the aggregated graph.

We say that a clique  $\lim_{\mathbb{R}^{r}} \operatorname{compact}$  (resp. uniform). It is then fully define

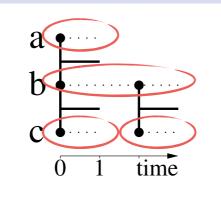
### STREAM set meaning that all pairs of nodes

#### **SG** - N & L

The number/quantity of nodes in a stream graph is defined as the total presence time of nodes divided by the dataset duration. In general, it isn't an integer.

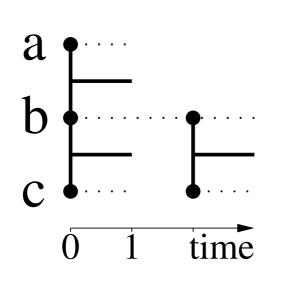
More formally:

 $N = \sum_{v \in V} N_v \text{ igure } 4$ : Examples of maximal pact cliques involving three nodes of



c .....d

# pact cliques involving STREAM $and Apples \{b, c, d\}$ . $[2,5] \times \{a,c\}, [1,8] \times$



For instance, in Fanot maximal as it is clique intersectsmanoth maximal compact clic clique  $[0,4] \times \{a,b\}$  is  $[0,4] \times \{a,b\} \cup [6,9] \times$ it is for instance inclu

A clique in S does

Indilicacing yifah Sugstatut a sang Versay/that a clique is com esp. uniform). It is then fully defin

OI ITOUES TO THE COST PIE

all pairs of nodes involved for instance, we de la la pairs of nodes involved for instance, we de la la la pairs of nodes involved for instance, we de la la la pairs of nodes involved for instance, we de la la pairs of nodes involved for instance, we de la pairs of nodes involved for instance, we de la pairs of nodes involved for instance, we describe the last of the last of

#### SG - L

$$V) \cap W, (T' \times V \otimes V)$$

The number of edges is defined as the toptalt presence on the top talt presence on the top talt presence of talt presence of the top talt presence of tall presence of talt presence of tall presence of talt presence of tall pr divided by the total dataset duration. Is  $([6,9],\{a,b,c\},[6,9])$ 

 $L = \sum_{(u,v),u,v \in V} L_{uv} = \frac{|E|}{|T|}$  betwo  $\text{denoted the problem of the problem o$ 

For instance, L=2 if there are 4 edges pleased that two edges present all the time.

lying three nodes of and  $|7,8| \times \{b,c,d\}$ . Its other maxim

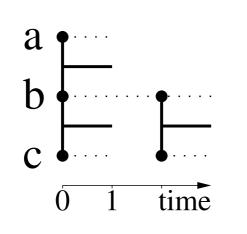
 $A^{[2,5]}$  in  $A^{[$ 

For instance, in Rigardinated Late L not maximal as it is included in displaying clique intersects another maximal gent

maximal compact clique involving t

Induced in What suppose to fairly that suppose the fairly that suppose

Figure 4: Examples of m STREAM pack chiques involving three and  $[7, 8] \times \{b, c, d\}$ . Its ot  $[2,5] \times \{a,c\}, [1,8] \times \{b,c\}$ 



For in  $\frac{1}{8}$  ance, in Figure  $L \equiv 1 \mod \max$  it is included clique intersects another ma maximal compact clique in clique  $[0,4] \times \{a,b\}$  is not a  $[0,4] \times \{a,b\} \cup [6,9] \times \{c,d\}$ 

A clique in S does not i  $[8,9] \times \{c,d\}$  is a clique for

it is for instance included in

betv deno

### STREAM GRASHigues

### SG - Edge domain - $L_{\mathrm{max}}$

In Stream Graphs, several possible definitions of  $L_{
m max}$  could exist ue of

- · Ignoring nodes duration:  $2 \frac{11}{2} pairs of nodes involved$
- · Ignoring nodes co-presence is  $\max$  in there is  $\sum_{t=1}^{t} \max_{t=1}^{t} \sum_{t=1}^{t} \sum_{t$
- Taking nodes co-presence into Weousay that a clique  $L_{\max}^{3} = \sum_{(u,v),u,v \in V} |T_{u} \bigcap T_{v}|$  uniform). It is then

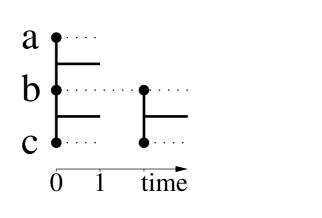
set) meaning that all pair

set) meaning that all pairs of

### STREAM GRAPHS

The density in static networks can be understood as the fraction of existing edges among all possible edges,

 $d = \frac{L}{\text{Trigure 4: Examples of maxin}}$  pact cliques involving three nod In the following, we will Triguize 4: Examples of maxin



For instance, instigure 4 the not maximal, as it is included included in clique intersects another maxim

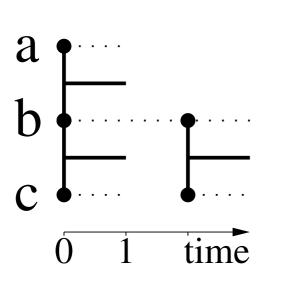
Pact eliques, involving, three, n

clique intersects another maxim Figure 4 clique intersects and there maxim Figure 4 clique involved different densities, respectively 0.75 (left) and 1 (right). It is include clique  $[0,4] \times \{a,b\}$  is not a maximal property of the clique intersects another maximal property of the clique intersects and the clique intersects are clique intersects.

b

C

d



For instance, in Figure 1. The stance in Figure 1. The stance in Figure 2. The

Examples of graphs with N = naccion[a] 1 diplompts with cliq different densities, respectively 0.75 (left) and 1 (right). clique  $[0,4] \times \{a,b\}$  is  $[0,4] \times \{a,b\} \cup [6,9] \times [0,4] \times [a,b] \cup [6,9] \times [a,b]$ 

A clique in S does

the same time space T as S. We therefore define the set V' of V as the substream induced by the node cluste  $(Y') \cap W, (T \times \mathcal{B}' \otimes \mathcal{A} \mathcal{B}) \text{ of } \mathcal{S}$  Likewise, we define the set T' of T as the substream induced by  $(T' \times V) \cap W$ ,  $\otimes V) \cap E Gof Gusters & Substreams$ In Stream Graphs, a clusters C is as subset of W, and the corresponding (induced) substream S(C) = (T, V, C, E(C)), with  $E(C) = \{(t, (u, v)) \in E, (t, u), (t, v) \in C\}.$ G is a cluster  $C_4$  of  $G_8$  in  $G_8$  time  $G_8$   $G_$  $e\ linked\ togeth$ examine GrsubghapalangunduGed isbenneaxi $mal\ if\ there\ is$ **lique** of stream graph S as a cluster C of S of density 1.

displayed in blue, is  $C = ([1, 4] \cup [5, 8]) \times \{a\} \cup [5, 9] \times \{b\} \cup [3, 8] \times \{c\}$ . Right: the substream induced by C is  $S(C) = ([0, 10], \{a, b, c, d\}, C, E(C))$  with  $E(C) = [6, 8] \times \{ab\} \cup [3, 4] \times \{ac\} \cup [5, 8] \times \{bc\}$ .

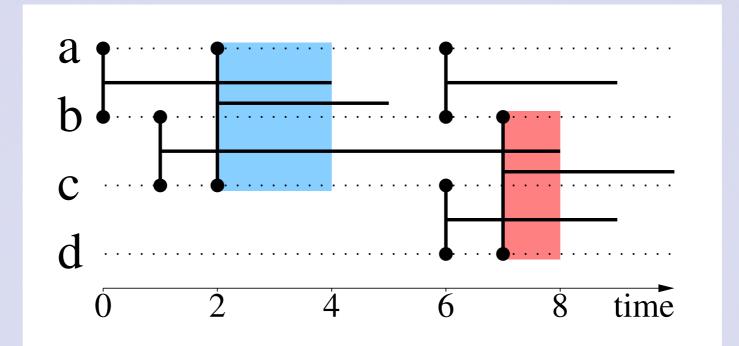
Given a cluster C, we say that the properties of its induced substream are the properties of C; for instance, we denote  $\delta(S(C))$  by  $\delta(C)$ . For any v in V, we also denote by  $T_v^C$  the set of times at which v is in C, and for any u and v in V we denote by  $T_{uv}^C$  the set of time instants at which u and v are in C and are linked together. For any t in T, we denote by

#### SG - Cliques

In Figure 3, for instance,  $T_a = [1,4] \cup [3,\delta]$ ,  $T_b = [3,9]$ ,  $T_c = [3,\delta]$  and  $T_d = [3,\delta]$ 

 $\emptyset$ ;

Having defined substreams and density, we can now naturally define a clique by analogy with static networks as a substream of density 1. A clique is said to be a **maximal clique** if it is not included in any other clique.



Red and Blue are the two maximal cliques of size three in this Stream Graph.

ques involving three nodes of the link stream L of Figure 1 (right):  $[2,4] \times 7$  $[8] \times \{b, c, d\}$ . Its other maximal compact cliques are  $[0, 4] \times \{a, b\}$ ,  $[6, 9] \times \{a, b\}$  $\{a,c\}, [1,8] \times \{b,c\}, [7,10] \times \{b,d\}, [6,9] \times \{c,d\}$  (involving two nodes each  $\inf_{s=1}^{a} \frac{1}{s}$  in Figu SG - Neighborhood N(u)ct clique. Howeve  $ext{ximal}$  as it is  $ext{inc}$  the neighborhood v(u) of node u is defined as the cluster combinate  $ext{are}$  in  $ext{chirp}$  and  $ext{chirp}$ posed of node-times such as an edge-time exists between it and its corresponding grant gr ntersectsmanother The maximal contact, chiefly tiny three nodes,  $[8,9] \times \{b,c,d\}$ . The maximal contact  $[8,0] \times \{b,c,d\}$ .  $0,4| imes\{a,b\}$  is not a maximal clique because it is 15° in Congrected the sinable SG - Degree k(u) $\{a,b\} \cup [6,9] \times \{c,$ is not maximal eit  $\begin{array}{c} A \ graph \ G = (V,E) \ is \ connected \ if \ for \ f$ the Neighborhood of node u, i.e. Figure in S does not in general induce a clique in  $\{c,d\}$  is a clique for the example in Figure 4 included in no other connected stuster of G, and they form a partition of Vs. where uv & F'dit uis not a Though of R.  $[b,e] \times X$  is a compact dused above we define the den **node** v in V, and the **density** Given a stream graph S = (T, V, V) $(\alpha, u)$ , which we denote by  $(\alpha, u)$  -  $-\frac{d}{dt}$ ...,  $(t_k, u_k, v_k)$  of elements of  $T \times V$  $\overline{\operatorname{tim}}(t_i, u_i v_i) \in E, \ [\alpha, t_0] \times \{u\} \subseteq W, \ [t_k, v_i]$  $\frac{1}{|V_v|} \stackrel{\text{4}}{=} \text{and} \stackrel{\text{6}}{=} \delta(t) \stackrel{\text{8}}{=} \frac{\text{tim} E_t}{|V_t \otimes V|}$ This sequence is similar to a path from Example, the neighborhood of node c is highlighted in blue. In the neighborhood of node c is highlighted in blue. The necessarily have  $t_0 \geq \alpha$ ,  $t_{i+1} \geq t_i$ k(c) = 1.3is symmetric: if  $(\alpha, u) - - - (\omega, v)$  then (|[1,8]|+|[2,5]|+|[6,9]|). 0, respectively, then we define  $\delta(uv)$ , (9,d) --- (3,g) through the sequence

 $tin(t_i, u_i v_i) \in E, [\alpha, t_0] \times \{$ 8 time This sequence is similar neighborhoods and degrees of modes have to be node under concern, and in grey the other links. Left: if  $(\alpha, u)$ -- $[4.5, 7.5] \times \{c\}$  is in blue, leading to  $d(a) = \frac{3}{10} + \frac{3}{10} = \frac{3}{5} = \frac{3$ SG - Ego-network connected. It is a weakl node de gree of S as follows. The Ego network  $G_u$  of node u is defined as the substream induced by its neighborhood, i.e.,  $G_u = (T, V, N(u), E(igur))14$  for an illustrat SG - Clustering coefficient

The clustering coefficient C(u) of node u is defined as the density of the ego-network of  $\underline{u}$  view  $u_{j+1}$  then  $P' = (u_0, v_0), \ldots, (u_{i-1}, v_{i-1}), (u_{j+1}, v_{j+1}), \ldots, (u_k, also is a path from <math>u$  to v. If one iteratively removes the cycles of P in this way, eventually obtains a G(up) e-path V(v) u to v.

The path P is a shortest path from u to v if there is no path in G of length lower t k. Then, k is called the distance between u and v and it is denoted by  $\partial(u,v)$ . If there no path between u and v then their distance is infinite. The diameter of G is the large finite distance between two nodes in V.

Figure 14: Weakly coefficients

# PATHS AND DISTANCES IN STREAM GRAPHS

### PATHS

#### SG - Paths

Figure 14: Weal has four weakly co

In a Stream Graph S=(T,V,W,E), a **path** P from note the  $x_{\alpha}$  [ $\{b, 3\}$ ]  $\cup$  [ $\{b, 3\}$ ]  $\cup$  [ $\{c, 4\}$ ] node-time  $y_{\omega}$  is a sequence  $(t_0, x, v_0), (t_1, v_0, v_1), .[\{c, t_k\}\}]$   $\cup$  [ $\{c, t_k\}\}$ ]  $\cup$  [ $\{c, t_k\}$ ]  $\cup$  [ $\{c, t_k\}\}$ ]  $\cup$  [ $\{c, t_k\}\}$ ]  $\cup$  [ $\{c, t_k\}\}$ ]  $\cup$  [

duration  $t_k - t_0$ .

and  $P_i \cap P_j = \emptyset$  for a because C and C are a second and C are a secon

Examples of two paths. The left one starts at 2, arrives at 5, has length 3 and duration 3. The right one starts at 2, arrives at 8, has length 4 and duration 6.

m and its corresponding grayshuahedmarpalehnissaveryche in restricted carr sponding path is a cycle in the graph aspects of a network using the two level reresentations introduced in Section II A 2 above. models as they all conserve the nodes  $\mathcal{V}$ , the tempor nnectedness and connected teamponents SG - Shortest -E)-Eyentshufflings furthermore conserve the multiset he =(V,E) is connected if for based up the line time representation with the feature Vis connected iength) is contributed for the static graph of a network but not the individual timelines, and time no other connected patricipation Thereof the the connected the continue of the connected the connect  $tey form a partition^3 of V siblication of the period of$ Furthermore, one can reproduct has a present the second of stance: Eventsinks in  $G^{\text{stat}}$ ). In practice they are implemented stream **Fastest** shortest paths which is introduced in the several more restricted class of minimal length -1 distributing the time times  $v_k$  of the hardest paths paths being the timest and the period  $v_k$  of elements of a to  $v_k$  of  $v_k$  of elements of a to  $v_k$  of  $v_k$   $E, [\alpha, t_0] \times \{u\} \subseteq W, [t_k$  resentations introduced in Section II.A.2 above  $t_k$  as  $x_0$ ice is similar to a path frequency of the individual timeling the similar to a path frequency of the practice they are implemented by frily have  $t_0 \geq \alpha$ ,  $t_{i+1} \geq t_i$ ; tributing the tinstant are one insequence between the

me

We display links. Left:

 $\frac{3}{10} + \frac{3}{10} = 0.6$ .  $=\frac{13}{10}=1.3.$  topological aspects of a network using the two level representations introduced in Section IIA2 above. network's static topology,  $G^{\text{scation}} = (1, \mathcal{L})$ , as well as any constraints on the content of the individual timelines  $\Theta_{(i,j)} \in \Theta$ . In practice they are implemented by redistributing the (instantaneous) events in or between the

Based on the link-timeline representation (Def. II.6), we Before place I had in ging which End to Bring it hoost altiforgraph SG - Connected mixing the static graph indicated at the mediates and on think shielding prossible ich pandom Eigthe. Simelines entos et the

Various definitions for the company of the beautiful by redisposed for temporal networks the instantant of the condense of 2018) for details. One of the main person is the weakly connected as Ins (b) ncomponent, defined suchias two the deatimes belongite the same afiguration connected component if anthous limit (Frette) is Impathotic motion to the plemented by randomizing the links  $\mathcal{L}$  in the static graph and reother, *ignoring time*. distributing the timelines  $\Theta_{(i,j)} \in \Theta$  on the new links without replacement. Based on the snapshot-sequence representation in the shuffings, on the other hand, constrain the

Hetwork's static topology, Generally, shufflings, which mire the order of the snapshots but not the shutsaust buy the conserve of the snapshots but the temporal removes a supplied by the conserve of the snapshot with the said the snapshot with the snapshot The shullings furthermore conserve the multiset of the representations (a) = 17 for evening the multiset of the representations (a) the representations (a) the representation of the representation o  $\ldots, (u_{i-1}, v_{i-1}), (u_{j+1}, v_{j+1}), \ldots, (u_k, v_k)$ 

ere is no path in G of length lower than and it is denoted by  $\partial(u,v)$ . If there is finite. The diameter of G is  $\mathsf{Example}^t$ of a Stream  $\mathsf{g}$ 

noves the cycles of P in this way, one

P from  $(\alpha, u) \in W$  to  $(\omega, v) \in W$  is a El, which conserves only the number of instantaneous elements of  $T \times V \times V$  such that  $u_0 = u$ ,

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## RANDOM MODELS FOR DYNAMIC NETWORKS

Laetitia Gauvin et al. "Randomized reference models for temporal networks". In: *arXiv preprint arXiv:1806.04032* (2018).

### RANDOM MODELS

- In many cases, in network analysis, useful to compare a network to a randomized version of it
  - Clustering coefficient, assortativity, modularity, ...
- In a static graph, 2 main choices:
  - Keep only the number of edges (ER model)
  - Keep the number of edges and the degree of nodes (Configuration model)
- In dynamic networks, it is more complex...

between two nodes is the same cycle in the stream if and only

## ANDOM MODELS

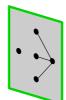
#### components

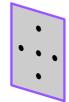
is a path between u h of G is the graph R = 1

#### **Snapshot Shuffling**

Snapshot Shuffling keeps the order of snapshots, randomize  $aximal\ connected\ c$  l edges inside snapshots. Any random model for static network can  $alled\ the\ connected\$  be used, such as ER random graphs or the Configuration Model.







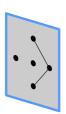












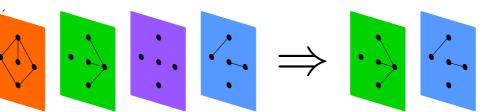
#### v) is weakly reachable from

equence  $(t_0, u_0, v_0), (t_1, u_1, v_1),$ 

 $v_k = v$ , for all  $i, v_i$ for all  $i, [t_i, t_{i+1}]$  Sequence Shuffling

igure 14 for instance, w

 ${
m ept\ for\ time\ constr}$ a **Sequence Shuffling** keeps each snapshot identical, switch ranonsequence, weak reachy their order.



 $d(\omega, v)$  in  $W, (\alpha, u)$ 

and gubatroom S((') is wookly

Link and timeline shufflings the tinstants recognise of the timeline shufflings

E (ink-timeline rejected at 14 foreinstance, we have

Belinging the Foliable Rivithe station raph static craphilatic doctor with estation raphilatic doctor with the static craphilatic doctor with the static property of the static propert

 $G^{\text{stat}}$ ). In practice they are implementing the links  $\mathcal{L}$  in the static graph and the timelines  $\Theta_{(i,j)} \in \mathbf{\Theta}$  on the new linear the snapshot-sequence represents the snapshot sna

The snapshot-sequence representations shufflings, on the other hand, constrain the atic topology,  $G^{equence}$ , L, L, as well as the order of the snapshots but not independent of the individual timelinapshot graphs and snapshot shufflings are implemented by mize individual snapshot graphs but the (installations) events in or between

shufflings constrain the content of instartates  $[0.8\,\mathrm{grb}]$  sing in the through  $[0.8\,\mathrm{grb}]$  sing in the through  $[0.8\,\mathrm{grb}]$  sing in the through  $[0.8\,\mathrm{grb}]$  static graph leads to the most random link sible,  $P[p_{\mathcal{E}}(\Theta)]$  frig. II.5(a)]. The most random link is sible,  $P[p_{\mathcal{E}}(\Theta)]$  frig. II.5(a)]. The most random link is sible,  $P[p_{\mathcal{E}}(\Theta)]$  is obtained by redistribution an instant-event and in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution and instant-event  $[0.8\,\mathrm{grb}]$  is an instant-event  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution and  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by redistribution  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{grb}]$  is obtained by  $[0.8\,\mathrm{grb}]$  in the state  $[0.8\,\mathrm{$ 

isplayed with a different color: [5,7]

 $[3,7] \times \{d\}$  in pink,  $([0,2] \cup [8,10])$  $[0] \stackrel{Sequence}{\times} \{f\} \stackrel{snapshot}{\cup} [5,10] \stackrel{shufflings}{\times} \{g\}$  in oran

the snapshot-sequence representate,  $P_1$  of k subsets of X such we define sequence shufflings, whethe order of the snapshots but not mapshot graphs, and snapshot shuffling omize individual snapshot graphs but

shufflings constrain the content of instartant graphs, i.e. the multiset  $p_{\mathcal{T}}(\mathbf{\Gamma}) = [\Gamma^t]_{t=\mathcal{T}}$ ,

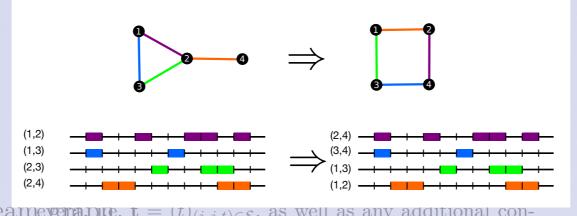
as well as possible additional constraints on the order of the snapshots. They are implemented simply by reshufting the order of the snapshots.

Snapshot shufflings instead constrain the time of each connected, i.e.  $\mathbf{t} = (t)_{(i,j,t) \in \mathcal{E}}$ , as well as any additional con-

snapshots.

#### Link Shuffling

Link Shuffling keeps activation time per node pairs, randomize the aggregated graph. For instance, a simple way to achieve this is to pick two node pairs at random (connected or not) of the aggregated graph, and to exchange activation time of these node pairs, e.g.:

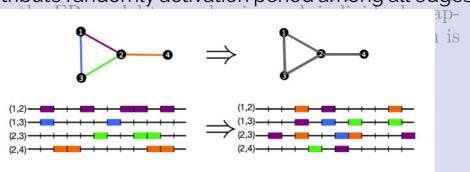


straints on the individual snapshot graphs  $\Gamma^t \in \Gamma$ . They are typically implemented by randomizing the snapshot [0])  $\times$ graphs individually and independently using any shuf-

#### **Timeline Shuffling**

shot

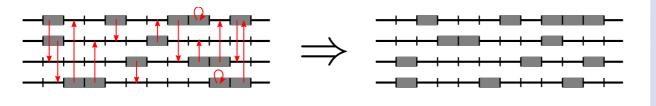
Timeline Shuffling keeps the aggregated graph, handomize hedges activation time. For instance, a simple way to achieve this gis to redistribute randomly activation period among all edges, e.g.:



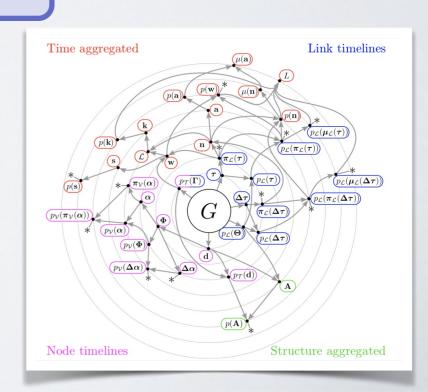
## RANDOM MODELS

### More constrained Shuffling

Variants of these shufflings with additional constraints have been proposed, for instance the **Local timeline shuffling**, randomizing events time edge by edge, or the **Weight constrained timeline shuffling**, randomizing events while conserving the number of observations for each edge. See (Gauvin et al. 2018) for details.



Laetitia Gauvin et al. "Randomized reference models for temporal networks". In: *arXiv preprint arXiv:1806.04032* (2018).

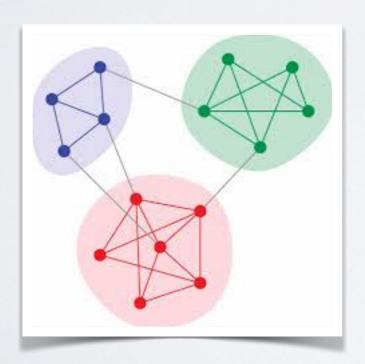


## DYNAMIC COMMUNITY DETECTION

## COMMUNITY DETECTION

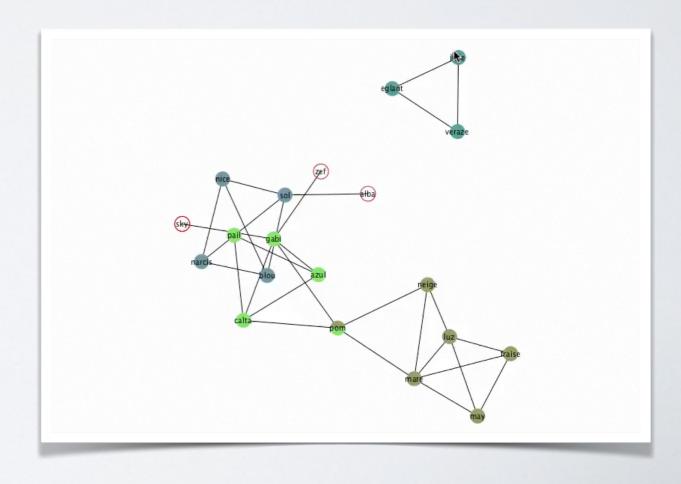
Static networks

Sets of nodes



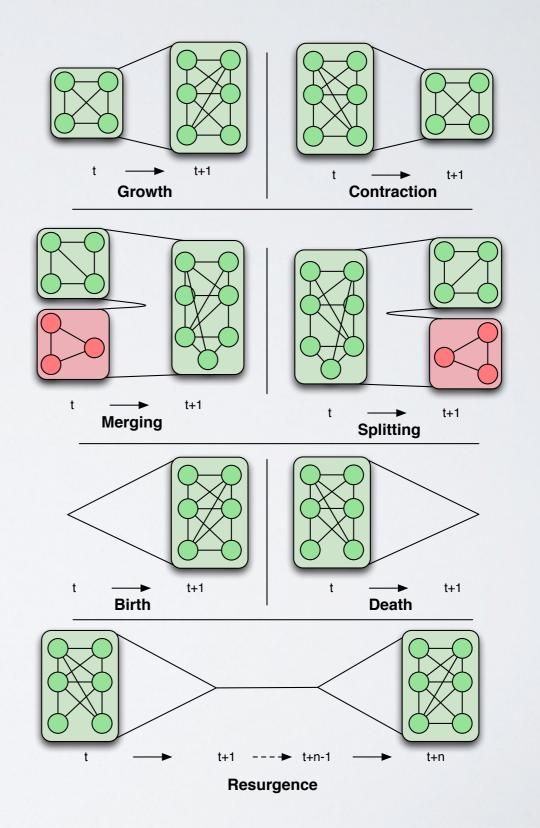
Dynamic Networks

Sets of periods of nodes



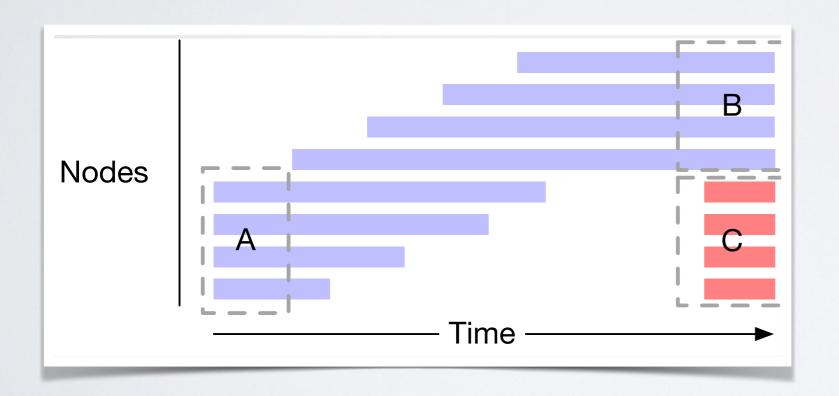
## COMMUNITY DETECTION

Community events (or operations)



## IDENTITY PRESERVATION

Ship of Theseus [Plutarch., 75]





# TO SUM UP ON DYNAMIC GRAPHS

### TO SUM UP

- · Currently, most practitioners still use the snapshot approaches
  - No widespread framework
  - No widespread coding libraries (pathpy, tnetwork, tacoma=>limited usage)
  - Datasets still relatively limited
- · But considered an important topic to work on
  - Dynamic is everywhere
  - Dynamic change everything in many cases