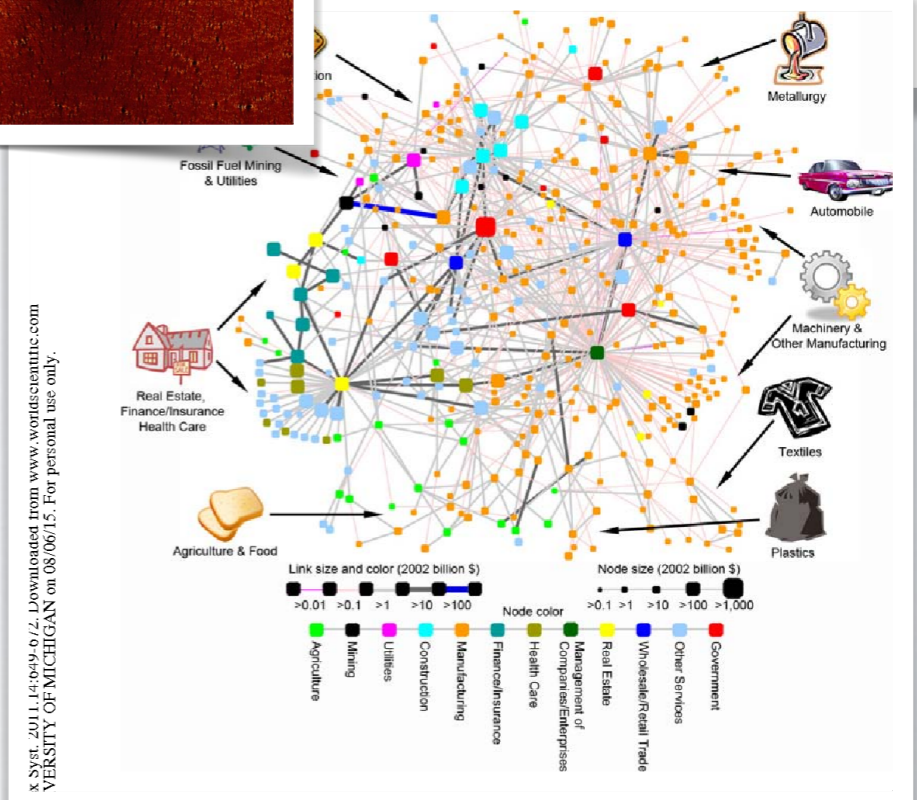
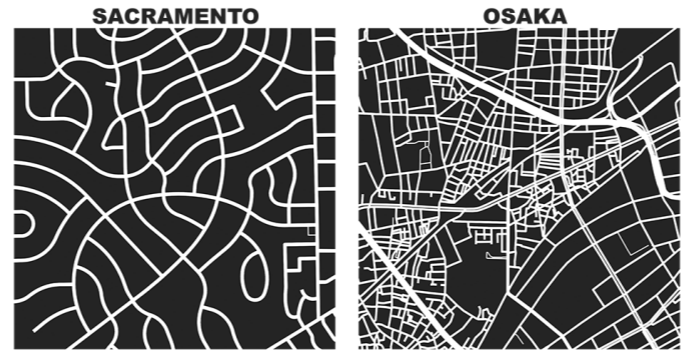
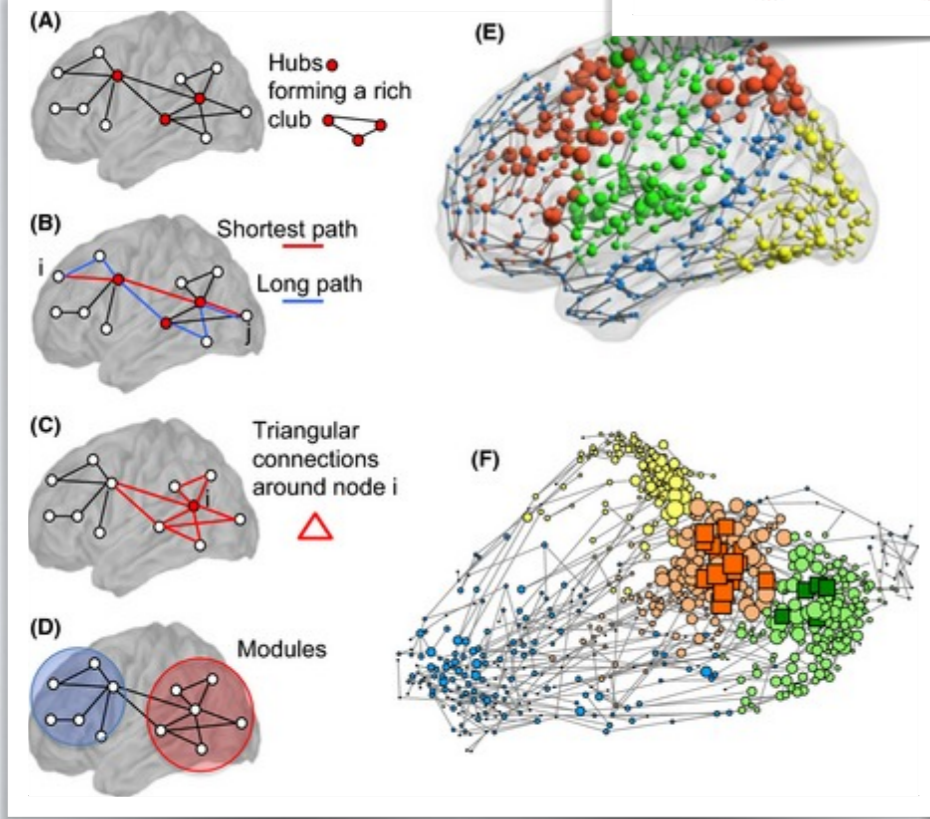
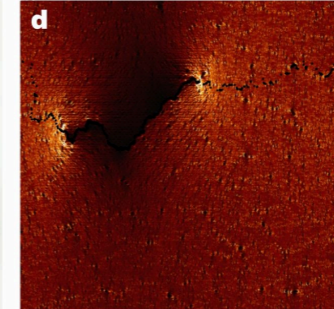
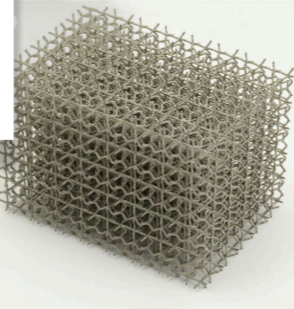
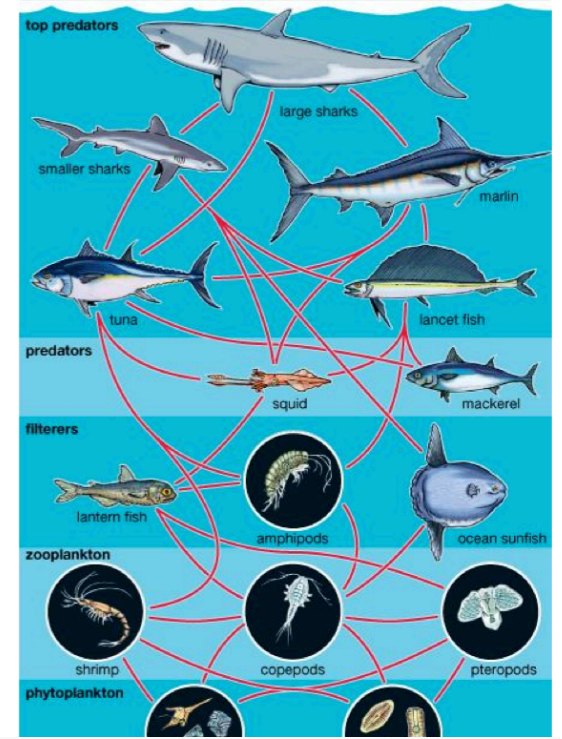
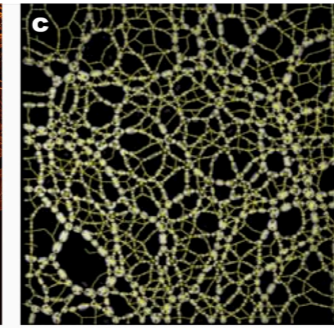
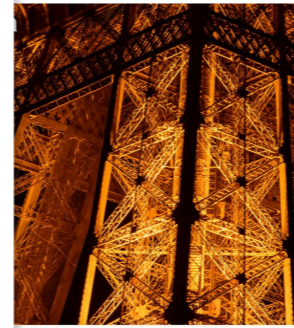
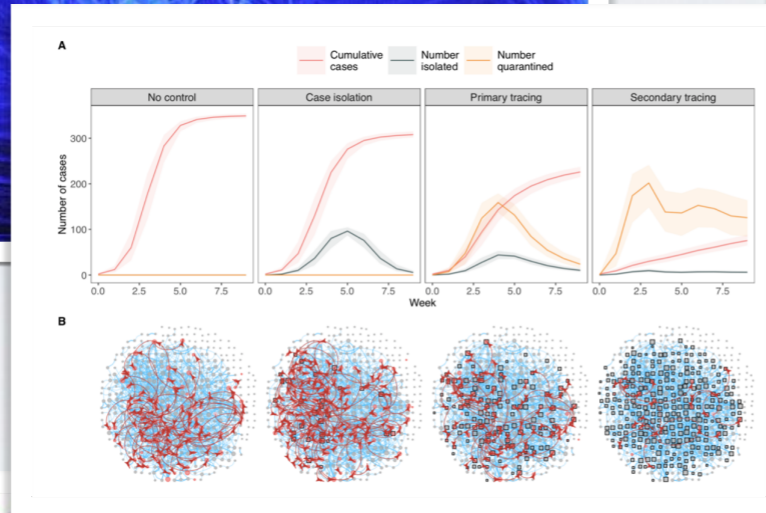
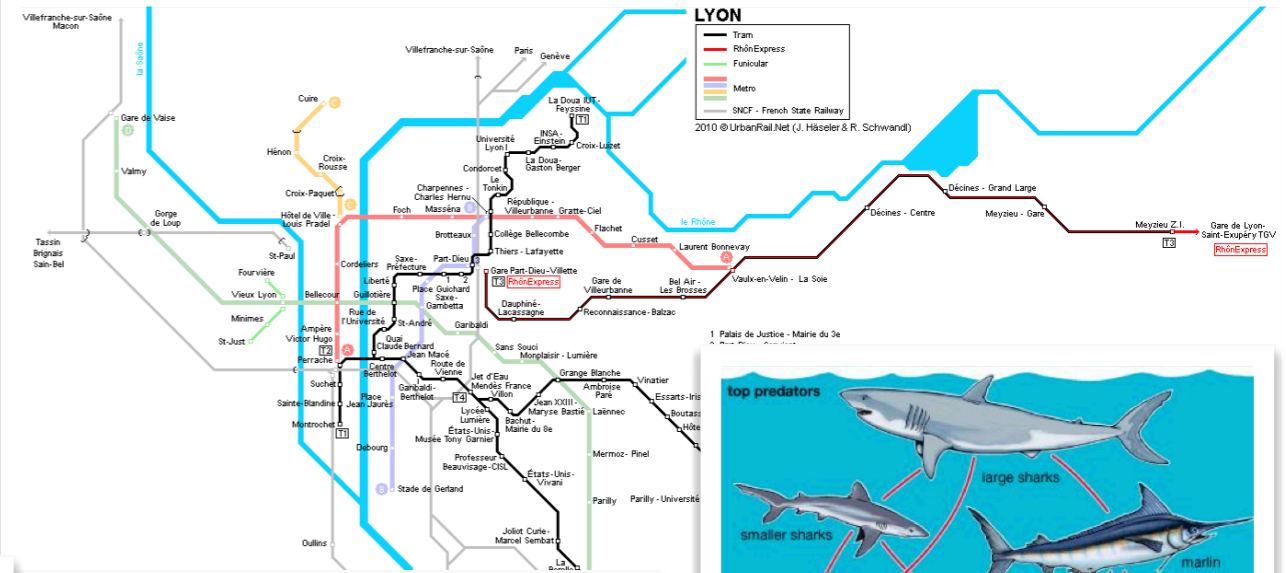
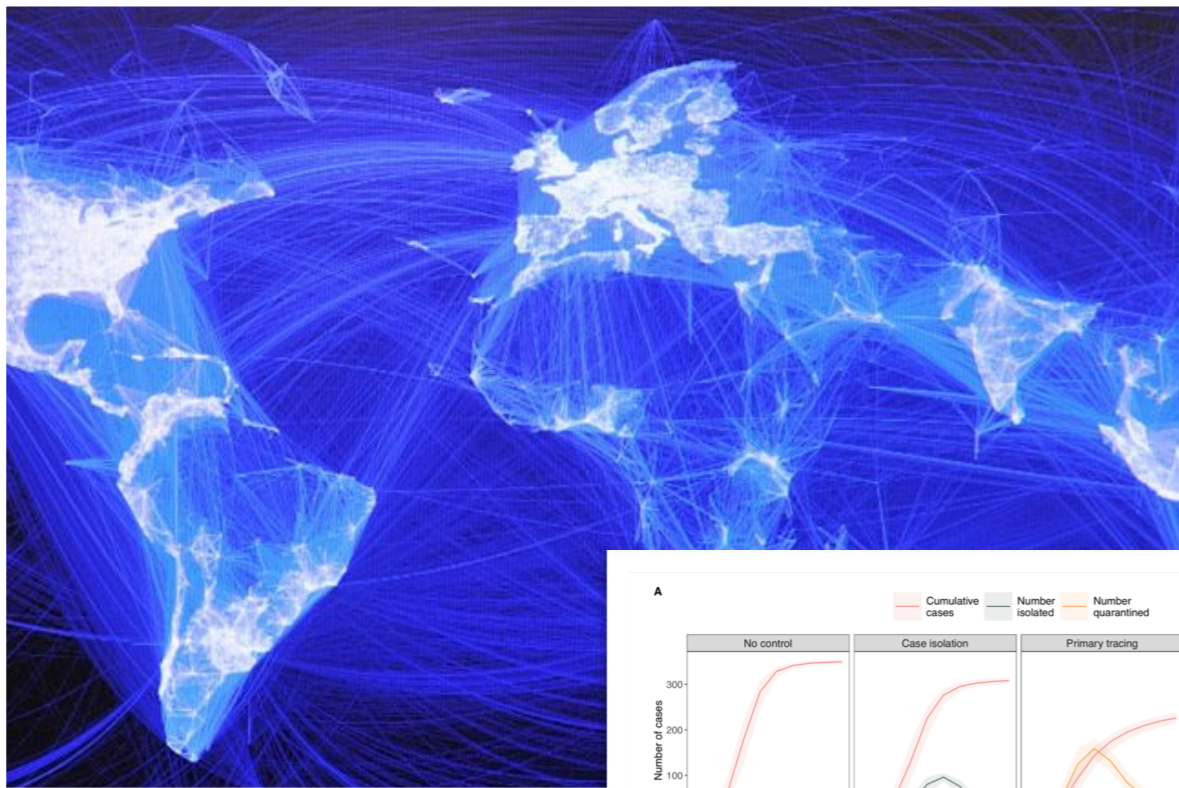


NETWORK DATA MINING

NETWORKS/GRAPHS

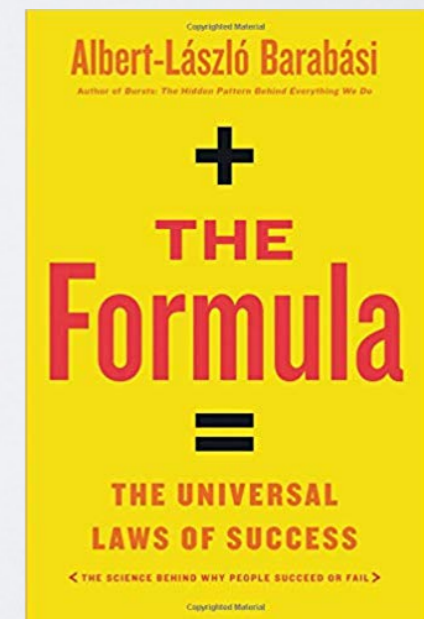
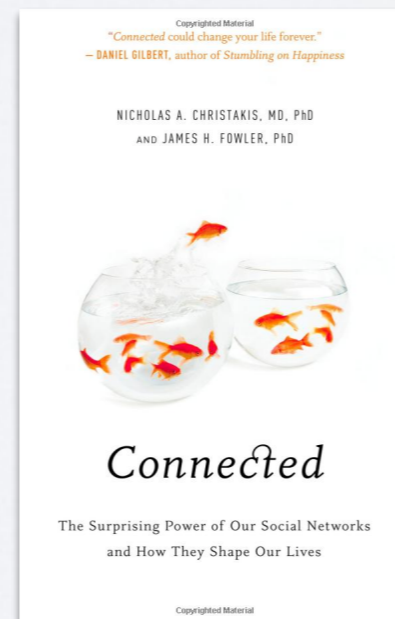
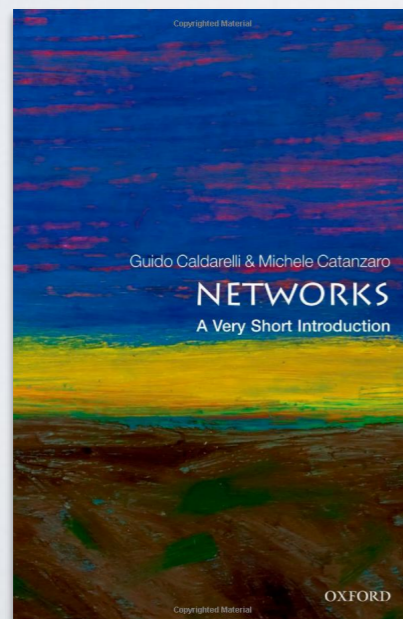
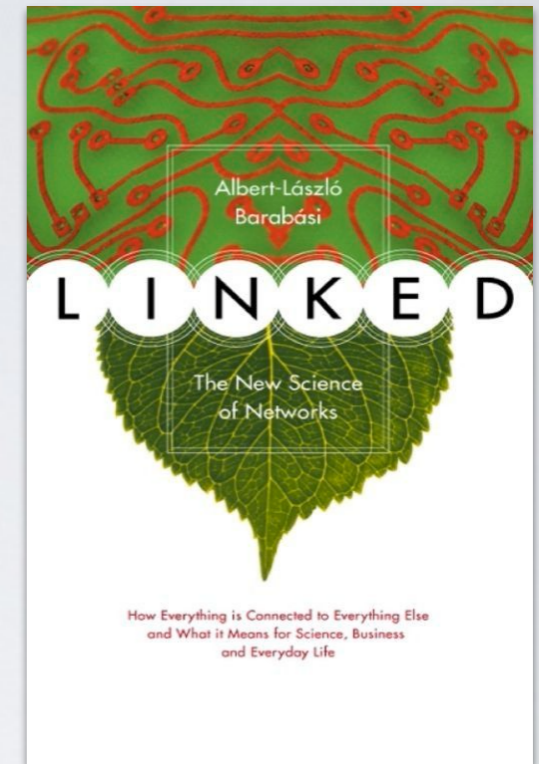
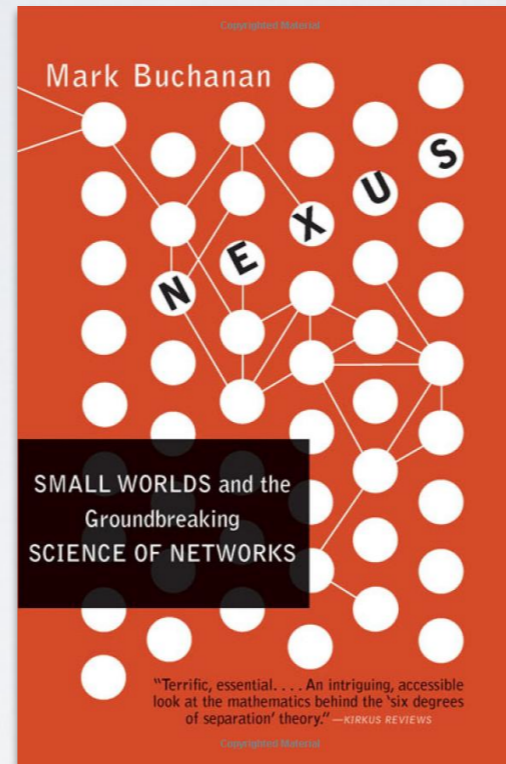
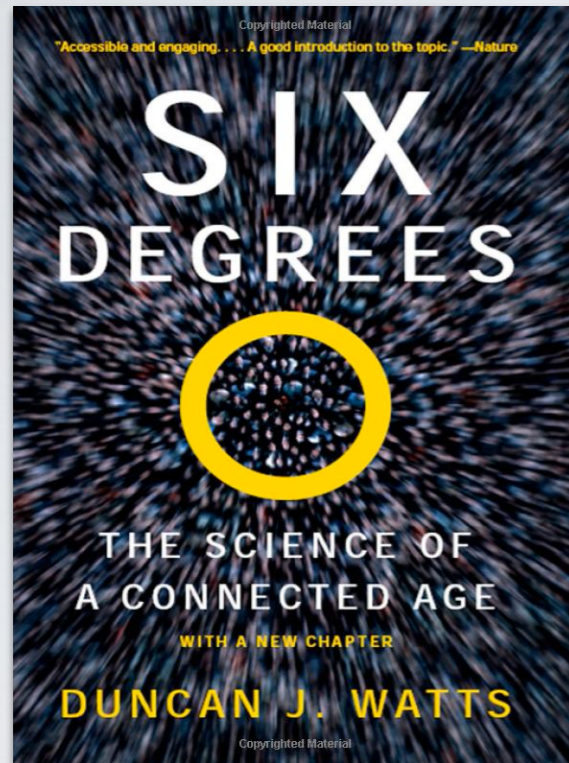
- Some data are not (only) “tabular”.
 - ▶ Items can have features that characterize them
 - Person: age, gender, etc.
 - ▶ Items also are linked/related to other items
 - This cannot be represented as a feature



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Materials

Pop-science books



I have a copy I can lend

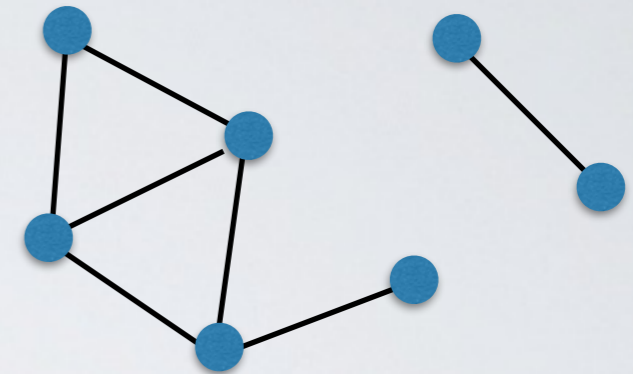
GRAPHS & NETWORKS

Networks often refers to real systems

- www,
- social network
- metabolic network.
- Language: (Network, node, link)

Graph is the mathematical representation of a network

- Language: (Graph, vertex, edge)



Vertex	Edge
person	friendship
neuron	synapse
Website	hyperlink
company	ownership
gene	regulation

In most cases we will use the two terms interchangeably.

NETWORK REPRESENTATIONS

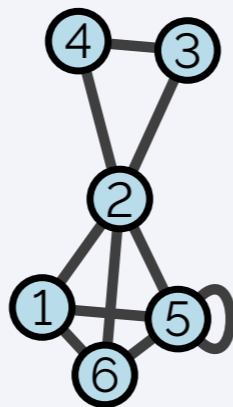
Networks: Graph notation

Graph notation : $G = (V, E)$

V	set of vertices/nodes.
E	set of edges/links.
$u \in V$	a node.
$(u, v) \in E$	an edge.

Network - Graph notation

Graph



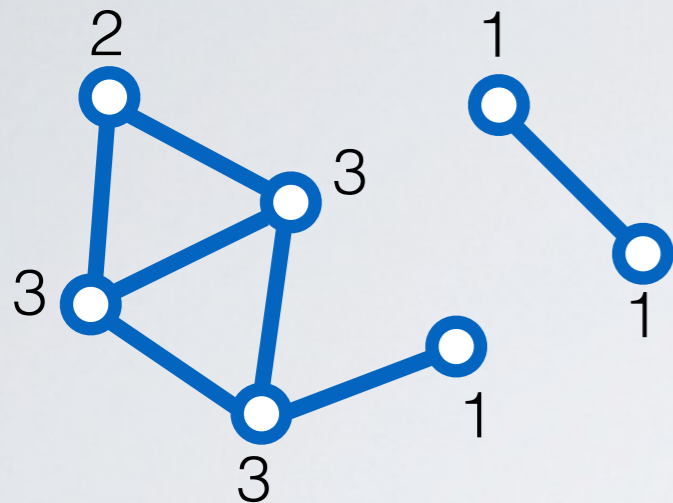
Graph notation

$G = (V, E)$
 $V = \{1, 2, 3, 4, 5, 6\}$
 $E = \{(1, 2), (1, 6),$
 $(1, 5), (2, 4), (2, 3), (2, 5),$
 $(2, 6), (6, 5), (5, 5), (4, 3)\}$

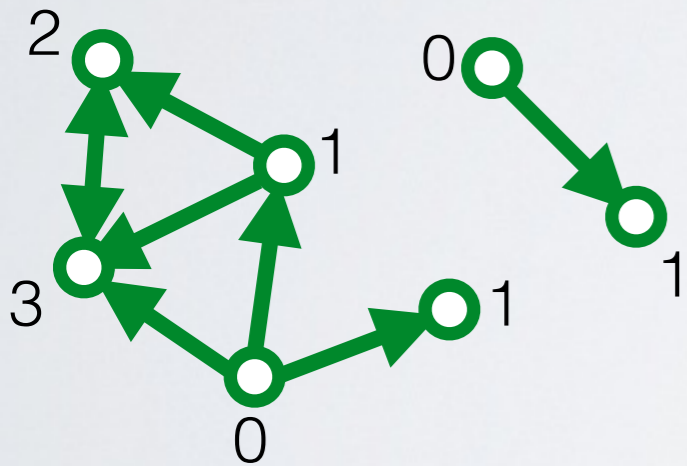
Node degree

Number of connections of a node

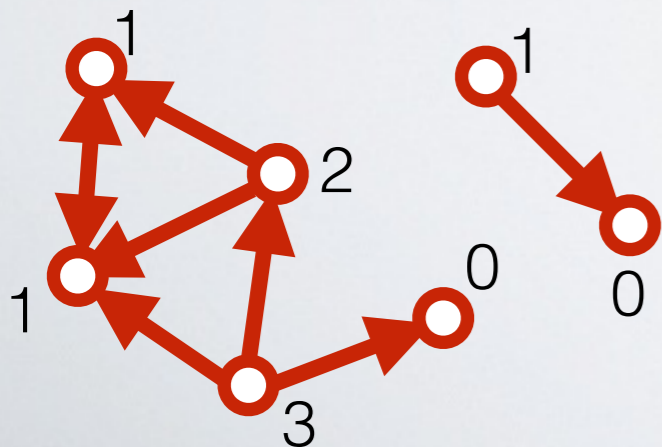
- Undirected network



- Directed network



In degree



Out degree

SIZE

Counting nodes and edges

N/n

L/m

L_{max}

size: number of nodes $|V|$.

number of edges $|E|$

Maximum number of links

Undirected network: $\binom{N}{2} = N(N - 1)/2$

Directed network: $\binom{N}{2} = N(N - 1)$

DENSITY

Network descriptors 1 - Nodes/Edges

$\langle k \rangle$

Average degree: Real networks are sparse, i.e., typically $\langle k \rangle \ll n$. Increases slowly with network size, e.g., $d \sim \log(m)$

$$\langle k \rangle = \frac{2m}{n}$$

$d/d(G)$

Density: Fraction of pairs of nodes connected by an edge in G .

$$d = L/L_{\max}$$

	#nodes	#edges	Densité	Deg. Moyen
Wikipedia	2M	30M	1.5×10^{-5}	30
Twitter 2015	288M	60B	1.4×10^{-6}	416
Facebook	1.4B	400B	4×10^{-9}	570
Brain c.	280	6393	0,16	46
Roads Calif.	2M	2.7M	6×10^{-7}	2,7
Airport	3k	31k	0,007	21

Attention: Densité difficile à comparer entre des graphes de taille différente

SUBGRAPHS

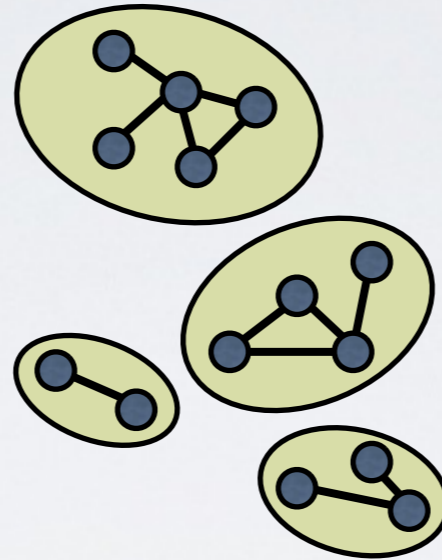


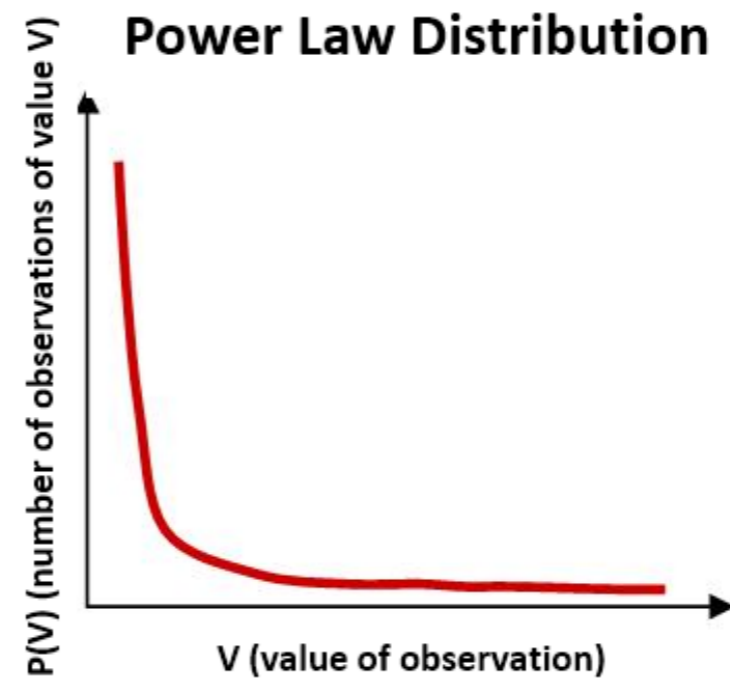
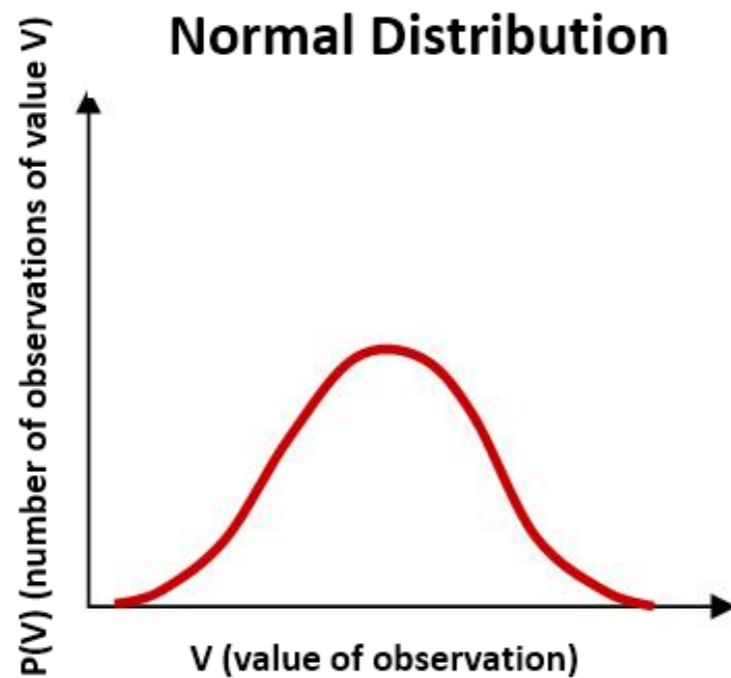
Figure after Newman, 2010

Clique: subgraph with $d = 1$

Triangle: clique of size 3

Connected component: a subgraph in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the supergraph

DEGREE DISTRIBUTION



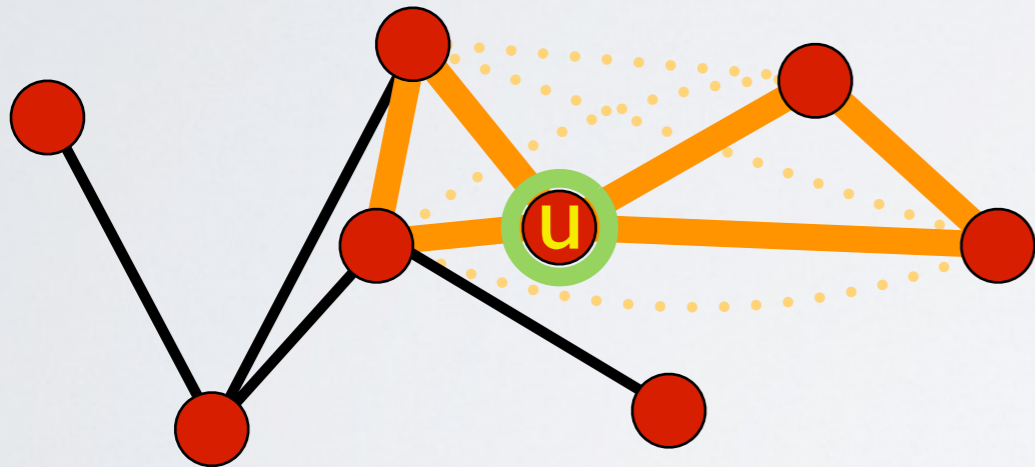
PDF (Probability Distribution Function)

CLUSTERING COEFFICIENT

- **Clustering coefficient** or **triadic closure**
- Triangles are considered important in real networks
 - ▶ Think of social networks: *friends of friends are my friends*
 - ▶ # triangles is a big difference between real and random networks

CLUSTERING COEFFICIENT

C_u - **Node clustering coefficient**: density of the subgraph induced by the neighborhood of u , $C_u = \frac{d(H(N_u))}{\binom{\delta_u}{2}}$. Also interpreted as the fraction of all possible triangles in N_u that exist, $\frac{\delta_u}{\delta_u^{\max}}$



Edges: 2
Max edges: $4 \cdot 3 / 2 = 6$
 $C_u = 2 / 6 = 1 / 3$

Triangles=2
Possible triangles = $\binom{4}{2} = 6$
 $C_u = 2 / 6 = 1 / 3$

PATH RELATED SCORES

Paths - Walks - Distance

Walk: Sequences of adjacent edges or nodes (e.g., **1.2.1.6.5** is a valid walk)

Path: a walk in which each node is distinct.

Path length: number of edges encountered in a path

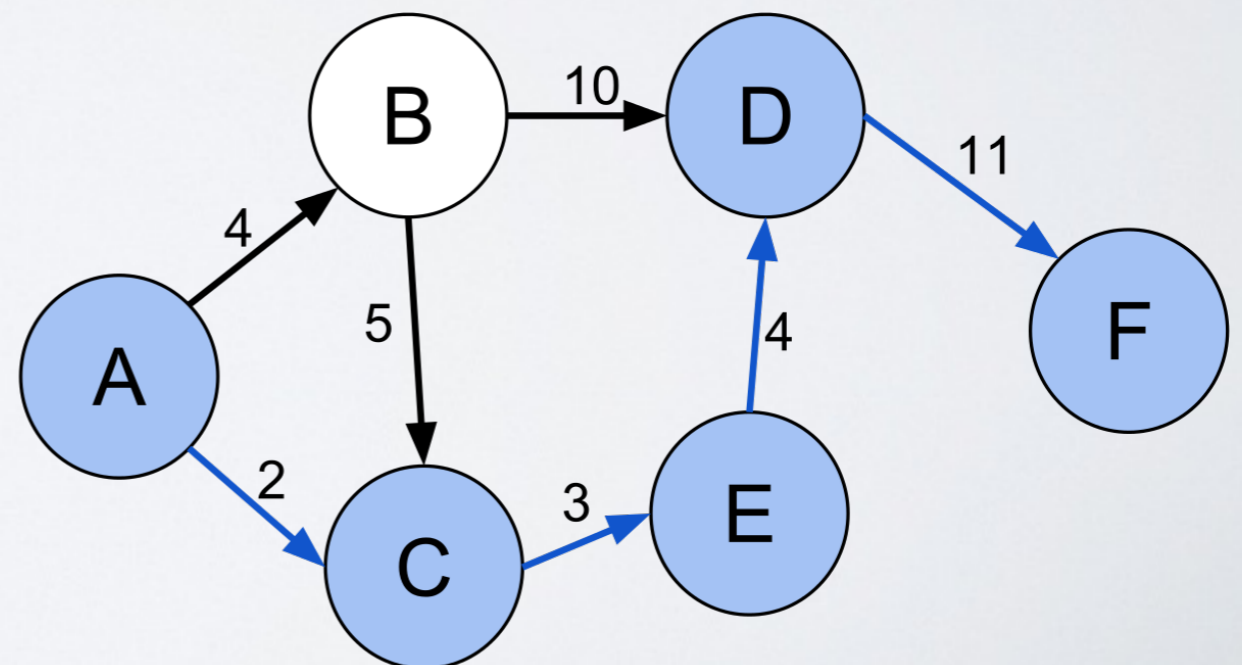
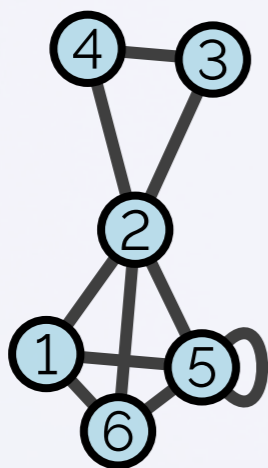
Weighted Path length: Sum of the weights of edges on a path

Shortest path: The shortest path between nodes u, v is a path of minimal *path length*. Often it is not unique.

Weighted Shortest path: path of minimal *weighted path length*.

$l_{u,v}$: **Distance:** The distance between nodes u, v is the length of the shortest path

Graph



PATH RELATED SCORES

Network descriptors 2 - Paths

l_{\max}
 $\langle l \rangle$

Diameter: maximum *distance* between any pair of nodes.

Average distance:

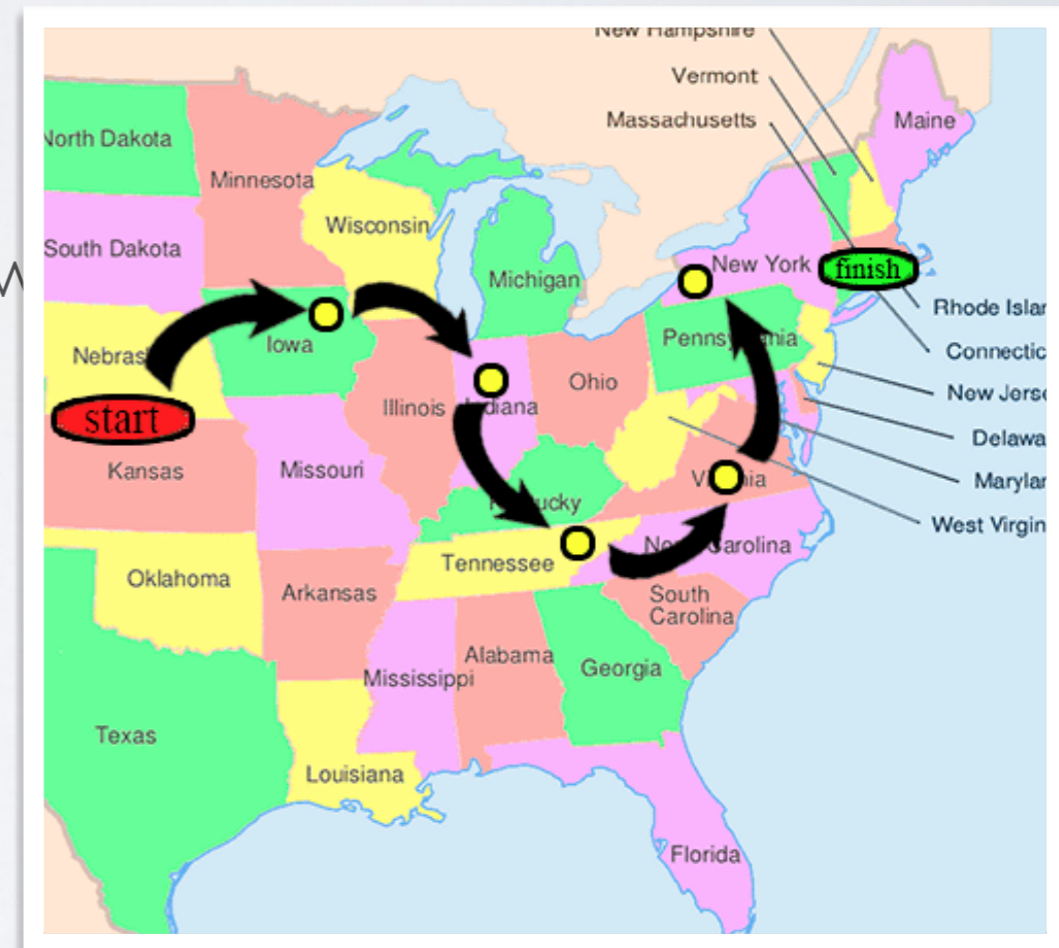
$$\langle l \rangle = \frac{1}{n(n-1)} \sum_{i \neq j} d_{ij}$$

AVERAGE PATH LENGTH

- The famous 6 degrees of separation (Milgram experiment)
 - (More on that next slide)
- Not too sensible to noise
- Tells you if the network is “stretched” or “hairball” like

SIDE-STORY: MILGRAM EXPERIMENT

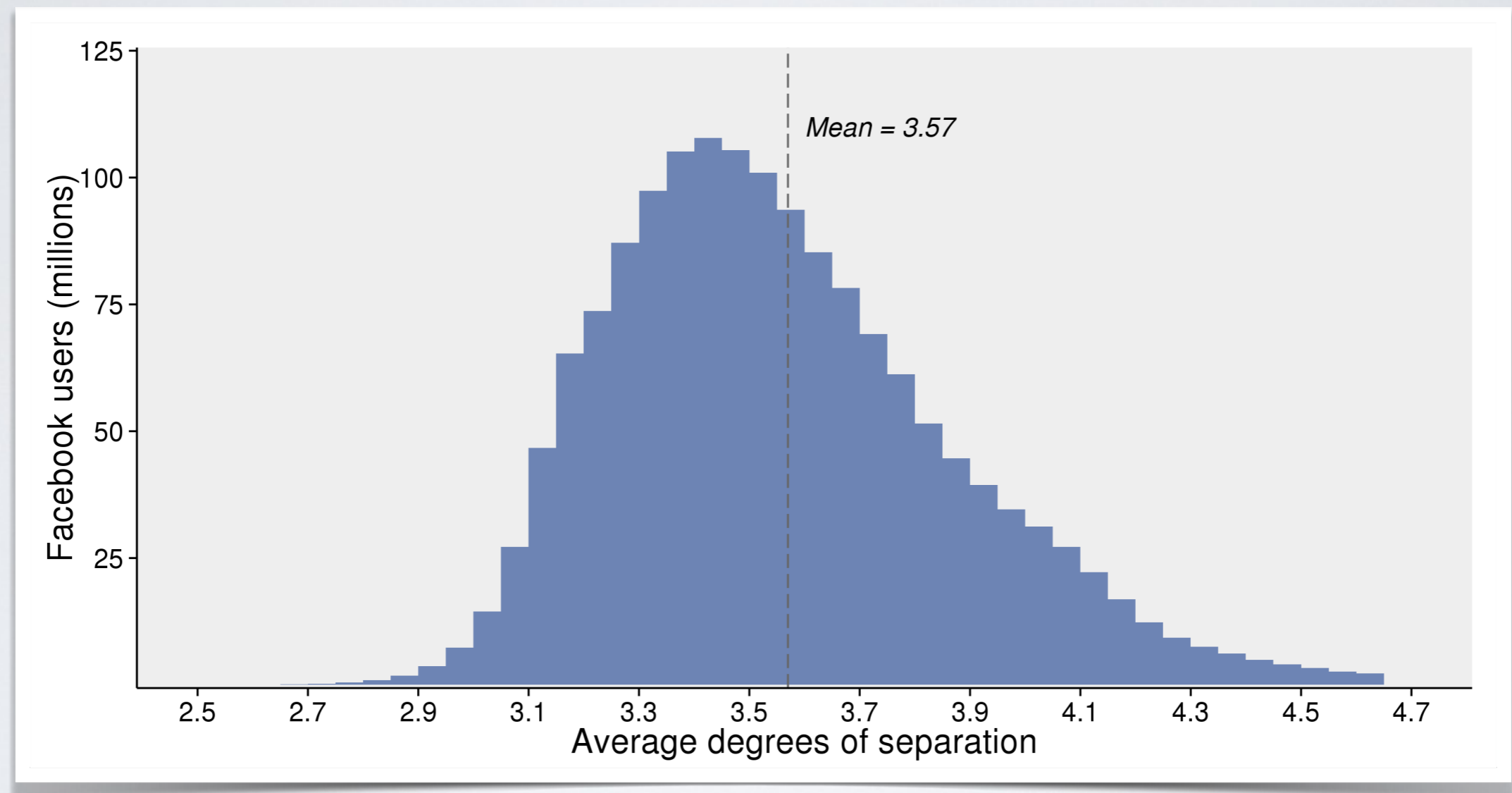
- Small world experiment (60's)
 - ▶ Give a (physical) mail to random people
 - ▶ Ask them to send to someone they don't know
 - They know his city, job
 - ▶ They send to their most relevant contact
- Results: In average, 6 hops to arrive



SIDE-STORY: MILGRAM EXPERIMENT

- Many criticism on the experiment itself:
 - ▶ Some mails did not arrive
 - ▶ Small sample
 - ▶ ...
- Checked on “real” complete graphs (giant component):
 - ▶ MSN messenger
 - ▶ Facebook
 - ▶ The world wide web
 - ▶ ...

SIDE-STORY: MILGRAM EXPERIMENT



Facebook

SMALL WORLD

Small World Network

A network is said to have the **small world** property when it has some structural properties. The notion is not quantitatively defined, but two properties are required:

- Average distance must be short, i.e., $\langle \ell \rangle \approx \log(N)$
- Clustering coefficient must be high, i.e., much larger than in a random network, e.g., $C^g \gg d$, with d the network density

ADJACENCY MATRIX

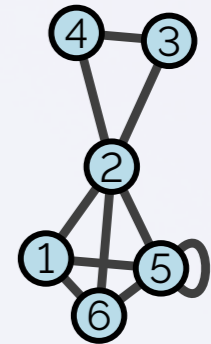
Typical operations on A

Some operations on Adjacency matrices have straightforward interpretations and are frequently used

Multiplying A by itself allows to know the number of walks of a given length that exist between any pair of nodes: A_{ij}^2 corresponds to the number of walks of length 2 from node i to node j , A_{ij}^3 to the number of walks of length 3, etc.

Multiplying A by a column vector W of length $1 \times N$ can be thought as setting the i th value of the vector to the i th node, and each node *sending* its value to its neighbors (for undirected graphs). The result is a column vector with N elements, the i th element corresponding to the sum of the values of its neighbors in W . This is convenient when working with **random walks** or **diffusion** phenomenon.

Graph



A - Adjacency Mat.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

A^2

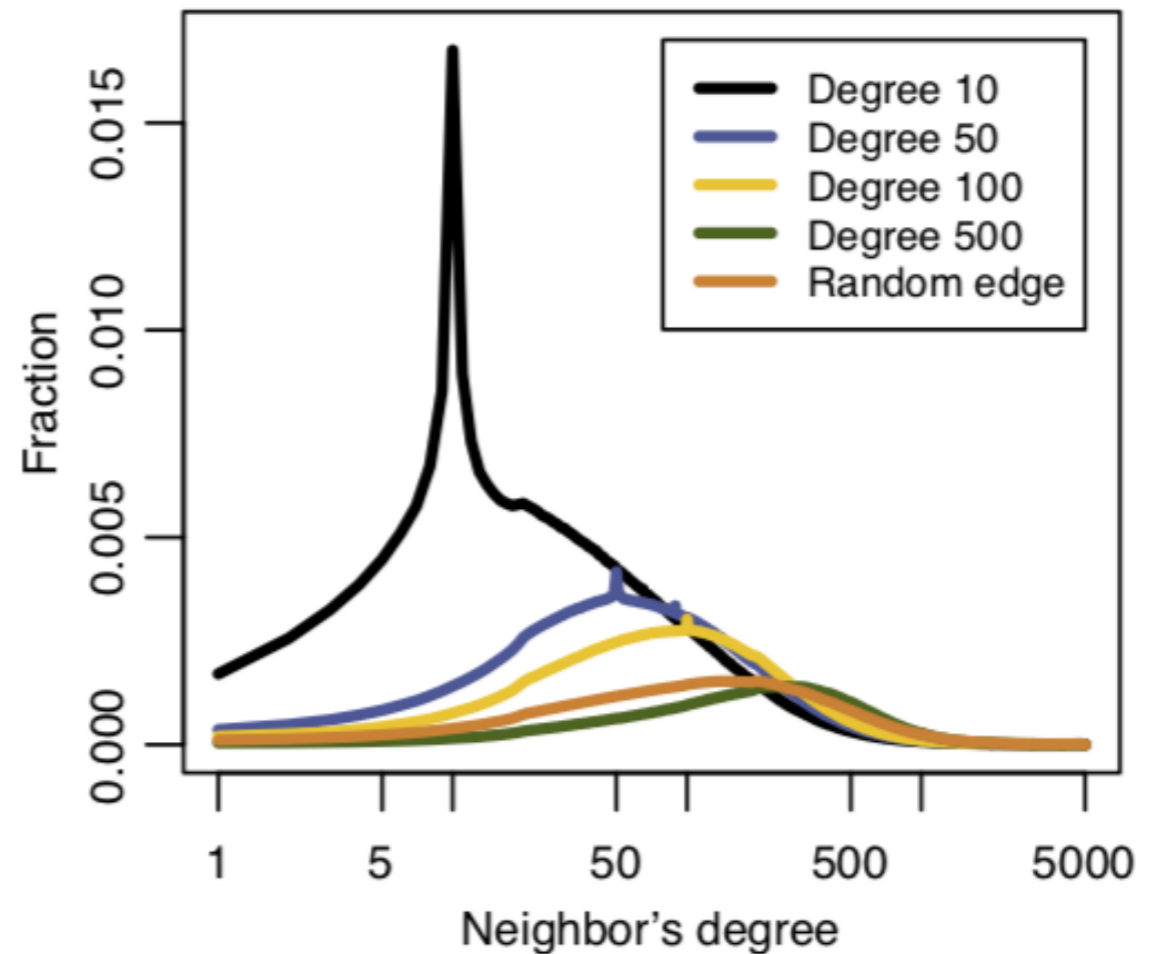
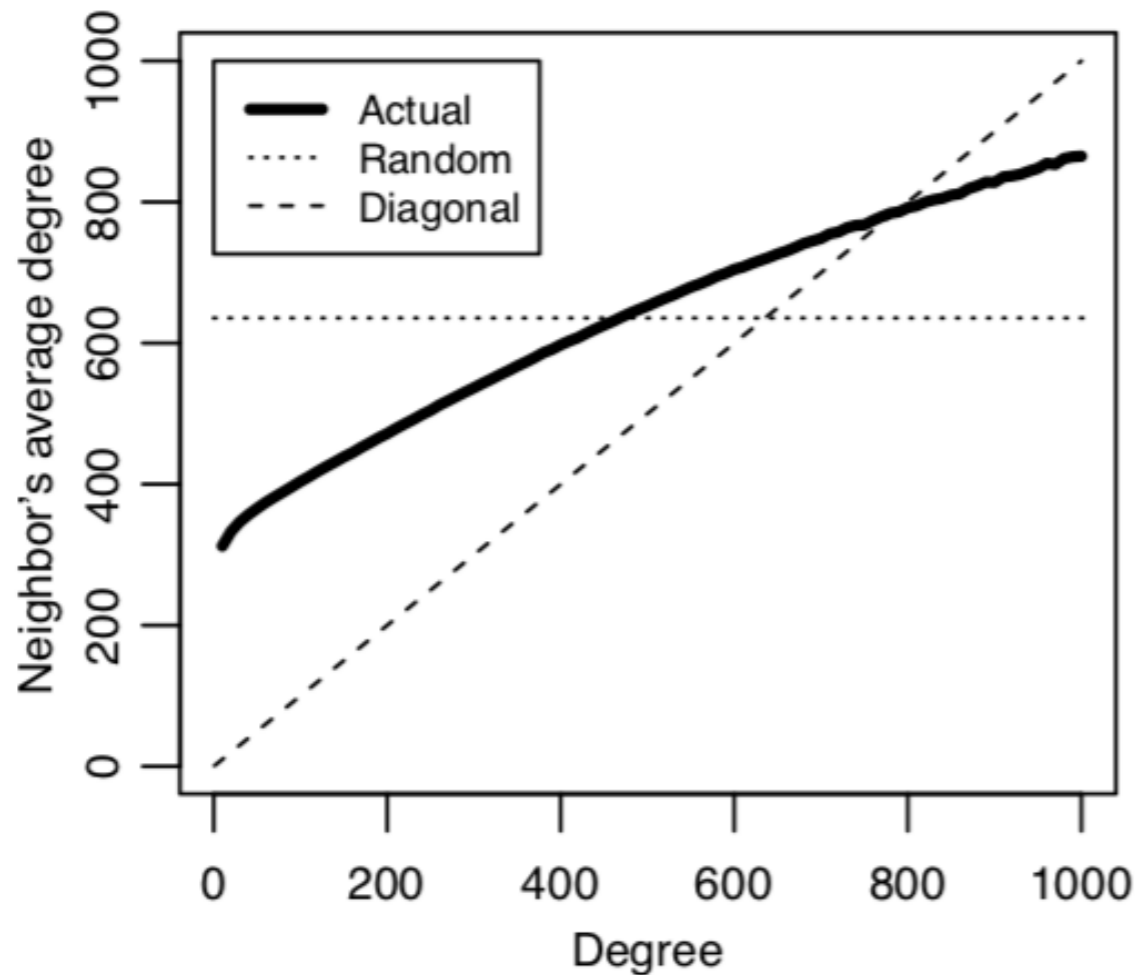
$$\begin{pmatrix} 3 & 2 & 1 & 1 & 3 & 2 \\ 2 & 5 & 1 & 1 & 3 & 2 \\ 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 \\ 3 & 3 & 1 & 1 & 4 & 3 \\ 2 & 2 & 1 & 1 & 3 & 3 \end{pmatrix}$$

EXAMPLE OF GRAPH ANALYSIS

- 721M users (nodes) (active in the last 28 days)
- 68B edges
- Average degree: 190 (average # friends)
- Median degree: 99
- Connected component: 99.91%

EXAMPLE OF GRAPH ANALYSIS

ANALYSIS

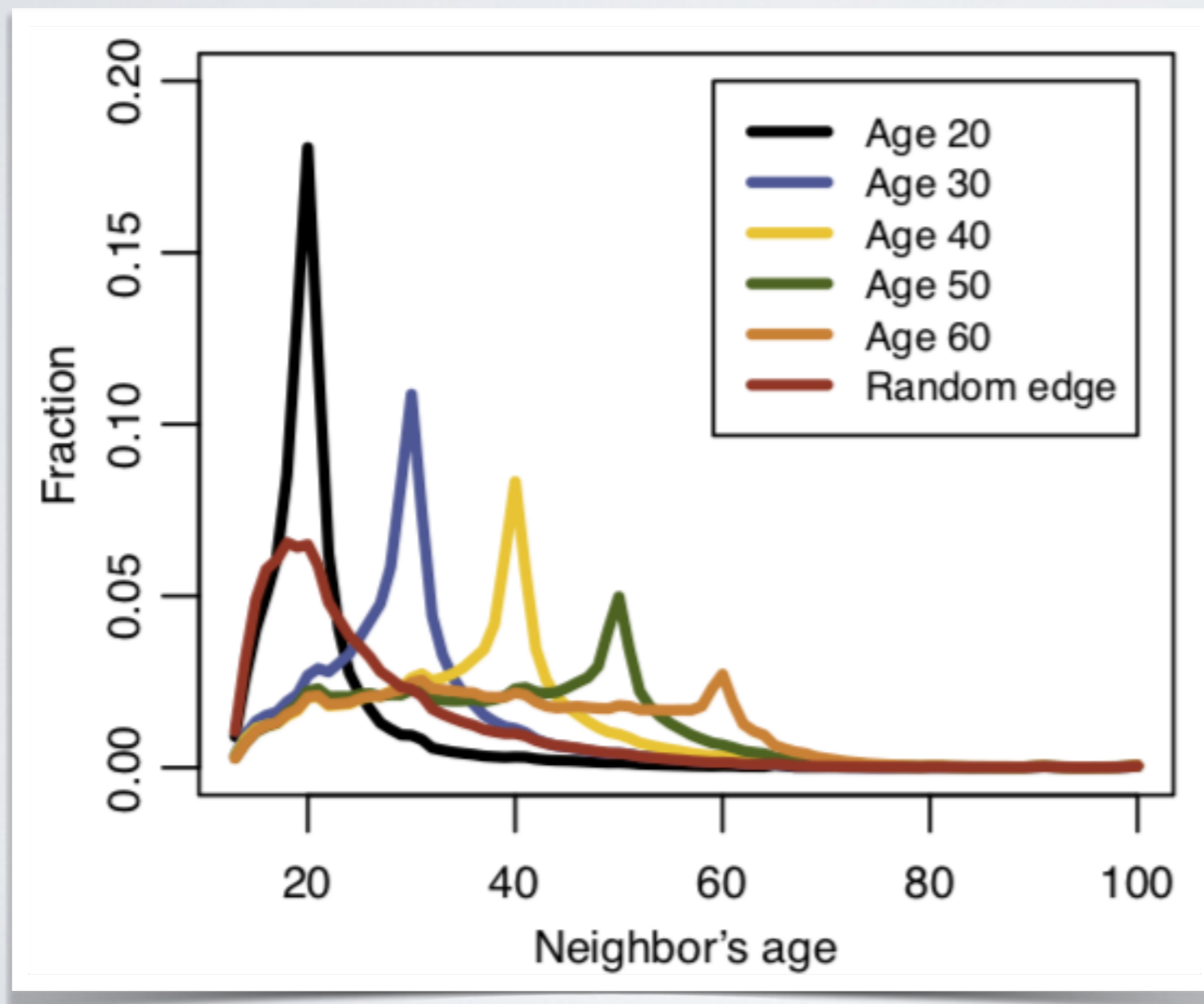


My friends have more Friends than me!

Many of my friends have the Same # of friends than me!

EXAMPLE OF GRAPH ANALYSIS

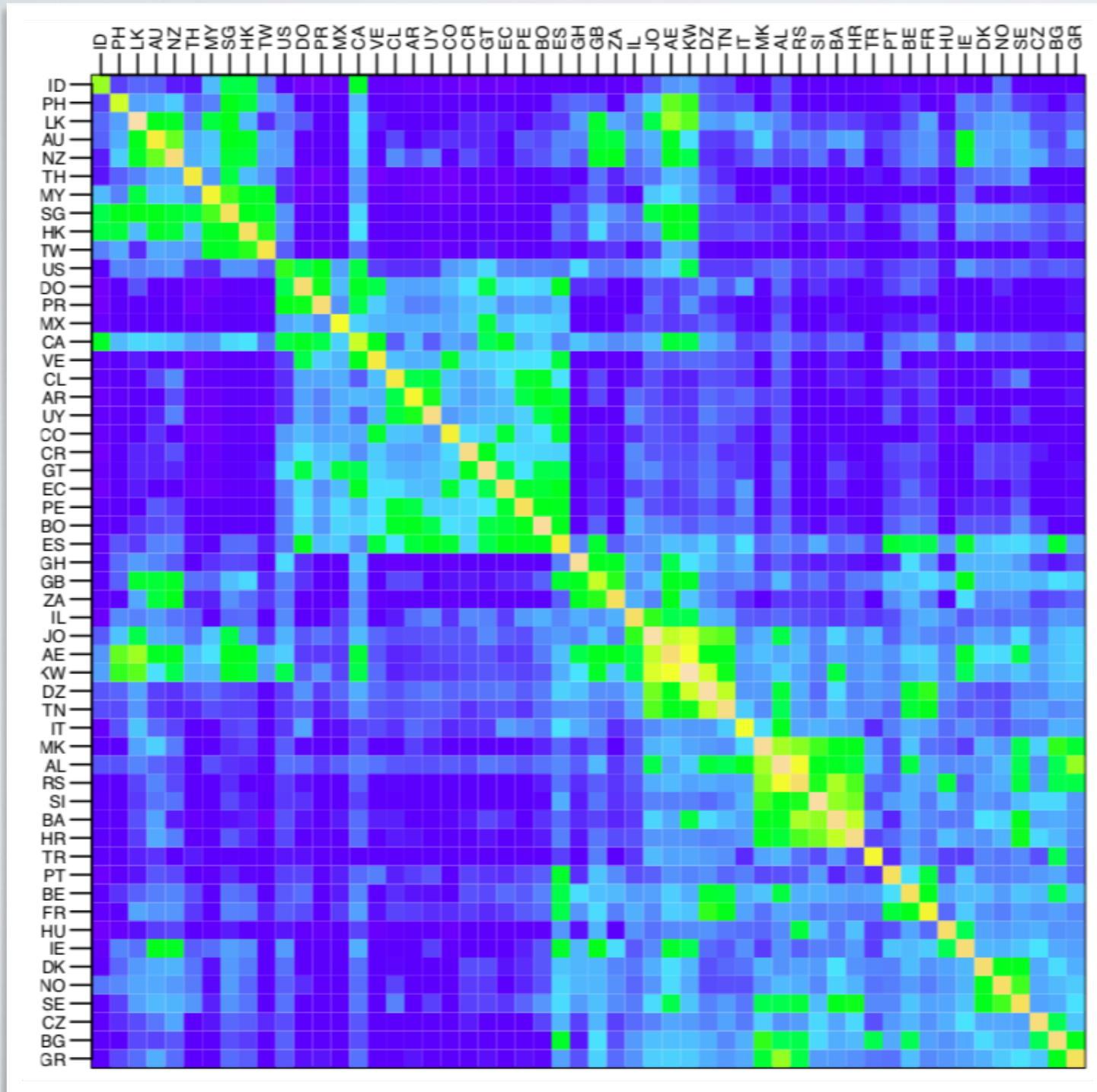
ANALYSIS



Age homophily

(More next class)

EXAMPLE OF GRAPH ANALYSIS



Country similarity

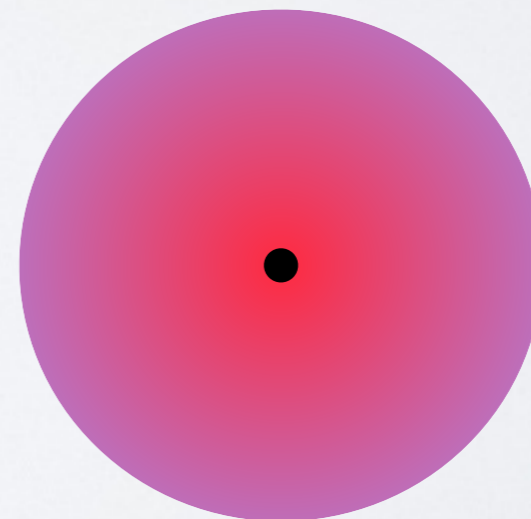
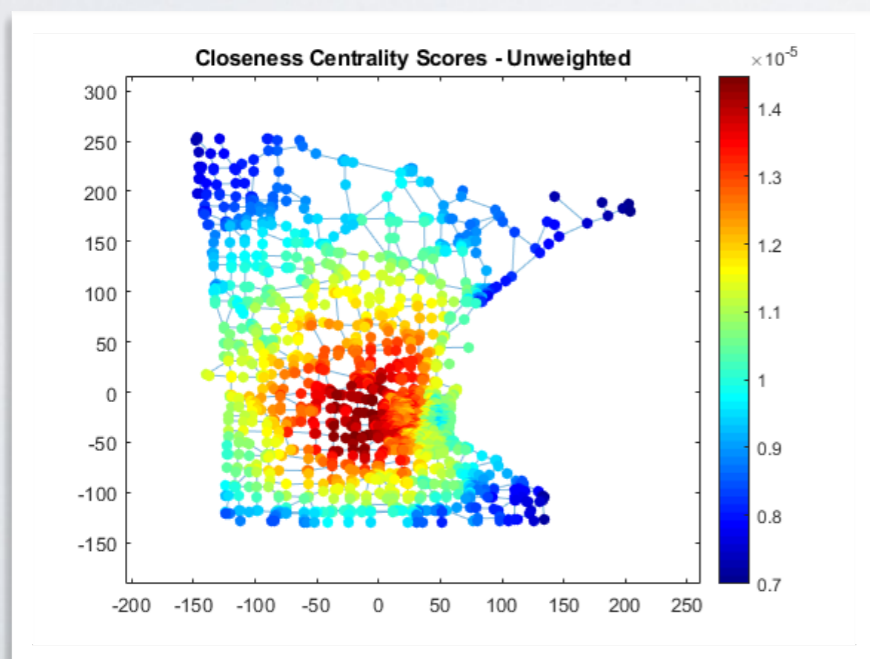
84.2% percent of edges are
within countries

(More in the community
detection class)

FARNESS, CLOSENESS
HARMONIC CENTRALITY

FARNESS, CLOSENESS

- How close the node is to all other nodes
- Parallel with the center of a figure:
 - Center of a circle is the point of shorter average distance to any points in the circle



FARNNESS, CLOSENESS

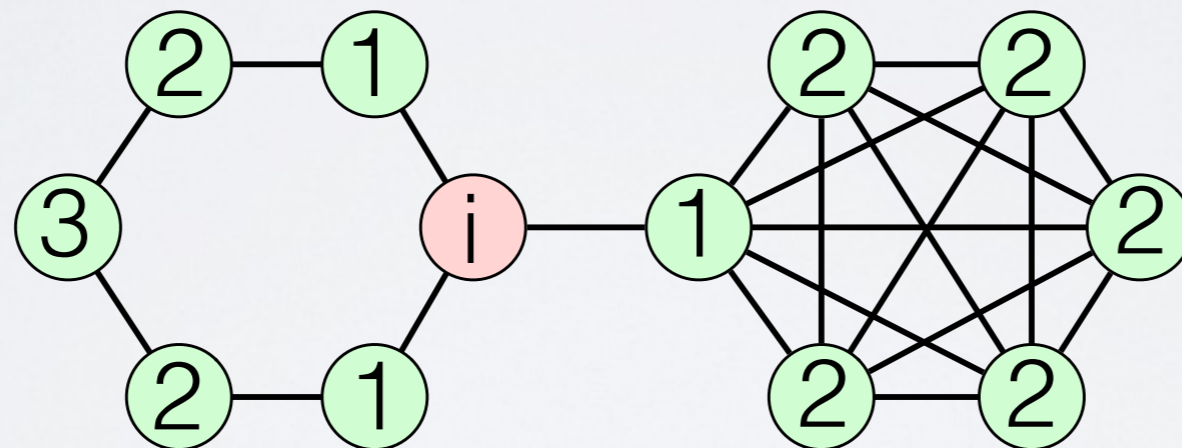
Farness: Average distance to all other nodes in the graph

$$\text{Farness}(u) = \frac{1}{N-1} \sum_{v \in V \setminus u} \ell_{u,v}$$

CLOSENESS CENTRALITY

Closeness: Inverse of the farness, i.e., how close the node is to all other nodes in term of shortest paths.

$$\text{Closeness}(u) = \frac{N - 1}{\sum_{v \in V \setminus u} l_{u,v}}$$



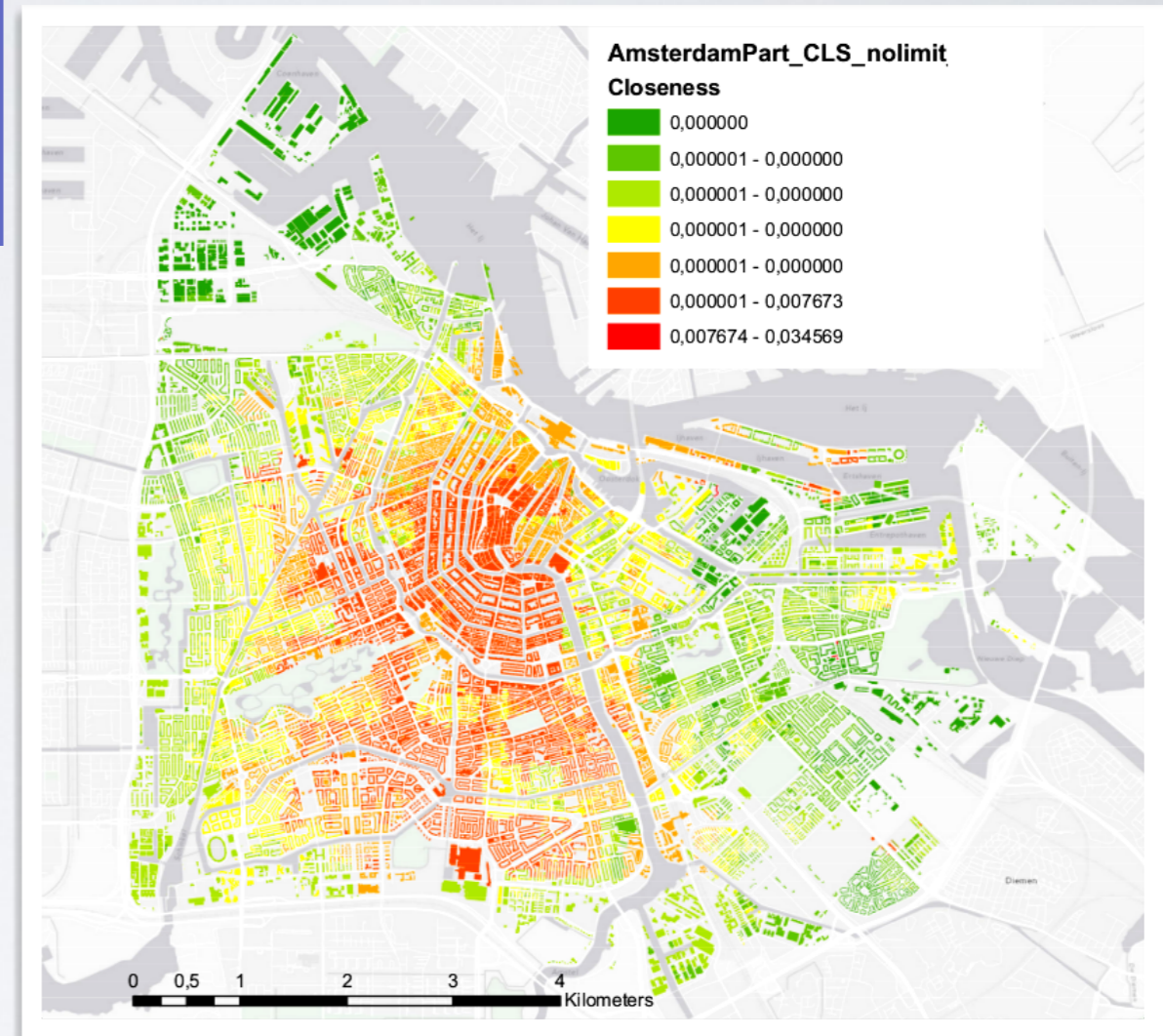
$$C_{cl}(i) = \frac{12 - 1}{(3 \times 1 + 7 \times 2 + 1 \times 3)} = \frac{11}{20} = 0.55$$

CLOSENESS CENTRALITY

Closeness: Inverse of the farness, i.e., how close the node is to all other nodes in term of shortest paths.

$$\text{Closeness}(u) = \frac{N - 1}{\sum_{v \in V \setminus u} \ell_{u,v}}$$

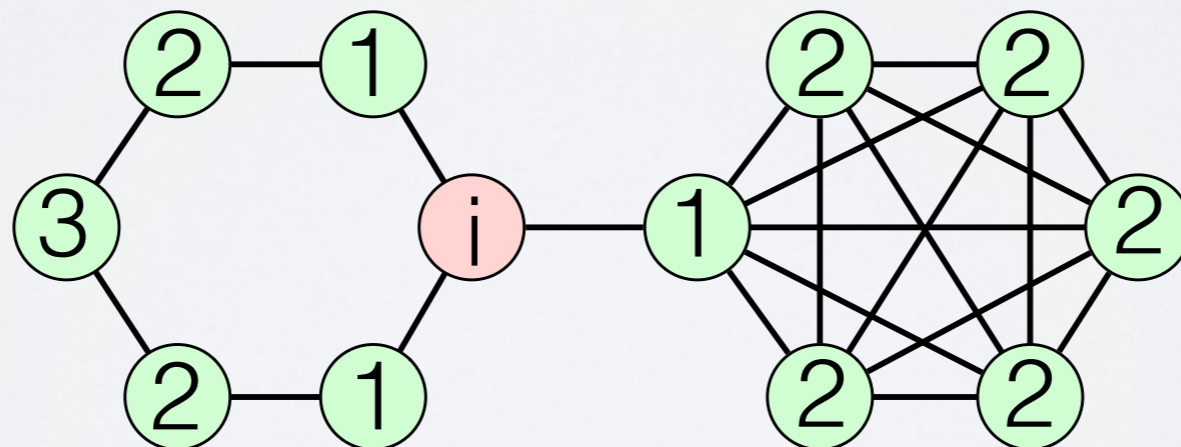
| =all nodes are at distance one



Harmonic Centrality

Harmonic centrality: A variant of the closeness defined as the average of the inverse of distance to all other nodes (Harmonic mean). Well defined on disconnected network with $\frac{1}{\infty} = 0$. Its interpretation is the same as the closeness.

$$\text{Harmonic}(u) = \frac{1}{N - 1} \sum_{v \in V \setminus u} \frac{1}{\ell_{u,v}}$$



$$C_h(i) = \frac{1}{12 - 1} \left(3 \times \frac{1}{1} + 7 \times \frac{1}{2} + 1 \times \frac{1}{3} \right) = \frac{41}{66} = 0.6212$$

BETWEENNESS CENTRALITY

- Measure how much the node plays the role of a bridge
- Betweenness of u : fraction of all the shortest paths between all the pairs of nodes going through u .

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

with σ_{st} the number of shortest paths between nodes s and t and $\sigma_{st}(v)$ the number of those paths passing through v .

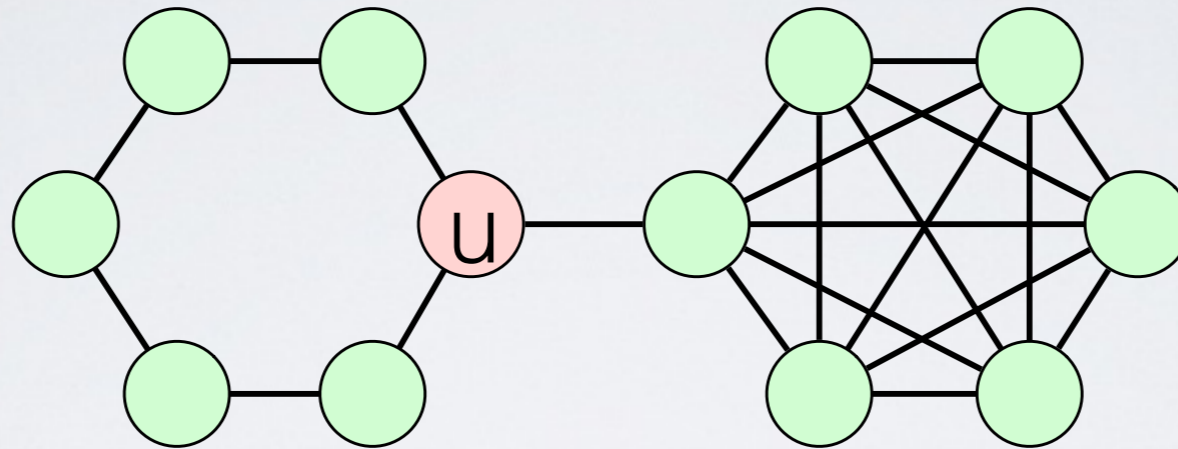
The betweenness tends to grow with the network size. A normalized version can be obtained by dividing by the number of pairs of nodes, i.e., for a

directed graph: $C_B^{\text{norm}}(v) = \frac{C_B(v)}{(N-1)(N-2)}$.

Betweenness Centrality

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

directed graph: $C_B^{\text{norm}}(v) = \frac{C_B(v)}{(N-1)(N-2)}$.



$$C_B(u) = 2 \frac{5 * 6 + 1 + \frac{1}{2} + \frac{1}{2}}{11 * 10} = \frac{64}{110}$$

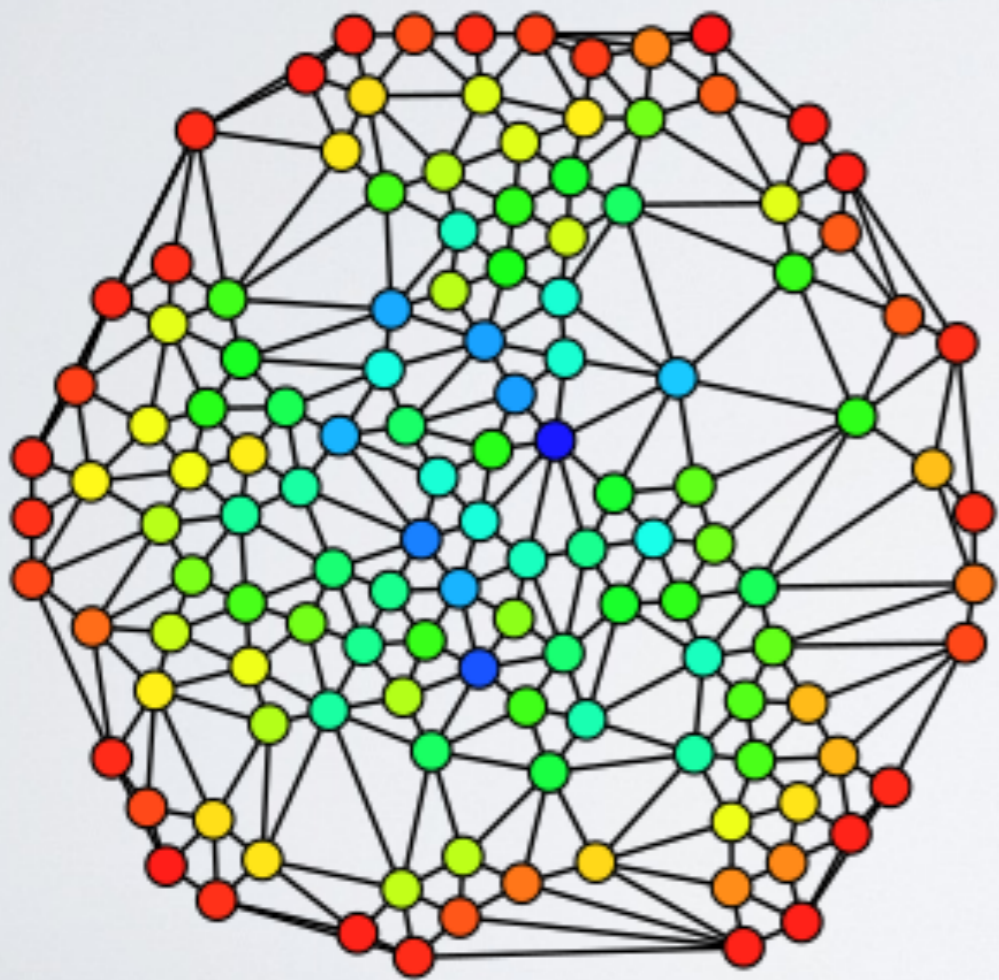
Exact computation:

Floyd-Warshall: $O(n^3)$ time complexity
 $O(n^2)$ space complexity

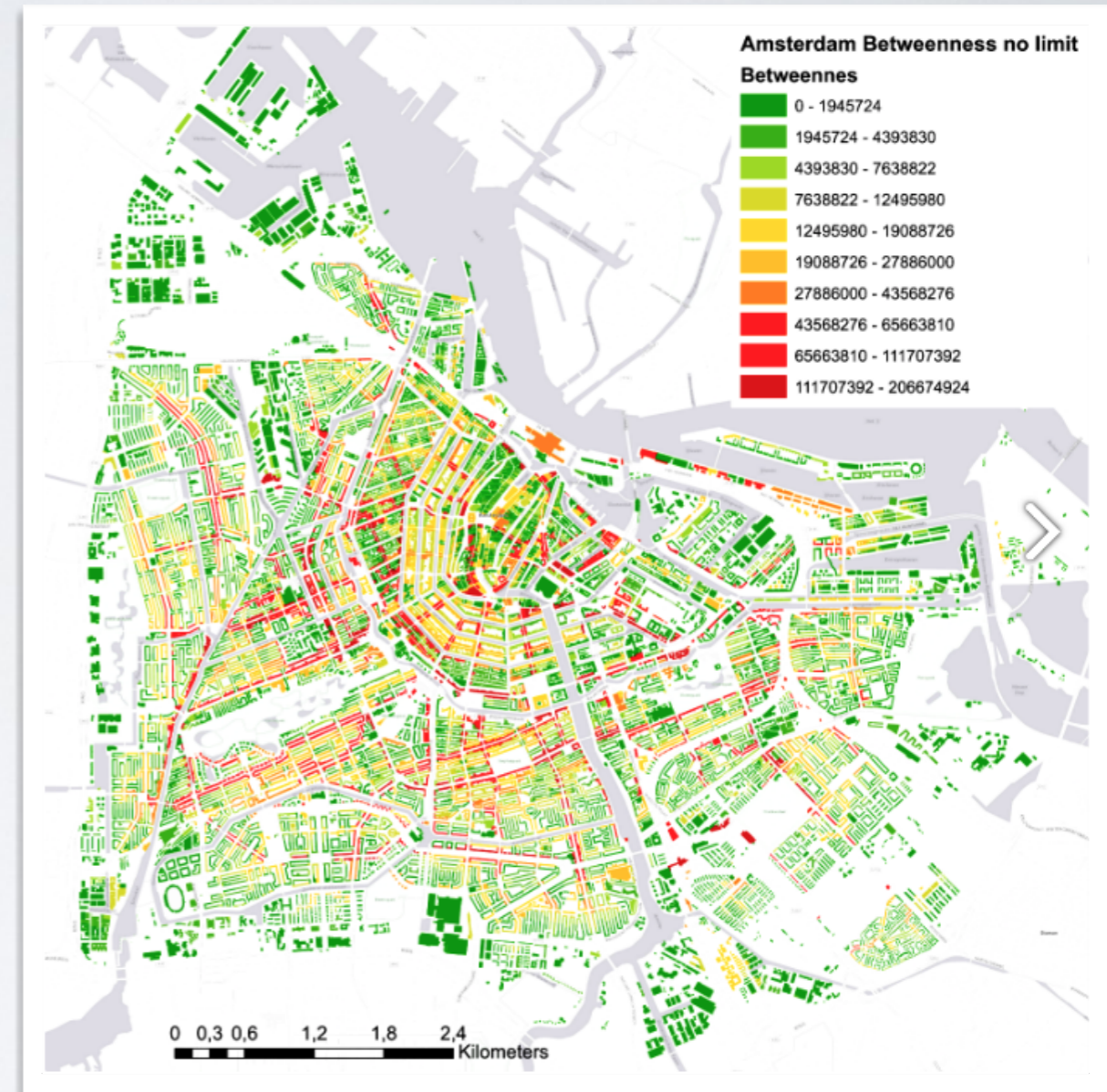
Approximate computation

Dijkstra: $O(n(m+n \log n))$ time complexity

BETWEENNESS CENTRALITY



(blue higher)



(red higher)

EDGE - BETWEENNESS

Same definition as for nodes

Can you guess the edge of highest betweenness in the European rail network?



RECURSIVE DEFINITIONS

RECURSIVE DEFINITIONS

- Recursive importance:
 - **Important nodes** are those connected **to important nodes**
- Several centralities based on this idea:
 - Eigenvector centrality
 - PageRank
 - ...

RECURSIVE DEFINITION

- We would like scores such as :
 - Each node has a score (centrality),
 - If every node “sends” its score to its neighbors, the sum of all scores received by each node will be equal to its original score

$$C_u^{t+1} = \frac{1}{\lambda} \sum_{v \in N_u^{in}} C_v^t \quad (1)$$

- With λ a normalisation constant

RECURSIVE DEFINITION

- This problem can be solved by what is called the *power method*:
 - 1) We initialize all scores to random values
 - 2) Each score is updated according to the desired rule, until reaching a stable point (after normalization)
- Why does it converge?
 - Perron-Frobenius theorem (see next slide)
 - => True for undirected graphs with a single connected component

EIGENVECTOR CENTRALITY

- What we just described is called the Eigenvector centrality
- A couple eigenvector (x) and eigenvalue (λ) is defined by the following relation: $Ax = \lambda x$
 - x is a column vector of size n , which can be interpreted as the scores of nodes
- What Perron-Frobenius algorithm says is that the power method will always converge to the *leading eigenvector*, i.e., the eigenvector associated with the highest eigenvalue

PageRank Centrality

- Eigenvector centrality generalised for directed networks

PageRank

The Anatomy of a Large-Scale Hypertextual Web Search Engine

Brin, S. and Page, L. (1998) The Anatomy of a Large-Scale Hypertextual Web Search Engine. In: Seventh International World-Wide Web Conference (WWW 1998), April 14-18, 1998, Brisbane, Australia.

Sergey Brin and Lawrence Page

*Computer Science Department,
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sergey@cs.stanford.edu and page@cs.stanford.edu*

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- Eigenvector centrality generalised for directed networks

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Abstract

In this paper, we present Google, a prototype of a large-scale search engine which makes heavy use of the structure present in hypertext. Google is designed to crawl and index the Web efficiently and produce much more satisfying search results than existing systems. The prototype with a full text and hyperlink database of at least 24 million pages is available at <http://google.stanford.edu/>

PageRank Centrality

(Side notes)

-“We chose our system name, Google, because it is a common spelling of googol, or 10^{100} and fits well with our goal of building very large-scale search “

-“[...] at the same time, search engines have migrated from the academic domain to the commercial. **Up until now most search engine development has gone on at companies with little publication of technical details. This causes search engine technology to remain largely a black art and to be advertising oriented (see Appendix A). With Google, we have a strong goal to push more development and understanding into the academic realm.**”

-“[...], we expect that advertising funded search engines will be inherently biased towards the advertisers and away from the needs of the consumers.”

PageRank Centrality

(Side notes)



Sergey Brin received his B.S. degree in mathematics and computer science from the University of Maryland at College Park in 1993. Currently, he is a Ph.D. candidate in computer science at Stanford University where he received his M.S. in 1995. He is a recipient of a National Science Foundation Graduate Fellowship. His research interests include search engines, information extraction from unstructured sources, and data mining of large text collections and scientific data.



Lawrence Page was born in East Lansing, Michigan, and received a B.S.E. in Computer Engineering at the University of Michigan Ann Arbor in 1995. He is currently a Ph.D. candidate in Computer Science at Stanford University. Some of his research interests include the link structure of the web, human computer interaction, search engines, scalability of information access interfaces, and personal data mining.

PAGERANK

- 2 main improvements over eigenvector centrality:
 - ▶ In directed networks, problem of source nodes
 - => Add a constant centrality gain for every node
 - ▶ Nodes with very high centralities give very high centralities to all their neighbors (even if that is their only in-coming link)
 - => What each node “is worth” is divided equally among its neighbors (normalization by the degree)

$$C_u^{t+1} = \frac{1}{\lambda} \sum_{v \in N_u^{in}} C_v^t$$

=>

$$C_u^{t+1} = \alpha \sum_{v \in N_u^{in}} \frac{C_v^t}{k_v^{out}} + \beta$$

With by convention $\beta=1$ and α a parameter (usually 0.85) controlling the relative importance of β

PAGERANK

Matrix interpretation

Principal eigenvector of the “Google Matrix”:

First, define matrix S as:

- Normalization by columns of A
- Columns with only 0 receives $1/n$

-Finally, $G_{ij} = \alpha S_{ij} + (1 - \alpha)/n$

$$(a) \quad A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$(c) \quad S = \begin{pmatrix} 0 & 1/2 & 1/3 & 0 & 1/5 \\ 1 & 0 & 1/3 & 1/3 & 1/5 \\ 0 & 1/2 & 0 & 1/3 & 1/5 \\ 0 & 0 & 1/3 & 0 & 1/5 \\ 0 & 0 & 0 & 1/3 & 1/5 \end{pmatrix}$$

$$(e) \quad G = \begin{pmatrix} 0.03 & 0.455 & 0.313 & 0.03 & 0.2 \\ 0.88 & 0.03 & 0.313 & 0.313 & 0.2 \\ 0.03 & 0.455 & 0.03 & 0.313 & 0.2 \\ 0.03 & 0.03 & 0.313 & 0.03 & 0.2 \\ 0.03 & 0.03 & 0.03 & 0.313 & 0.2 \end{pmatrix}$$

Graph	A - Adjacency Mat.	Random W. mat.
	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{1}{5} & 0 & 0 & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{3} \\ 0 & \frac{1}{5} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{5} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{5} & 0 & 0 & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{5} & 0 & 0 & \frac{1}{4} & 0 \end{pmatrix}$

PageRank - as Random Walk

Main idea: The PageRank computation can be interpreted as a Random Walk process with restart

Teleportation probability: the parameter α gives the probability that in the next step of the RW will follow a Markov process or with probability $1-\alpha$ it will jump to a random node

Pagerank score of a node thus corresponds to the probability of this random walker to be on this node after an infinite number of hops.

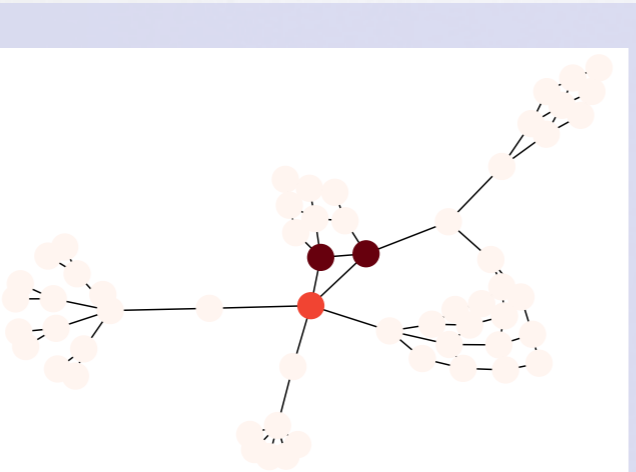
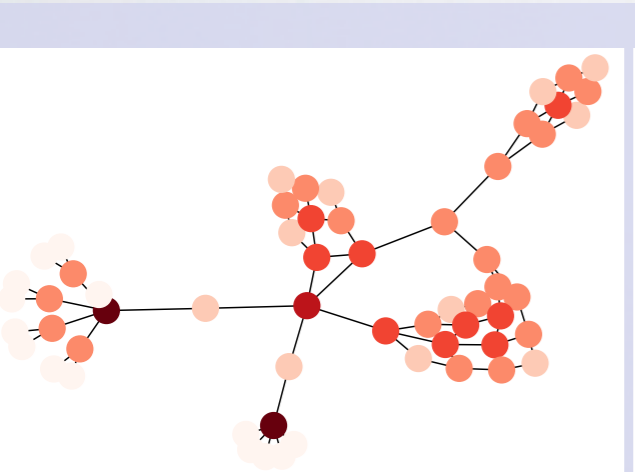
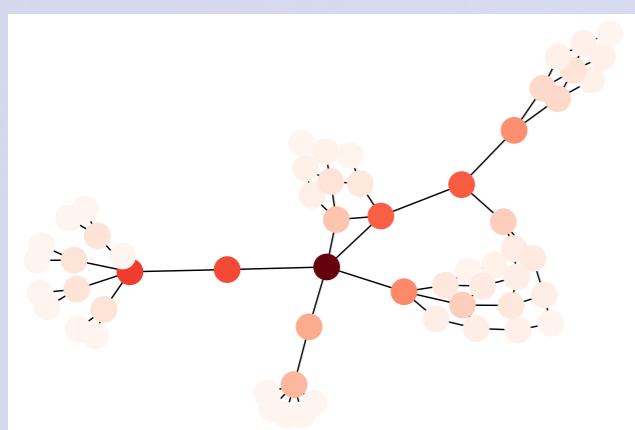
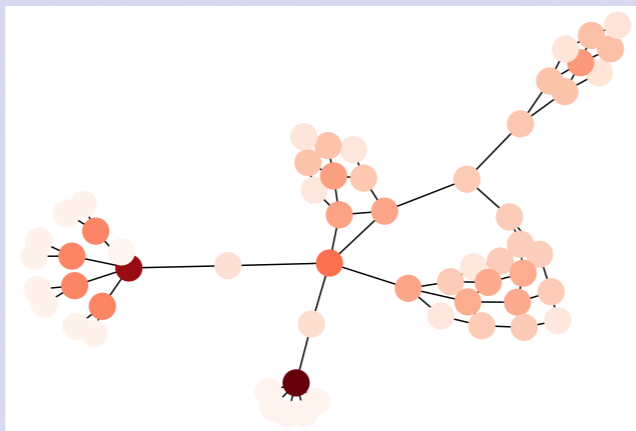
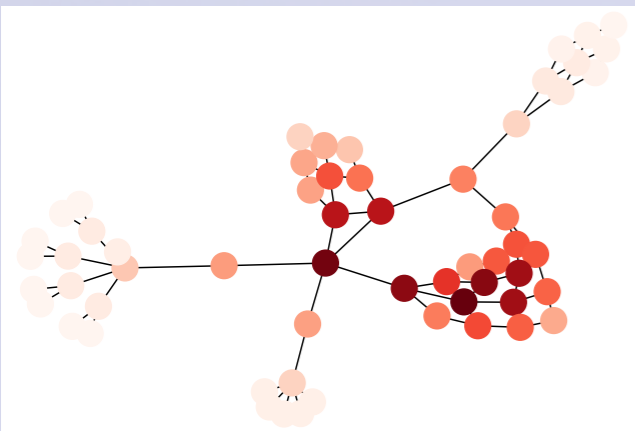
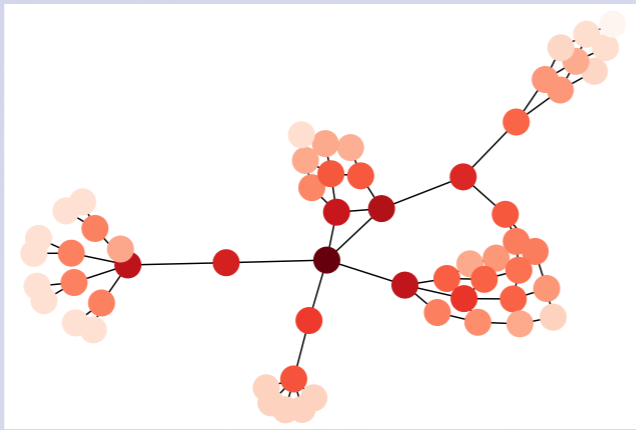
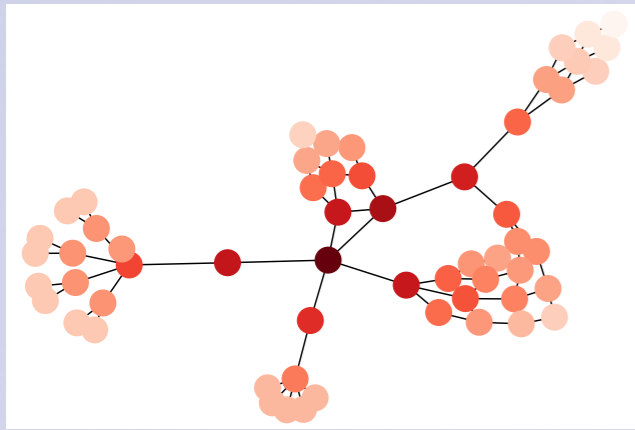
PAGERANK

- Then how do Google rank when we do a research?
- Compute pagerank (using the power method for scalability)
- Create a subgraph of documents related to our topic
- Of course now it is certainly much more complex, but we don't really know:
“Most search engine development has gone on at companies with little publication of technical details. This causes search engine technology to remain largely a black art” [Page, Brin, 1997]

OTHERS

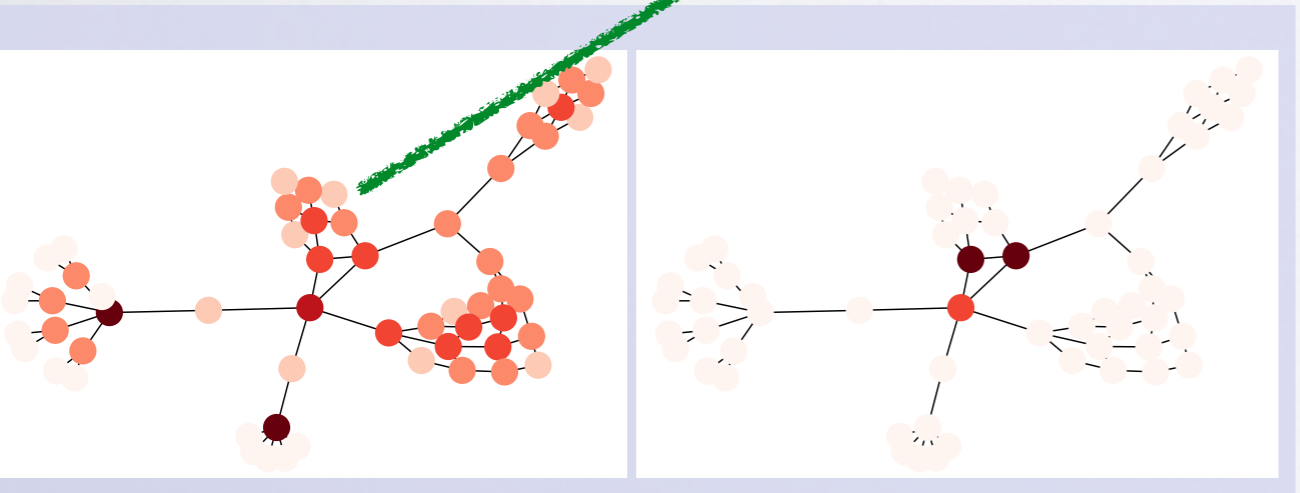
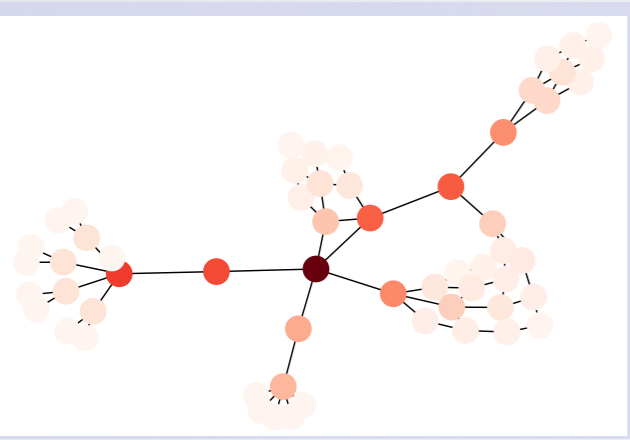
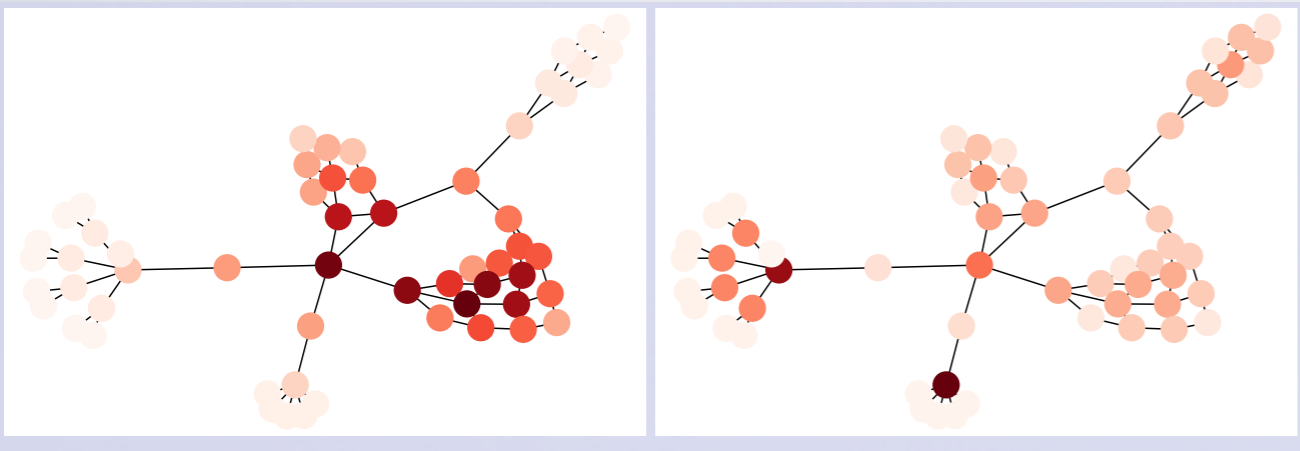
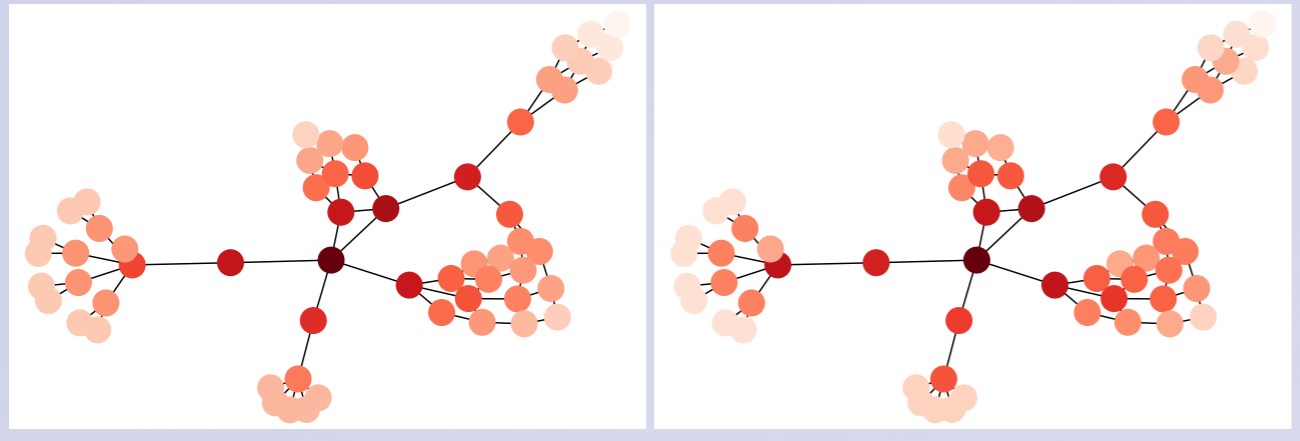
- Many other centralities have been proposed
- The problem is how to interpret them ?
- Can be used as supervised tool:
 - Compute many centralities on all nodes
 - Learn how to combine them to find chosen nodes
 - Discover new similar nodes
 - (roles in social networks, key elements in an infrastructure, ...)

Which is which ?



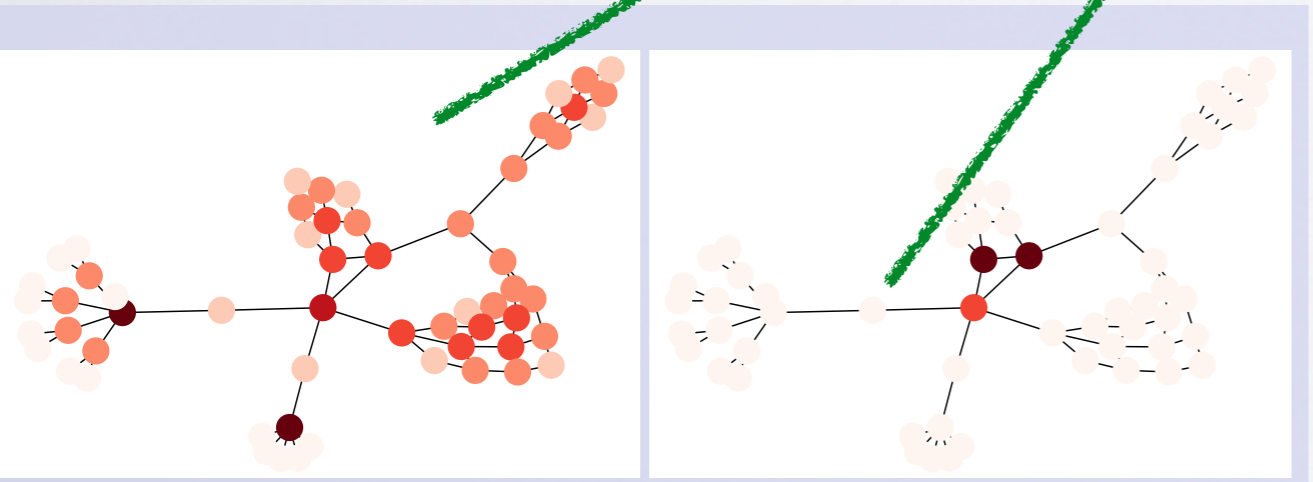
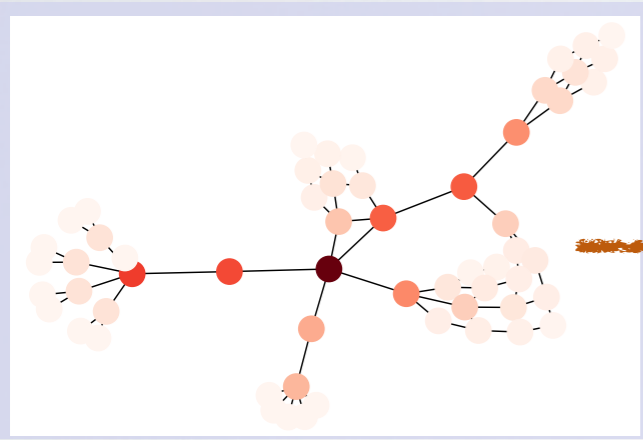
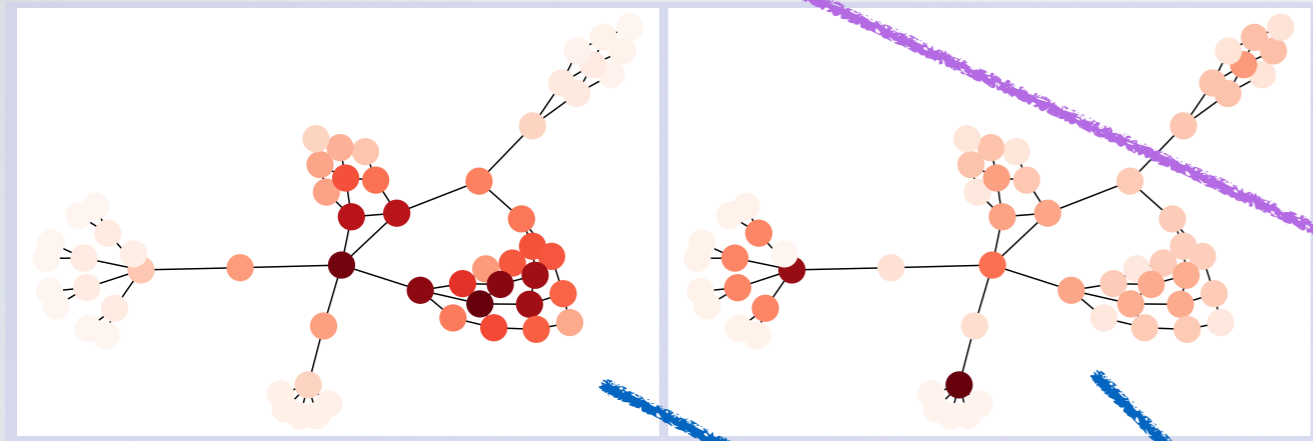
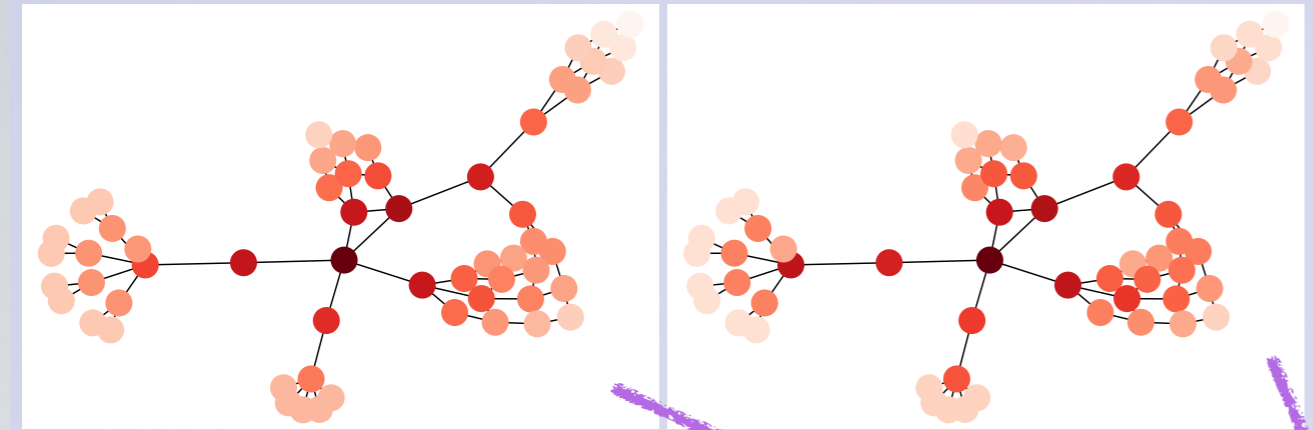
Degree
Clustering coefficient
Closeness
Harmonic Centrality
Betweenness
Eigenvector
PageRank

Which is which ?

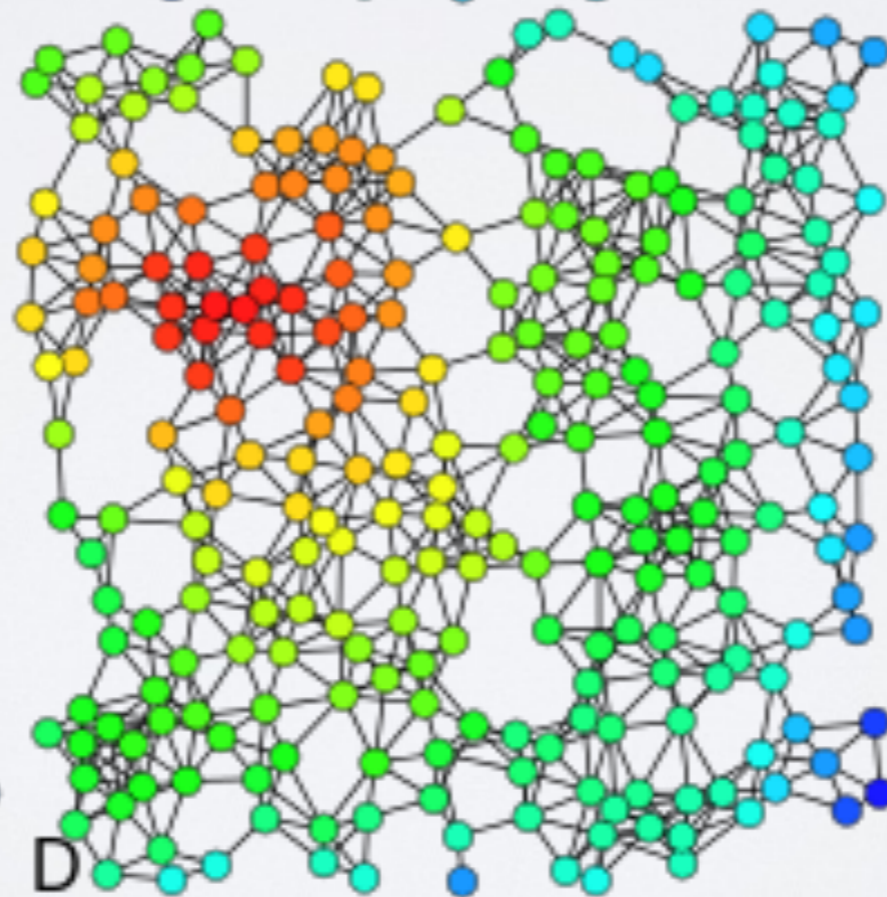
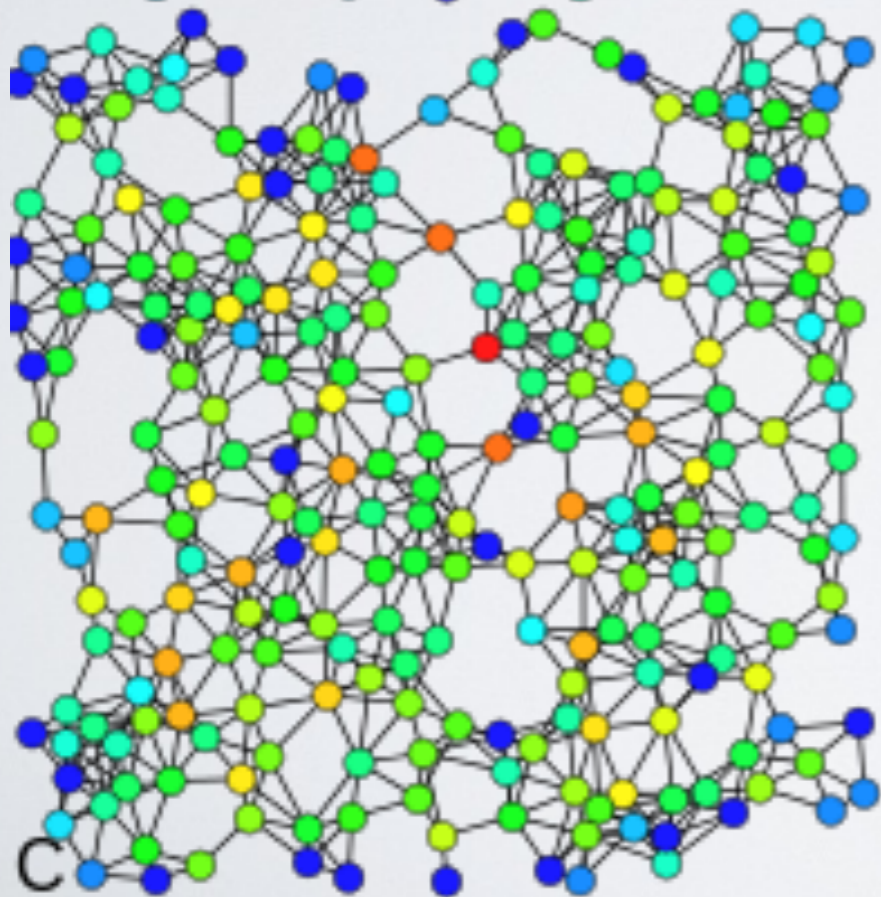
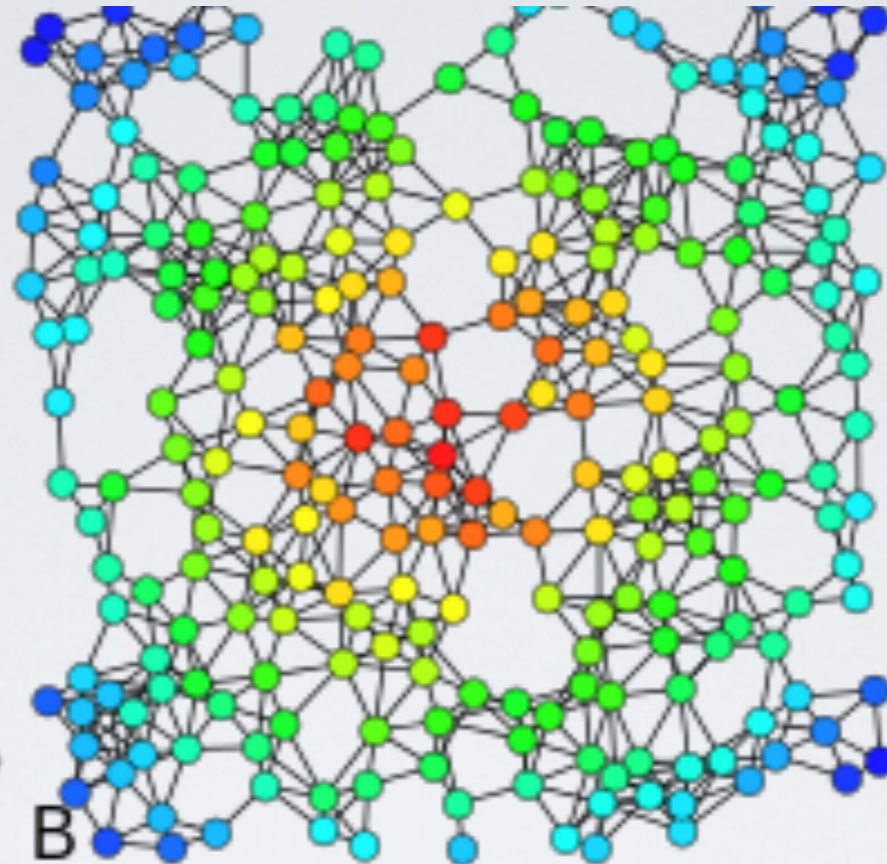
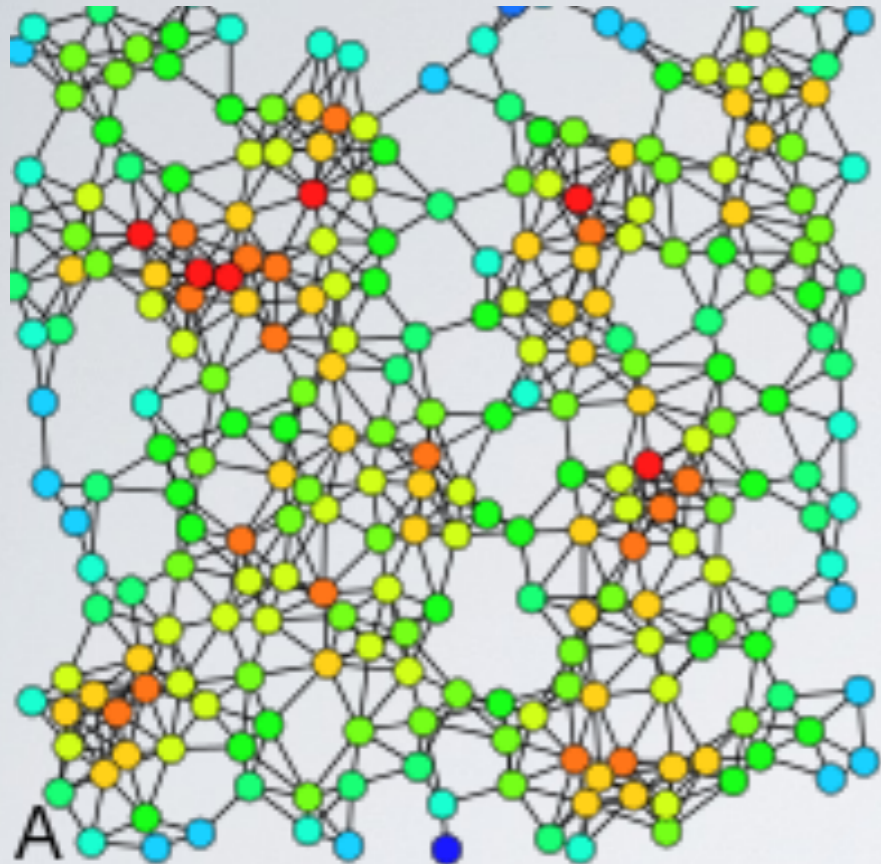


- Degree
- Clustering coefficient
- Closeness
- Harmonic Centrality
- Betweenness
- Eigenvector
- PageRank

Which is which ?

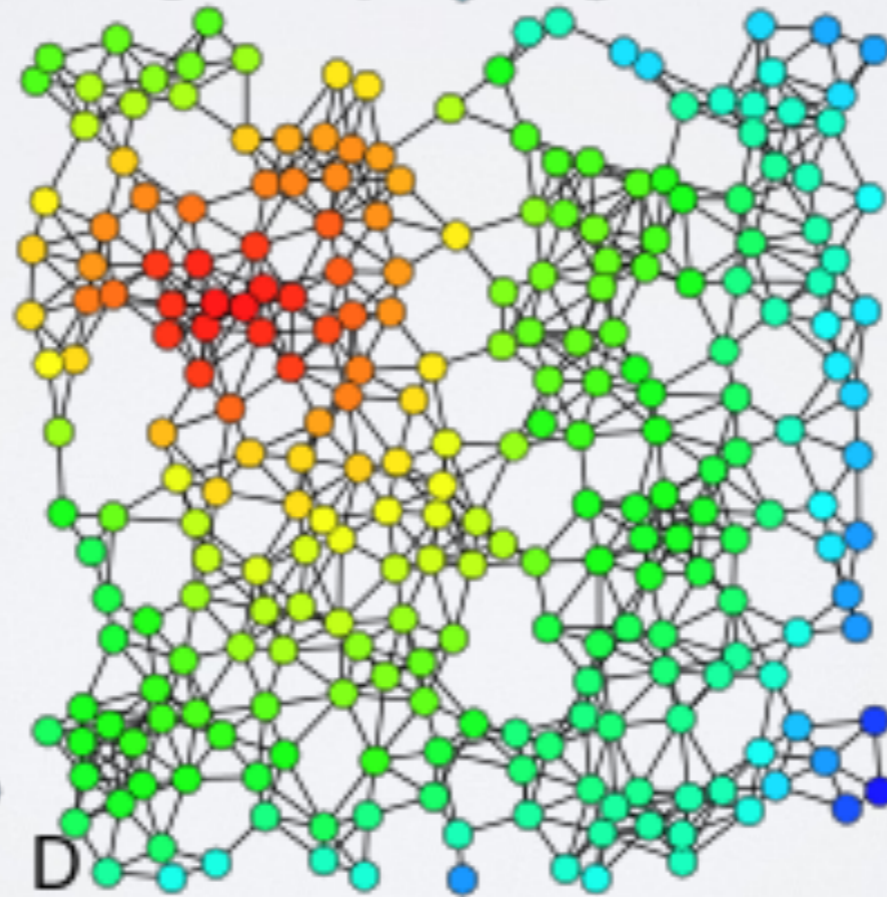
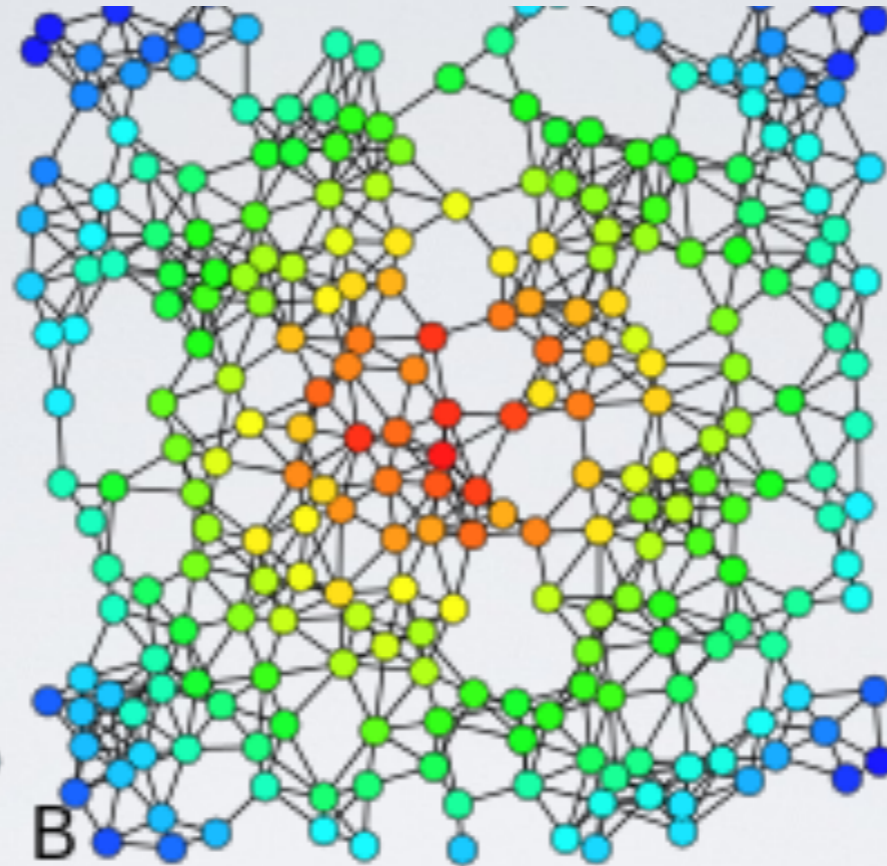
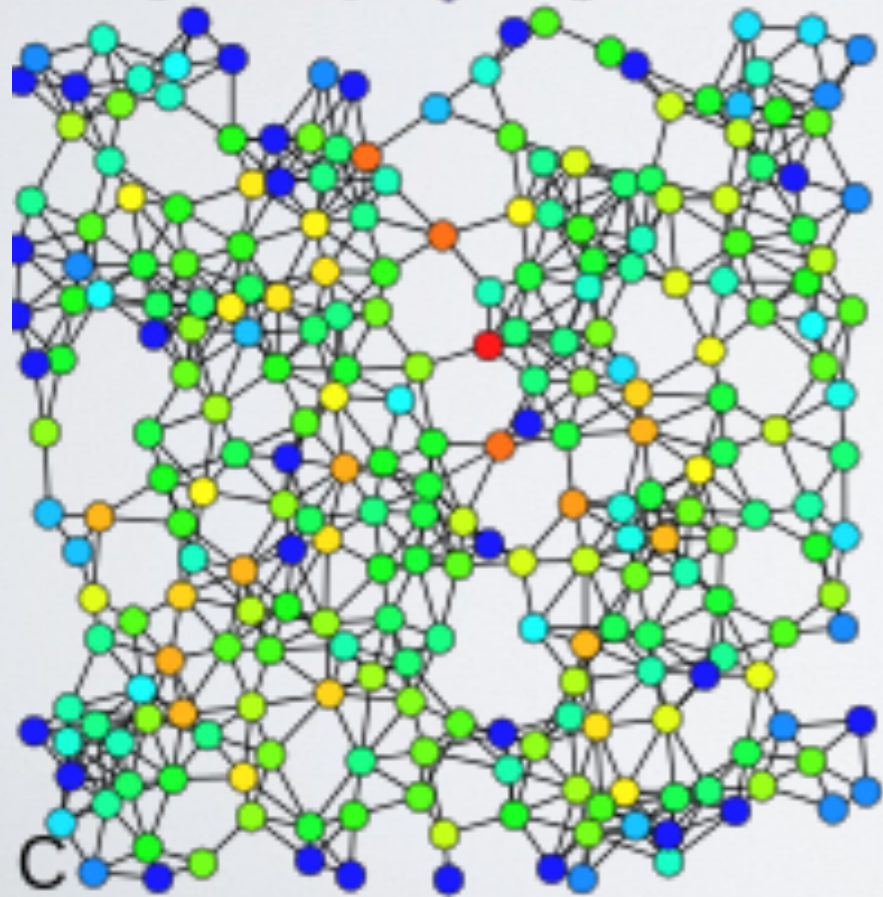
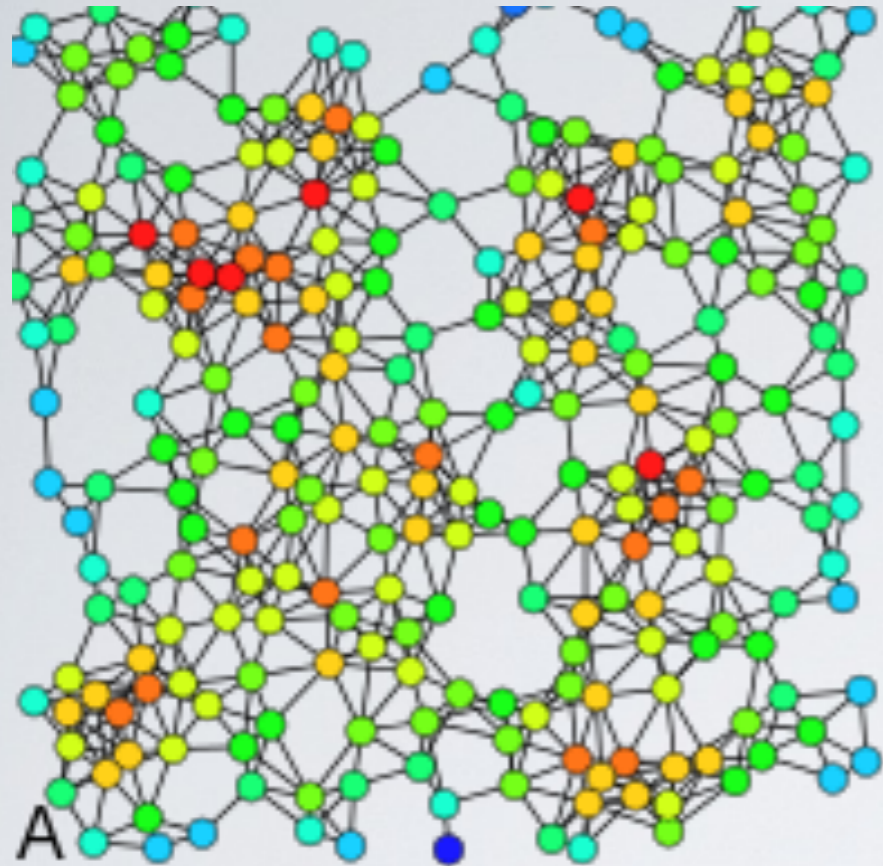


- Degree
- Clustering coefficient
- Closeness
- Harmonic Centrality
- Betweenness
- Eigenvector
- PageRank



Try again :)

Degree
Betweenness
Closeness
Eigenvector



Try again :)

A: Degree

B: Closeness

C: Betweenness

D: Eigenvector