RECOMMENDER SYSTEMS

And matrix factorization

RECOMMENDER SYSTEMS

- A popular problem in Data Mining with many commercial/industrial applications
- Given a user and its past interaction with items, recommend them some new items
  - Movies, Music, Book, Video Games, etc.
  - Products on Amazon or any shop with past information
  - Posts/contents on Twitter, Facebook, Youtube, news media
  - …

CONTENT-BASED

- **Content-based recommendation**
  - We describe all our items using features
    - Movies genre, length, age rate, topics…
    - Objects categories, price range, etc.
  - We recommend to users items having similar features to the ones they like
    - For instance, using supervised machine learning (classification or score regression)

- Often disappointing in practice
  - Finding useful descriptors is usually very hard
    - What makes you like/dislike a music/movie is more than a list of keywords
    - Somewhat arbitrary (is movie M a comedy? Book B a child book? 2 people might disagree)
  - Very costly on large catalogs
    - Impossible for social medias, but also Amazon, YouTube..

COLLABORATIVE FILTERING

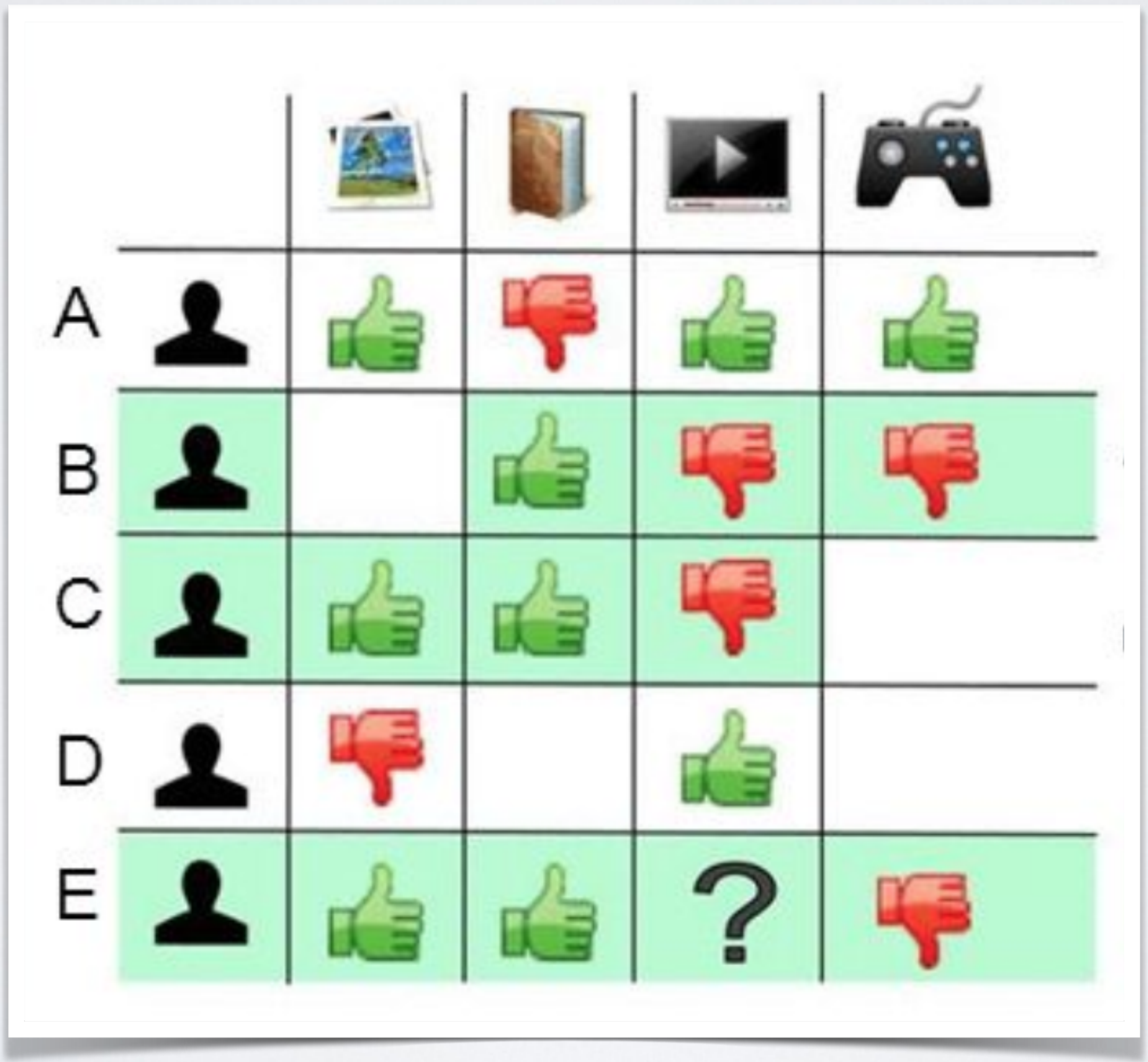
- Solution: **Collaborative filtering**

- Principle:
  - To evaluate if two items are similar, instead of comparing manually chosen descriptors (genre, etc.), we compare the users who have interacted with them
  - =>Users themselves become the features

- The definition of similarity **emerges** from the **collaborative** efforts of all users

- *Tell me what you like, I'll tell you who you are*

COLLABORATIVE FILTERING



DATA

- We model past data a matrix of size U x I
  - U users
  - I items

- X(u,i)=user/item interaction
  - Buy, watch, clic, like, vote, etc.

- Users could be treated as any feature, but they have some specificities
  - Values are sparse:
    - Missing values in all rows and columns (no user rates all items, no item is rated by every user)
  - Both Users or Items can be used as variables or observations (rows/columns)

DATA COMPLEXITY

- Data form:
  - Binary vote
    - 1 and 0 are both reliable (rare)
  - Like, Heart, Watched, Bought, Listened, etc.
    - 1 is a reliable information, but 0 and nan are not differentiable.
  - Note (e.g., 1 to 5 stars, etc.)
    - Often imbalanced between 4/5 (frequent), 1/2 (less frequent)
    - Missing values and 0 are correlated (people rate what they watch, and watch what they like)

- Users can have different labelling standards
  - "Good" for one might correspond to "excellent" for another
    - Some users put a like/share everything they find above average
    - Other users will only like/share what they find exceptional
    - Same for scores: some never give maximal note, while others use only the maximal note

DATA COMPLEXITY

- User note diversity => Normalize/Standardize scores for each user

- Normalizing by item ?
  - We don't care anymore if the score is good, we want to know if its better than for other users

- Considering both aspects: subtracting a *baseline*
  - We estimate the baseline score (u, i) based on 2 constants, b_u and b_i
    - b_u captures the tendency of u to give high or low marks
    - b_i captures the tendency of i to have low or high marks
    - e.g., minimize by gradient descent a regularized baseline
    - $\sum_{r_{ui} \in R_{train}} \left(r_{ui} - (\mu + b_u + b_i)\right)^2 + \lambda \left(b_u^2 + b_i^2\right).$
    - μ: average note for a random user on a random item

USER-BASED KNN

USER-BASED KNN

- KNN: K-Nearest-Neighbors
  - Simple yet powerful method popular in classification task
    - 1)Find k most similar items (neighbors) to item i.
    - 2)Each neighbor "vote" for its target => average/mode of targets of neighbors

- Application to **user-based** collaborative filtering
  - 1) Find k most similar users (neighbors)
  - 2) Each neighbor "vote" for the products they liked
    - Average notes
    - Count of 1 for binary data (like, etc.)
  - Usually, votes weighted by similarity to the original user

USER-BASED KNN

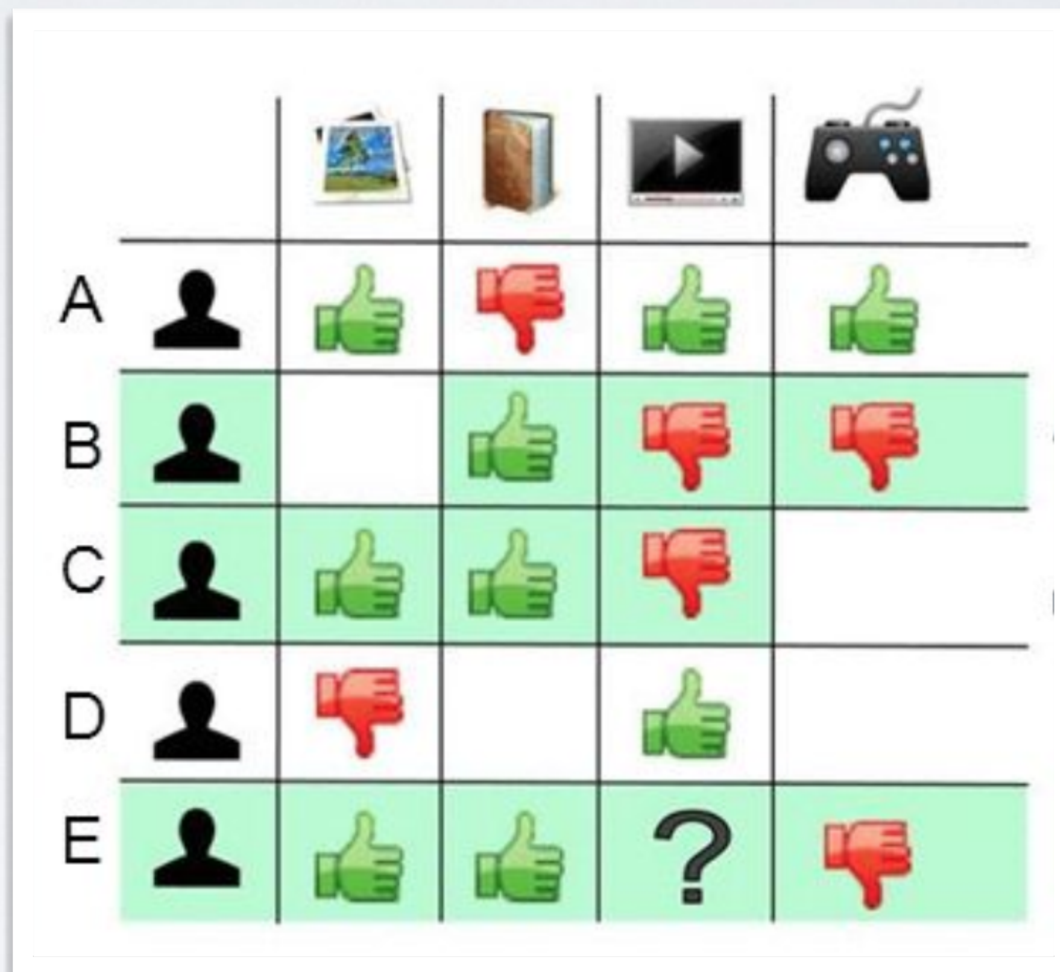
Similarity to E

-1

2

2

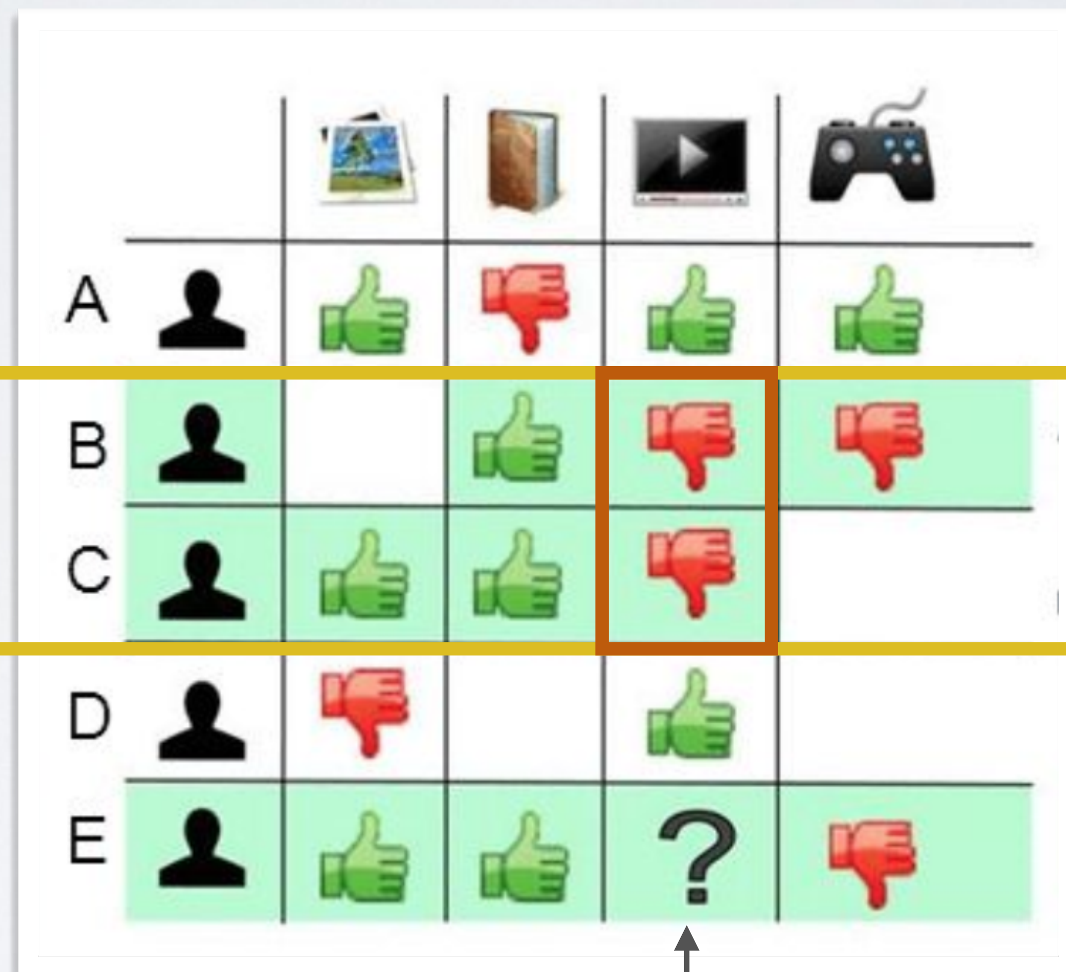
-1



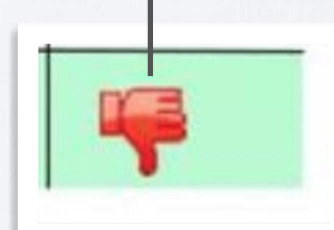
USER-BASED KNN

Similarity to E

-1
2
2
-1



(2*-1)/2=-1



SIMILARITY

- How to compute the similarity between users ?
  - Euclidean distance => Poor results
    - Think of a user with few likes {0,1}. They are very distant from users having many like, since each difference adds distance.
  - Number of similar votes only ?
    - Now users with many likes are similar to everyone

- Solution:
  - (Binary) Jaccard Similarity => | likes(u&v) | / (| likes(u) | + | likes(v) |)
  - (Notes) MSD=>Means Squared Difference <u>when both notes present</u>
  - (Binary & Notes) Cosine Similarity

SIMILARITY

$$\cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} = \frac{\sum\limits_{i=1}^{n} A_i B_i}{\sqrt{\sum\limits_{i=1}^{n} A_i^2} \sqrt{\sum\limits_{i=1}^{n} B_i^2}}$$

For binary:

$$\frac{|\,\text{likes}(u \,\&\, v)\,|}{\sqrt{|\,\text{likes}(u)\,|}\sqrt{|\,\text{likes}(v)\,|}}$$

Similar Principle than Jaccard Coefficient

ITEM-BASED
COLLABORATIVE FILTERING

ITEM-BASED

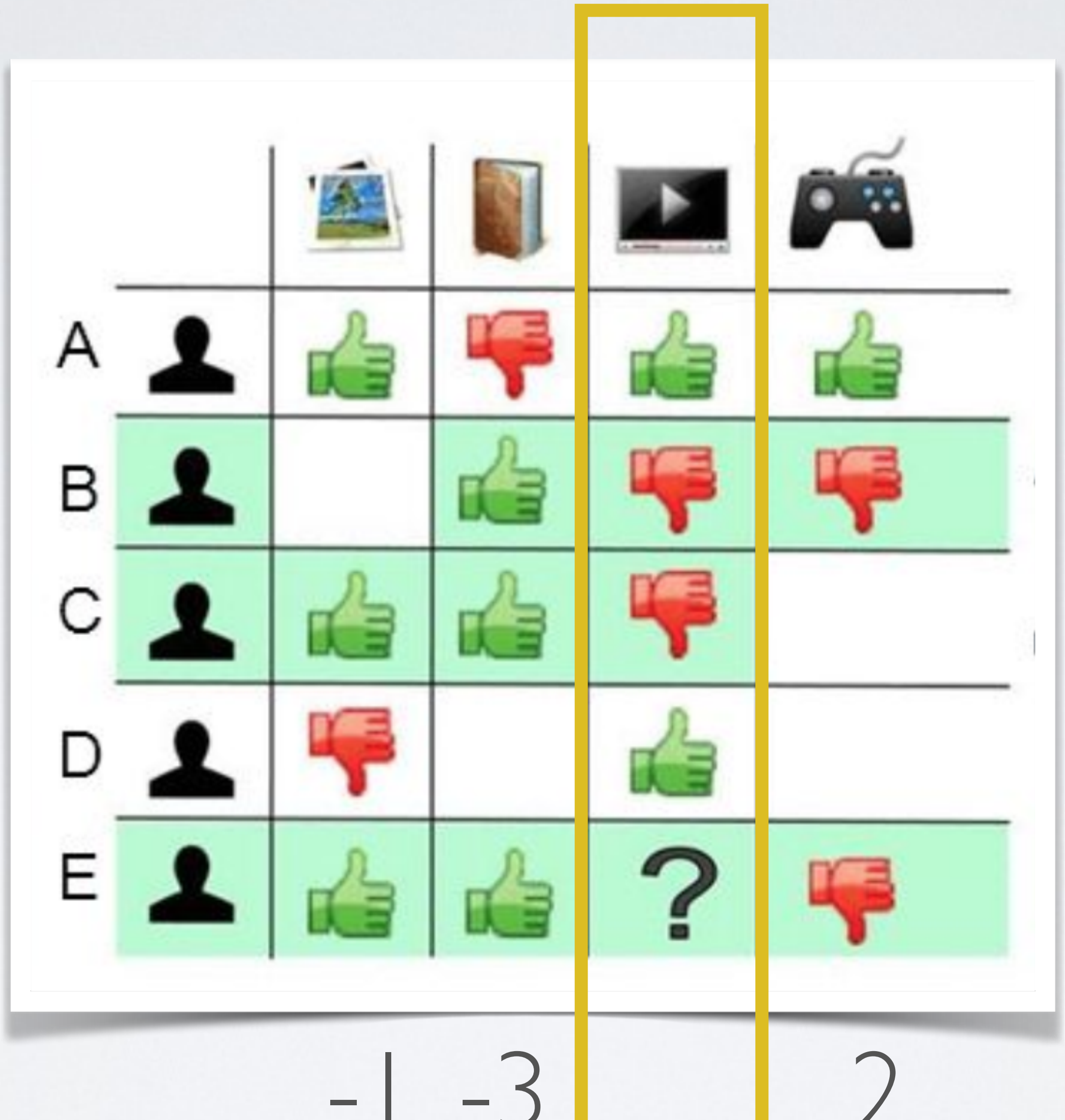
- User-based collaborative filtering has weaknesses in practice
  - Users with little info will have neighbors with little info too
    - =>We will learn based on few info
    - Imagine you liked movies M1 and M2. The 20 most similar users will like exactly M1 and M2, maybe 1 movie more.
  - Users change a lot =>Need to recompute KNN on whole database very frequently

- => Move to Item-based Collaborative filtering

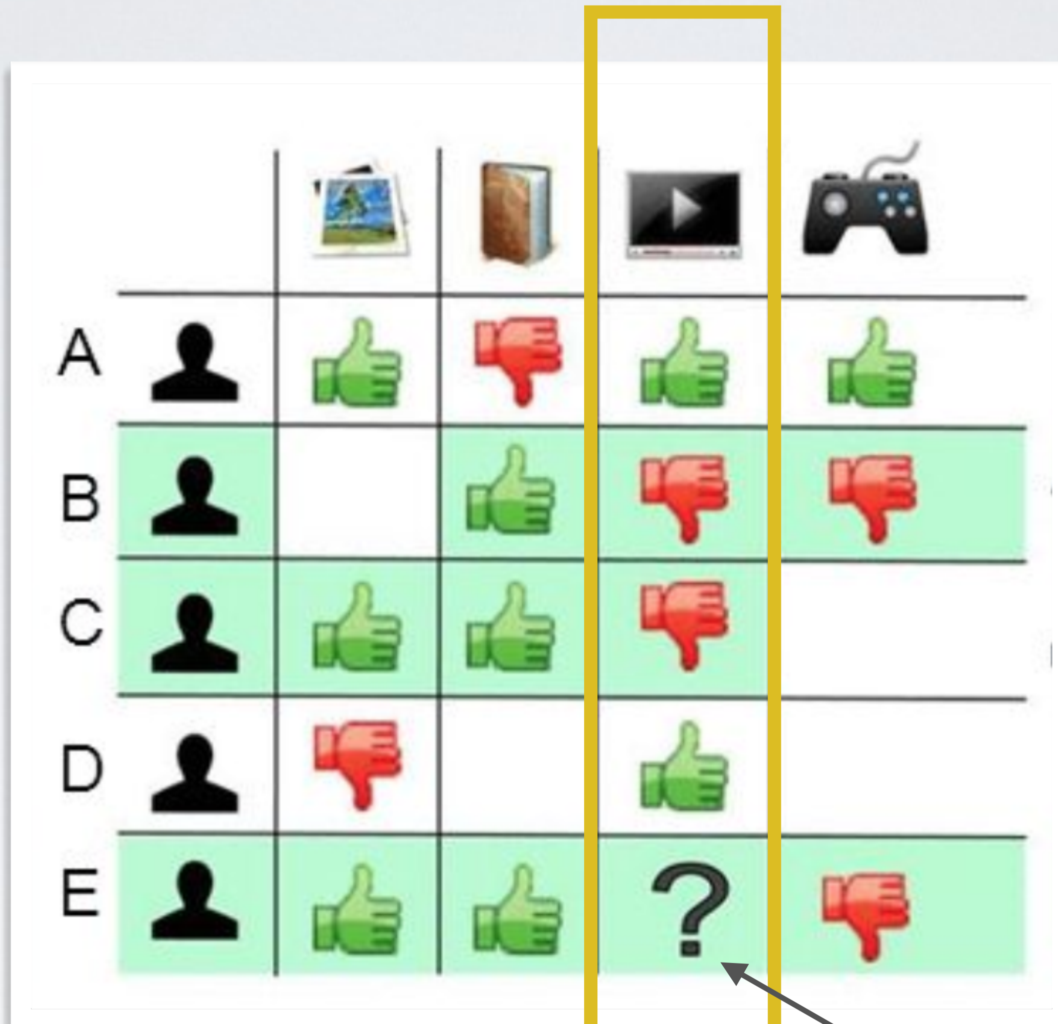
ITEM-BASED

- We want to evaluate the interest of (u,i)
  - 1)For each item x liked by u
    - Compute the similarity between x and i
  - 2)(u,i) is the average similarities (x,i) for x liked by u
- We compute score (u,i) for every unknown item

ITEM-BASED

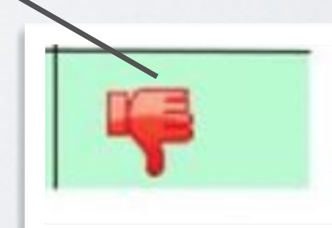


ITEM-BASED



-1 -3 2

$=(1*(-1)+1*(-3)-1*2\ )/3=>-2$



ITEM-BASED

- Original Amazon patented method introduced in 1998
- Strengths
  - Distances between items can be precomputed at fix interval, do not change too quickly
  - Distances between items robust, lot of information (appart from new items)

MATRIX FACTORIZATION COLLABORATIVE FILTERING

NETFLIX PRIZE

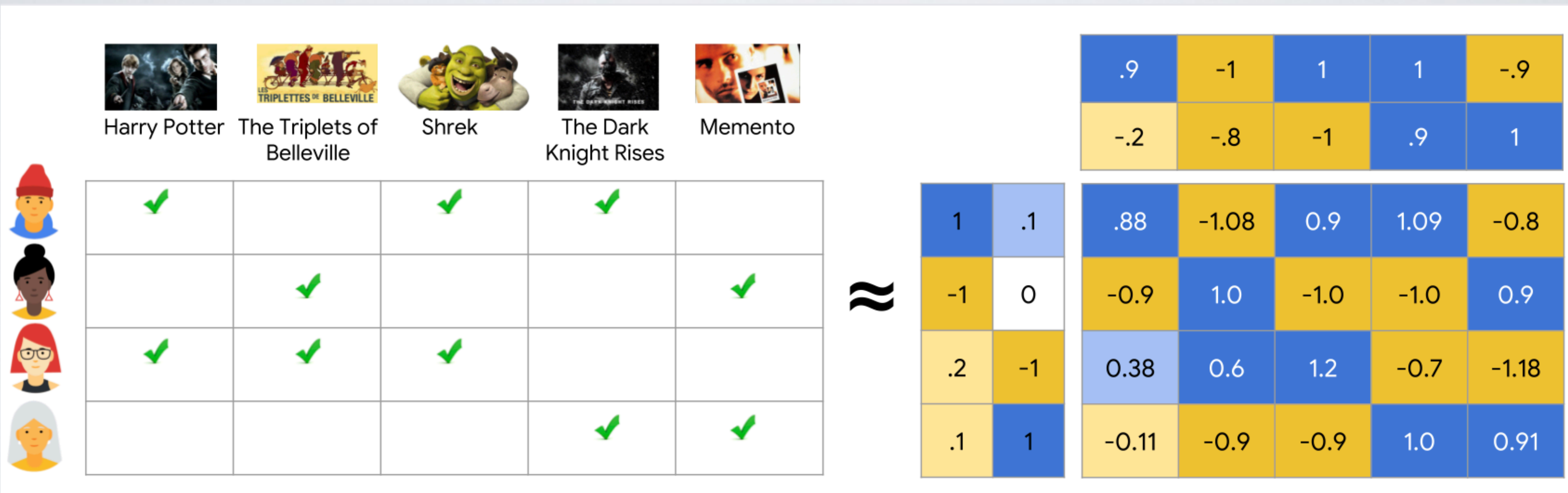
- Worldwide competition to improve Netflix recommendation
  - Cash prize, 1 Million $
  - 2006 to 2009 (Objective of reducing RMSE on scores by 10% compared with Netflix own method)
- Winning method: Stacking of multiple recommendation systems
- Yet, one new popular approach attracted lot of attention: SVD
  - /!\ Singular Value Decomposition(SVD) is a classic linear algebra matrix decomposition. But in recommendation literature, SVD is also the name of an algorithm related but different to the original SVD.

https://intoli.com/blog/pca-and-svd/

MATRIX FACTORIZATION

- **Matrix Factorization** is a name given to a general approach of data mining
  - We start with an original matrix A, typically item/user matrix
  - We search for 2 matrices U, V, such as to minimize a cost function $L(A, UV)$
    - With UV a matrix multiplication
- If the dimension of A is $X \times Y$
  - Then $U => X \times D, V => D \times Y$
    - With D a parameter, corresponding to a number of *latent variables*
  - The process is a type of dimensionality reduction

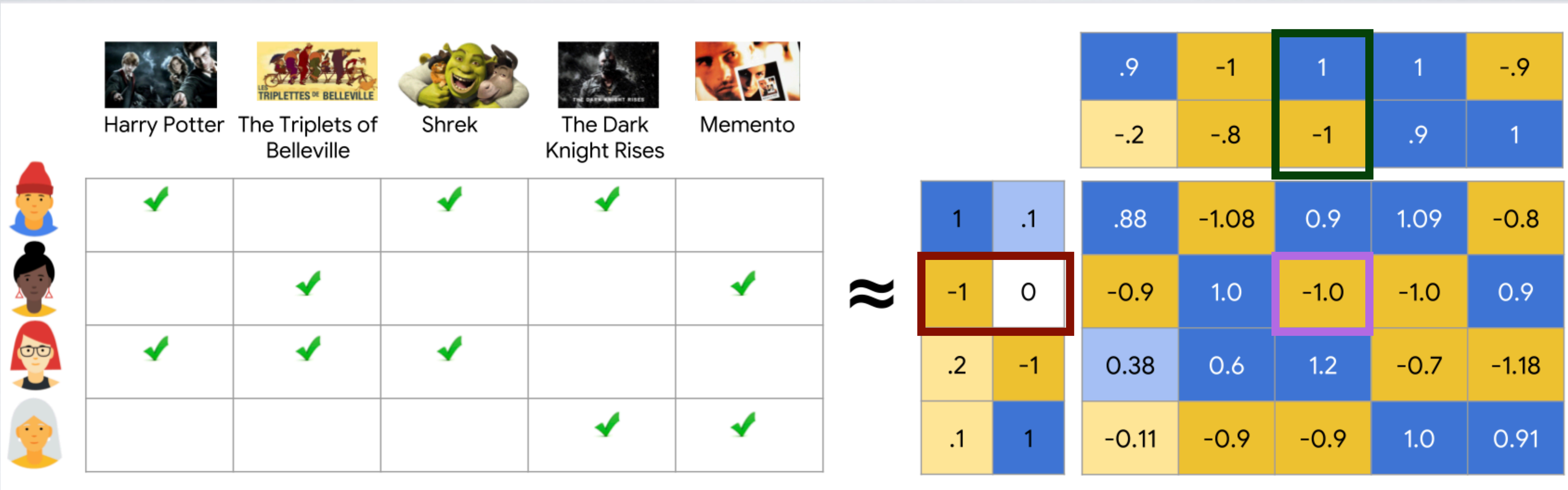
MATRIX FACTORIZATION



2 latent variables

https://developers.google.com/machine-learning/recommendation/collaborative/matrix

MATRIX FACTORIZATION



Vector representing user 2, u2

Vector representing item 3, i3

Multiply the two vectors to reconstruct estimated value(u2,i3)

https://developers.google.com/machine-learning/recommendation/collaborative/matrix

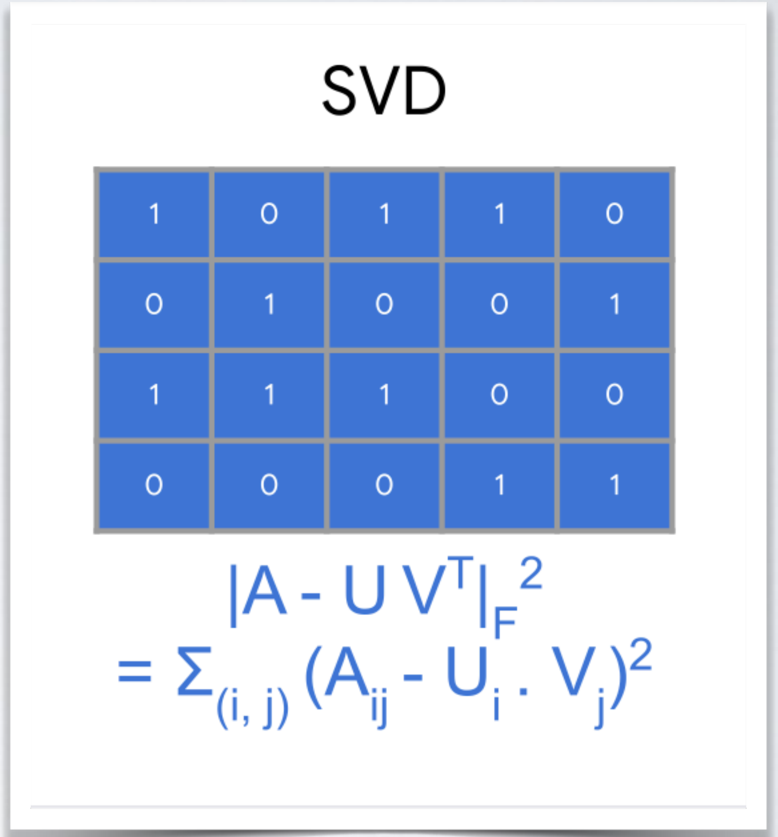
MATRIX FACTORIZATION

- As with word embedding approaches (word2vec, etc.), dimensions can be understood as *latent variables*, i.e., features representing some semantic notion

- For instance, in movies, latent variables could capture
  - Horror-ness, comedy-ness, adult-ness, etc.
  - Each user has a score in each of these features (enjoy horror=1, comedy=0.2)
  - Each movie too (is horror=1, is comedy=1.5)
  - =>(user, movie)=>combination of match in each category

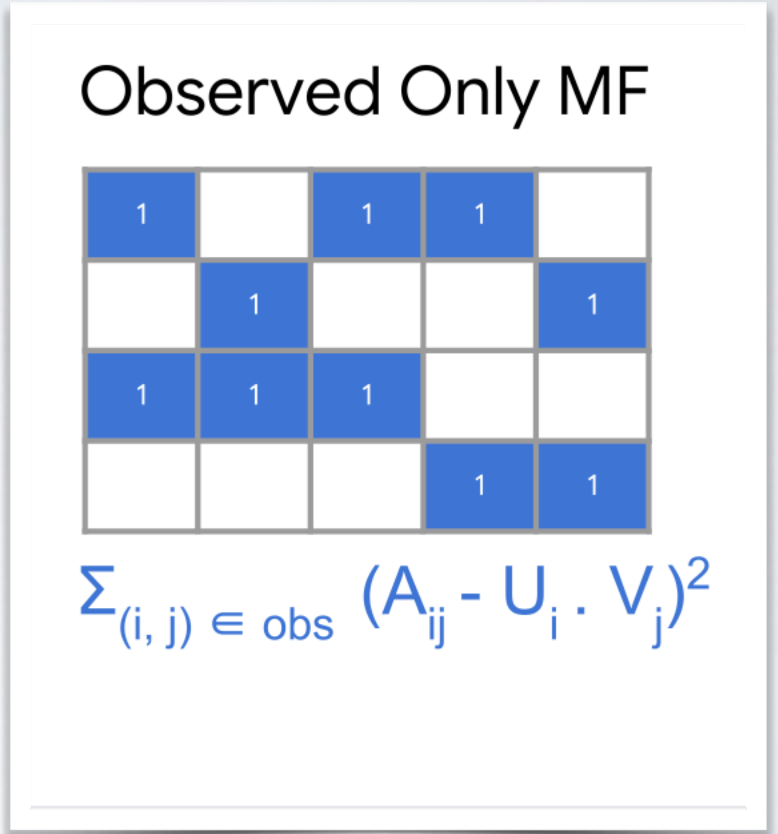
OBJECTIVE FUNCTION

- The *classic* SVD would correspond to using as a loss the means squared error
  - **Having 0 where we have no data** (like/rating)

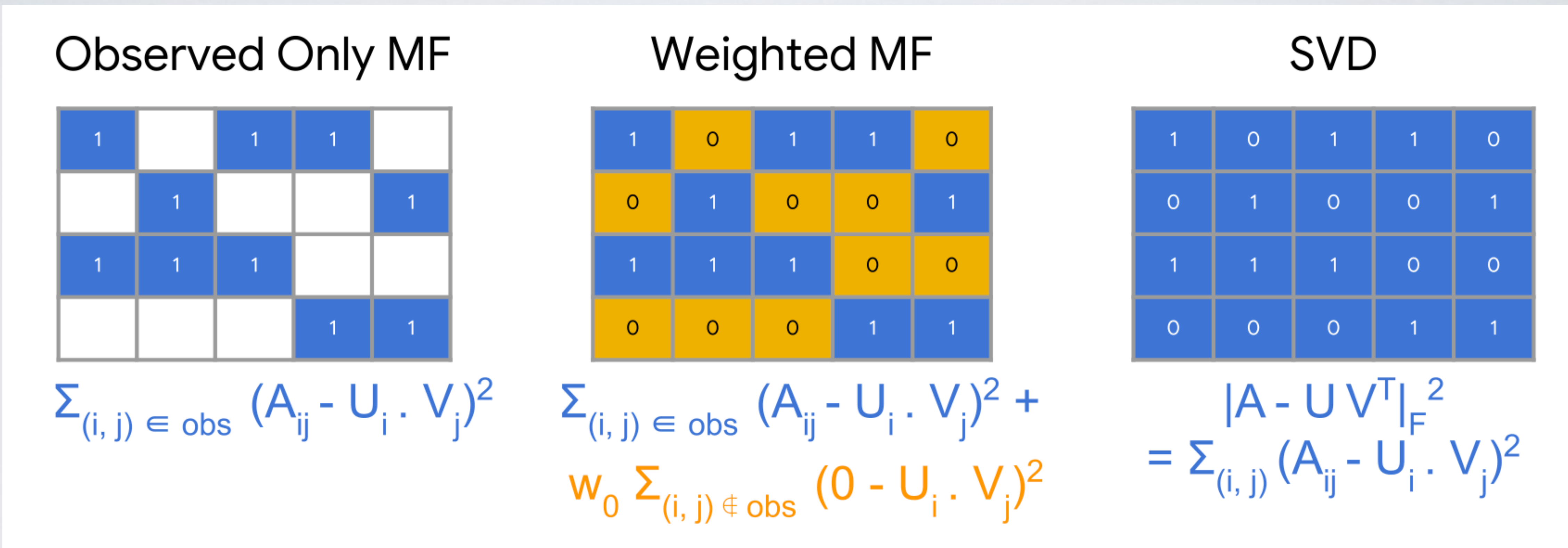


OBJECTIVE FUNCTION

- The recommendation based Matrix Factorization has an adapted loss,
  - **Computed only on non-zero values**



OBJECTIVE FUNCTION



A variant has a parameter to combine both
(Weighted Matrix Factorization)

https://developers.google.com/machine-learning/recommendation/collaborative/matrix

OPTIMIZATION

- To find the two matrices, we use a greedy approach
  - Typically the Weighted Alternating Least Square (WALS)
    - 1)Initialize values at random
    - 2)Fix $\boldsymbol{U}$ and solve for $\boldsymbol{V}$
    - 3)Fix $\boldsymbol{V}$ and solve for $\boldsymbol{U}$
    - Repeat 2 and 3 until convergence
  - Solving in 2 and 3 is equivalent to doing linear regression for each component

OPTIMIZATION

$$\begin{array}{cc} & \begin{array}{ccc}\text{Product 1} & \text{Product 2} & \text{Product 3}\end{array} \\ \begin{array}{c}\text{user 1}\\ \text{user 2}\end{array} & \begin{bmatrix} 0.5 & ? & 4 \\ 1 & 3 & 5 \end{bmatrix} \end{array} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc}\text{Product 1} & \text{Product 2} & \text{Product 3}\end{array} \\ \begin{array}{c}\text{user 1}\\ \text{user 2}\end{array} & \begin{bmatrix} 0.5 & ? & 4 \\ 1 & 3 & 5 \end{bmatrix} \end{array} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$

Arbitrary initialization

$$p_1^* = \operatorname{argmin}\,(0.5 - p_1)^2 + (1 - p_1)^2 \tag{6}$$

$$p_2^* = 3$$

$$p_3^* = \operatorname{argmin}\,(4 - p_3)^2 + (5 - p_3)^2 \tag{7}$$

$$P = \begin{bmatrix} 0.75 & 3 & 4.5 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.7461 \\ 1.7966 \end{bmatrix} \qquad P = \begin{bmatrix} 0.758 & 2.5431 & 4.7999 \end{bmatrix}$$

MF + REGULARIZATION

- As with many machine learning tasks, we can introduce regularization to avoid overfitting
  - Due to the large number of parameters, regularization is important
- The objective to solve becomes:
  - $\sum_{r_{ui} \in obs} (r_{ui} - \hat{r}_{ui})^2 + \lambda \left(||q_i||^2 + ||p_u||^2 \right)$
    - q_i, p_u are latent vectors
  - λ controls the strength of the regularization

MF + BASELINE

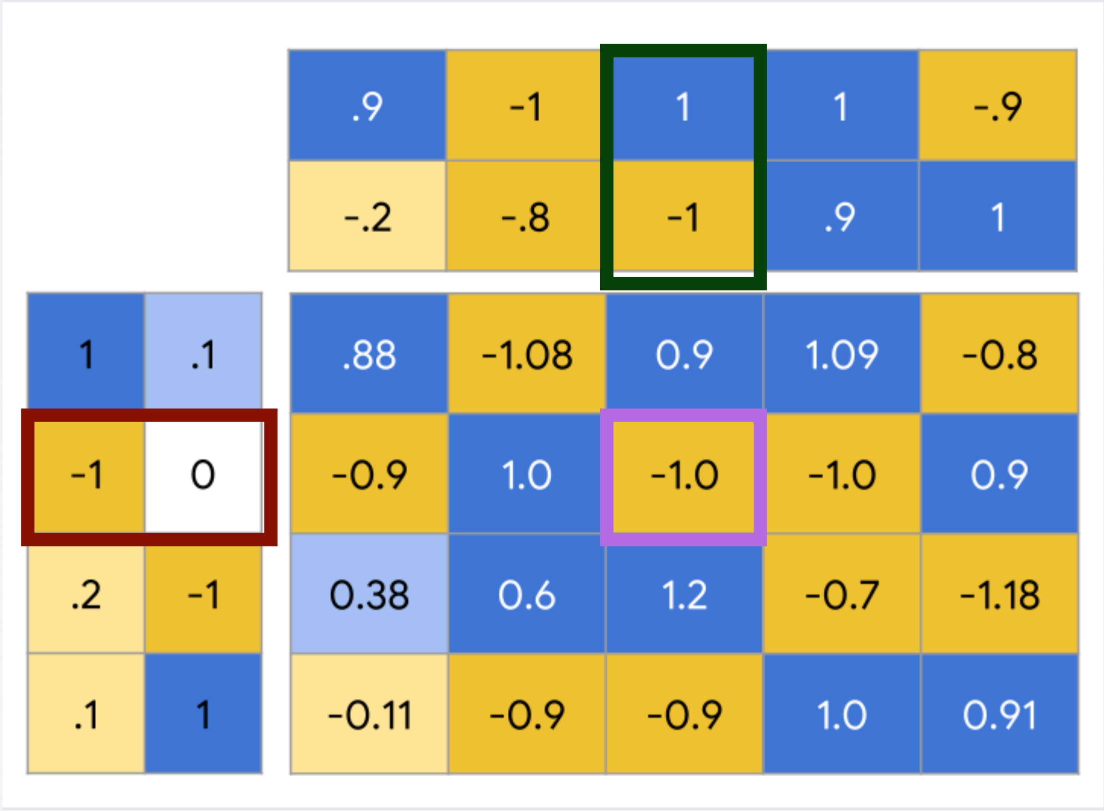
- As mentioned before, it is also important to take into account the variability of users and of items
  - We want to predict what cannot be simply predicted by
    - Movies being good/bad
    - Each actor tendency to give good/bad scores
    - => If most users give good marks to movie M1, and user U1 tend to always give maximal scores to movies they rate, the fact that (U1,M1)=maximal note is "expected"

- The objective to solve becomes:
  - $$\sum_{r_{ui} \in obs} (r_{ui} - \hat{r}_{ui})^2 + \lambda \left(b_i^2 + b_u^2 + ||q_i||^2 + ||p_u||^2 \right)$$
  - With b_i and b_u representing items and users expected scores, respectively

MF RECOMMENDATION

- From the two partial matrices, we can compute any value by multiplying the corresponding vectors

- Recommending for a user consists in picking
  - In the user row
  - The highest computed values



NETFLIX PRIZE

- A few other elements were taken into account in the Netflix Prize winning strategy
  - Temporal aspects (how long since this product was rated…)
  - Sequential aspects
    - Watch episode1 then episode 2. Contrary unlikely.
- Fine parameter tuning, clever stacking…

NEW USER

- If a new user requests a recommendation, the complexity to provide one depends on the method
  - User based=>Compute distance to all other users
    - Then direct answer for all items
  - Item based=>Precomputed distances betweeen all items
    - Naive approach, need to compute for all candidate items, but in reality, faster tricks
    - e.g., Find items that are "close" to the ones liked by that user
  - Matrix Factorization
    - In theory, not possible to make recommendation to a new user without recomputing everything
    - In practice, an approximation can be obtained quickly, doing 1 step of the Alternating Least Square: we consider the item latent matrix fixed, updating the user matrix. Similar in nature to solving a linear regression

EVALUATION OF RECOMMENDER SYSTEMS

EVALUATION

- Recommendation evaluation use similar quality scores as supervised machine learning evaluation
  - RMSE, Precision@k, AUC, etc.
- The specificity of recommender systems is the way the train and test sets are built
  - General principle: For one *test* user,
    - We **show** part of their scores/votes to the trained recommender
    - We **hide** part of them, to use as ground truth
  - The problem is thus either:
    - A regression: how accurately do we predict the scores of hidden items
    - A classification: how many of the positive items in the test set do we recommend? Or, more realistically, AUC=Do we assign high scores to positive items?

EVALUATION

- In practice, two ways to evaluate, hiding users or hiding pairs(u,i)

- Hiding users
  - Rarer, but more realistic
    - If possible, even keep the most recent users hidden: prediction at time *t*
  - 1)We train with full data on a fraction of users
  - 2)We validate on other users, considered "new"

- Hiding pairs (u,i)
  - Hide random (u,i) pairs, ensuring a minimal number of visible ratings per users and items
  - Evaluate the recommendation on those removed pairs.

OTHER RECOMMENDATION QUALITY CRITERIA

- Diversity of recommendation
  - e.g., average cosine distance between 2 items recommended to a same user (among top-5)
- Coverage
  - e.g., fraction of all items recommended at least once, or information entropy…
- Personalization
  - e.g., average cosine distance between users recommendation

MF VARIANT: NMF

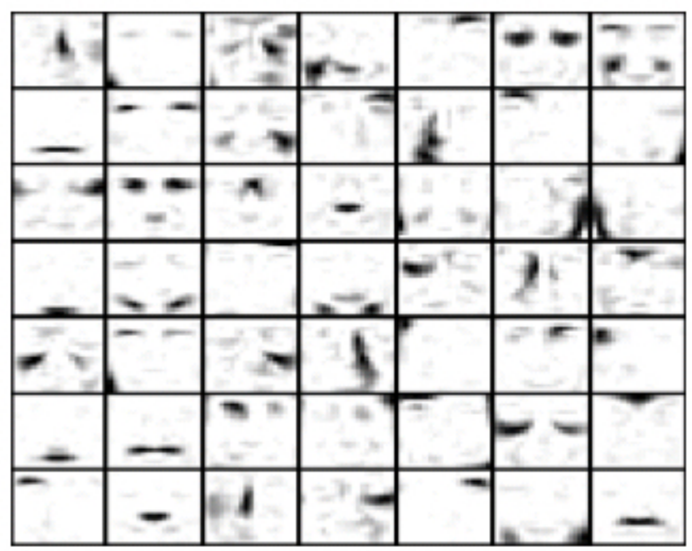
Non-negative Matrix Factorization

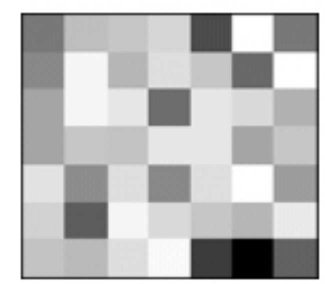
NMF

- A strength of Matrix Factorization is that it produces latent variables which, in theory, can be interpretable.
- A weakness of classic MF is that these variables can cancel each other, if one is positive and the other negative
- In NMF (Non-negative MF), we impose that all variables values must be positive. Of course, the Matrix to decompose must be positive too.
  - Imposes additive combinations

NMF

$$\underbrace{X(:,j)}_{j\text{th facial image}} \approx \sum_{k=1}^{r} \underbrace{W(:,k)}_{\text{facial features}} \underbrace{H(k,j)}_{\substack{\text{importance of features}\\ \text{in } j\text{th image}}} = \underbrace{WH(:,j)}_{\substack{\text{approximation}\\ \text{of } j\text{th image}}} .$$





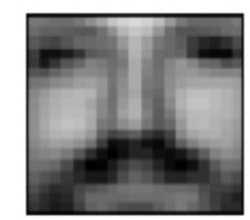


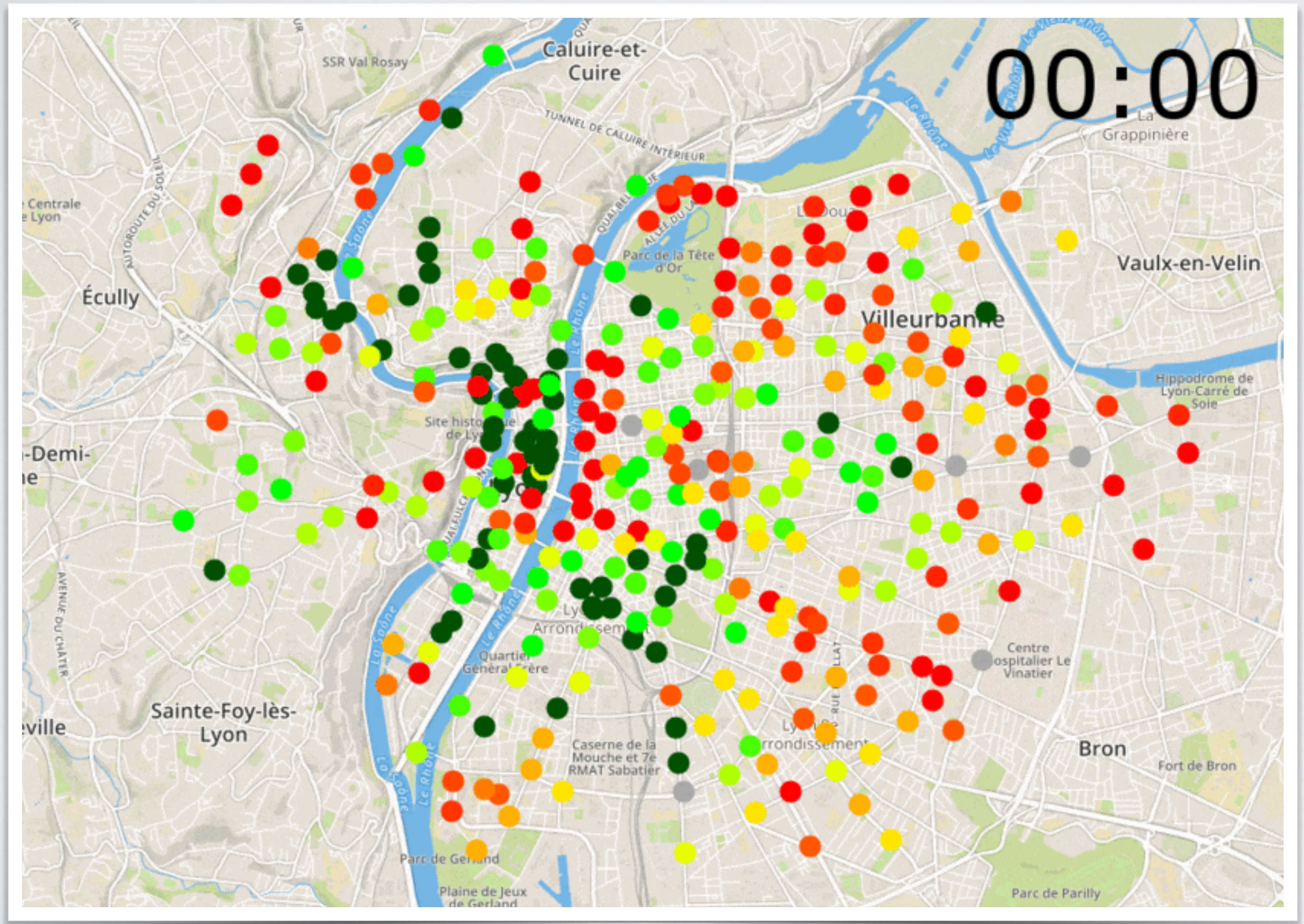
Figure 1: Decomposition of the CBCL face database, MIT Center For Biological and Computation Learning (2429 gray-level 19-by-19 pixels images) using $r = 49$ as in [79].

BICYCLE SHARING SYSTEMS

Docking stations

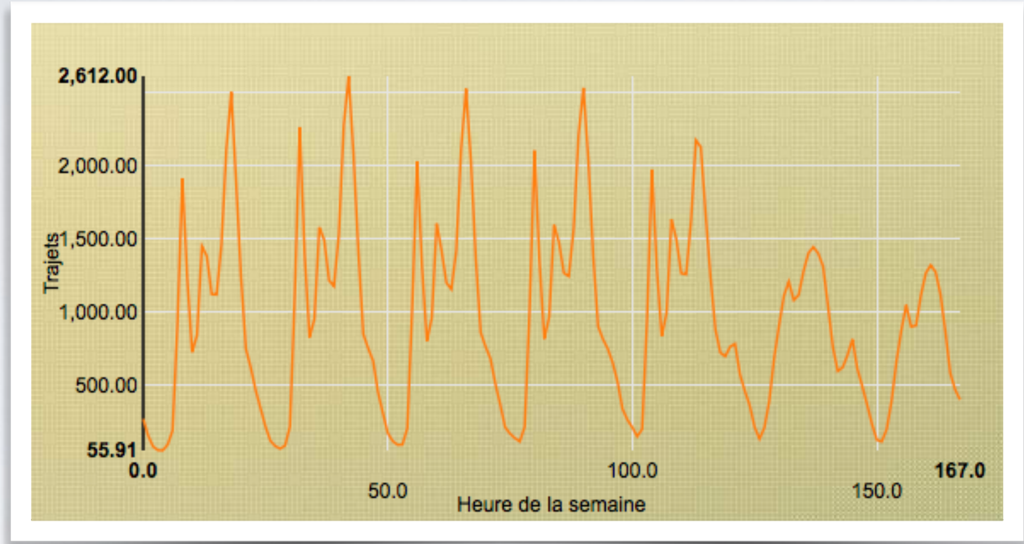
Bicycle trips

DATA

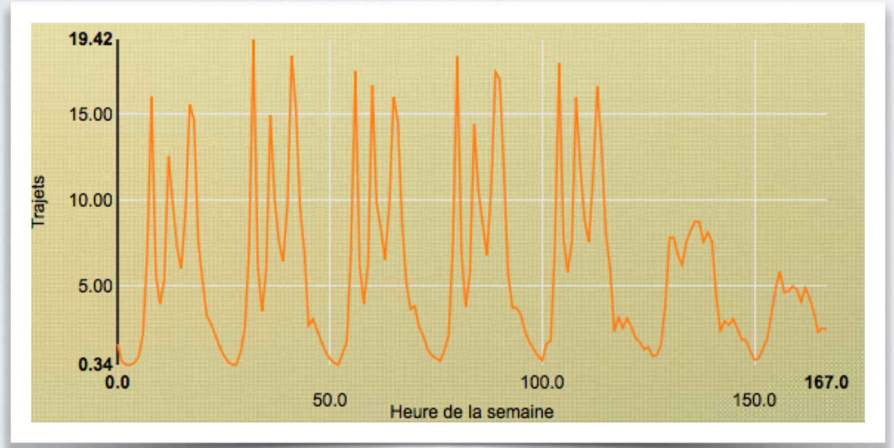


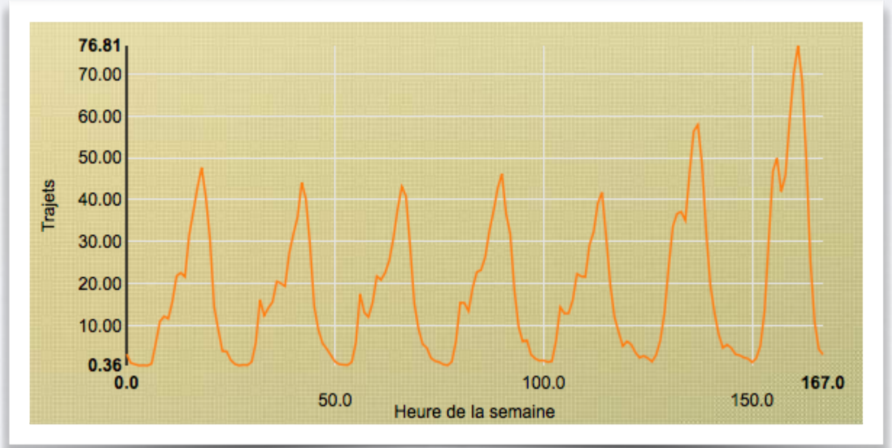
Red: empty

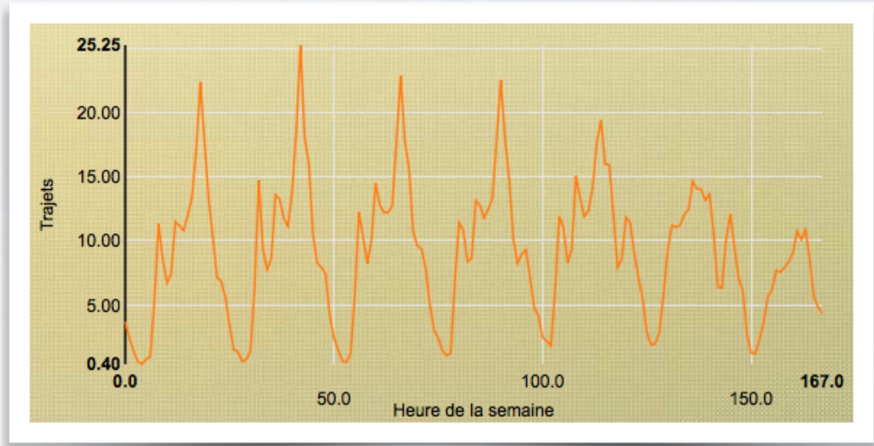
Green: full



Cumulated







Part Dieu

Tête d'or

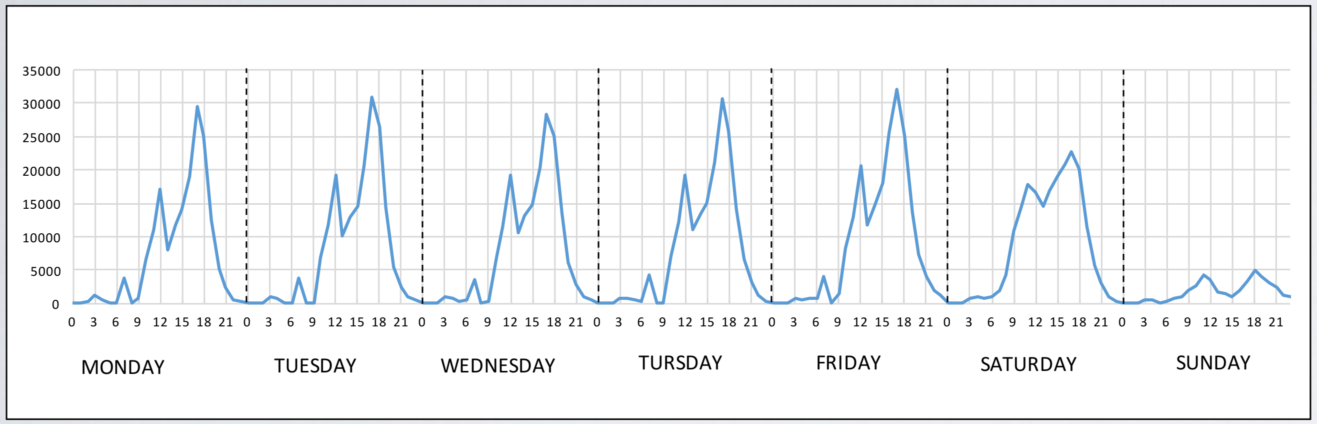
Guillotière

Hours of the typical week

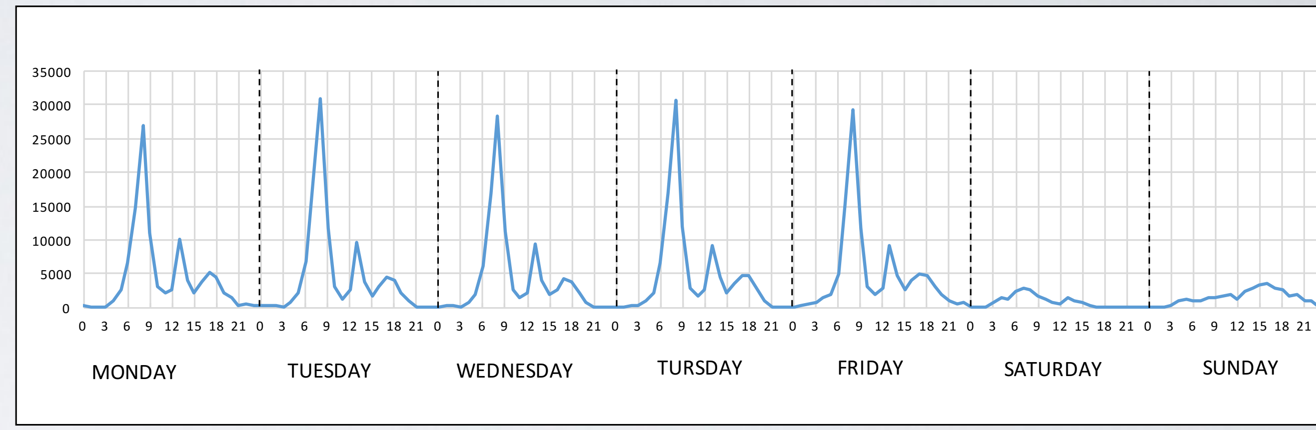
Entities (station)

	t1	t2	t3	t4	t5	t6	…		t168
e1									
e2									
e3									
e4									
…									

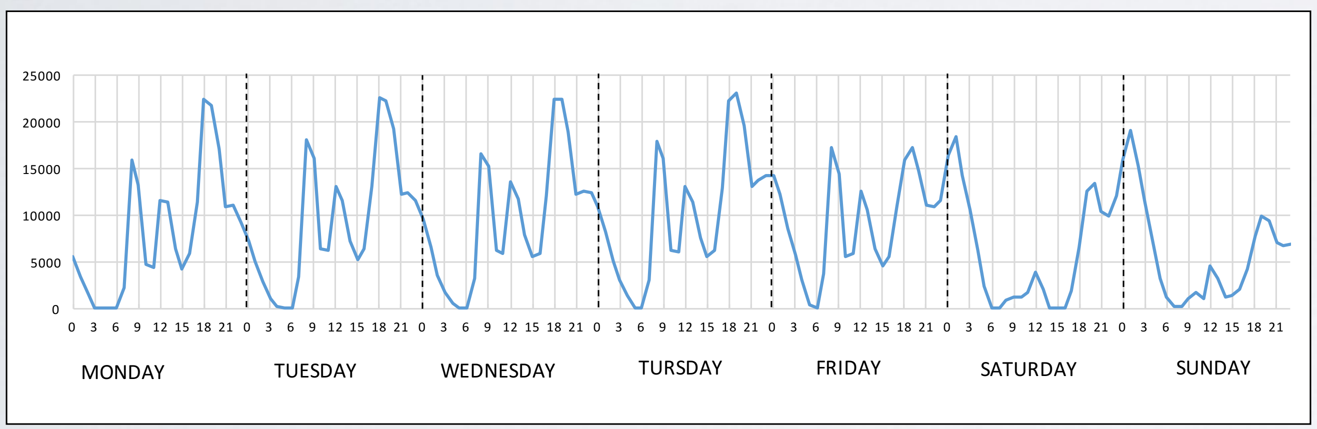
Automatically discovered patterns



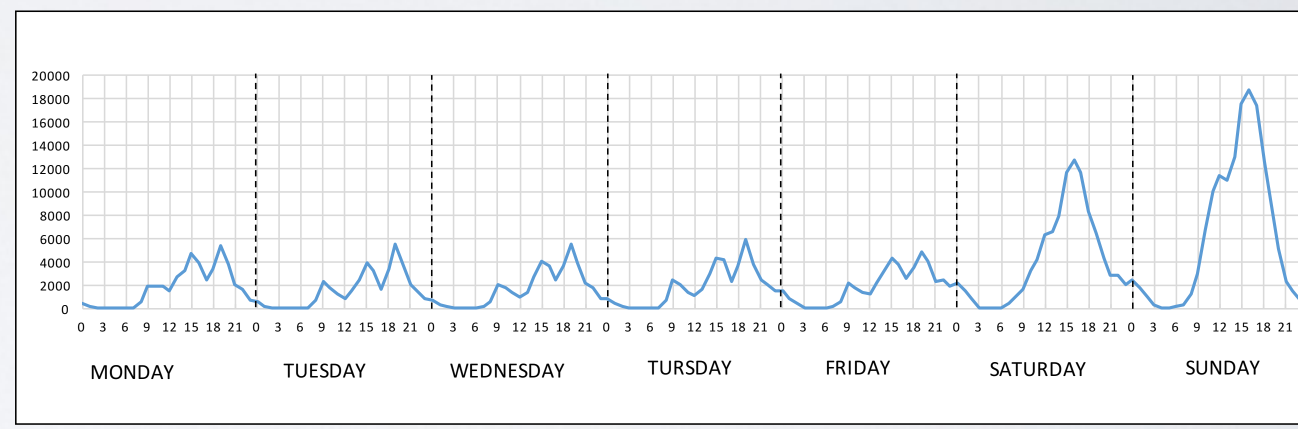
"Commercial" ?



"Work" ?



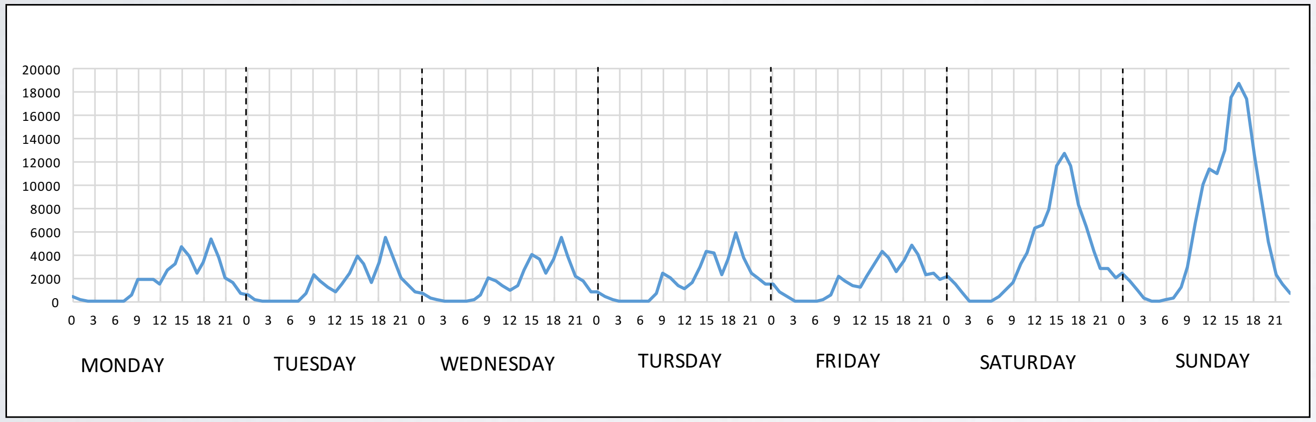
"Bars-Restaurants" ?



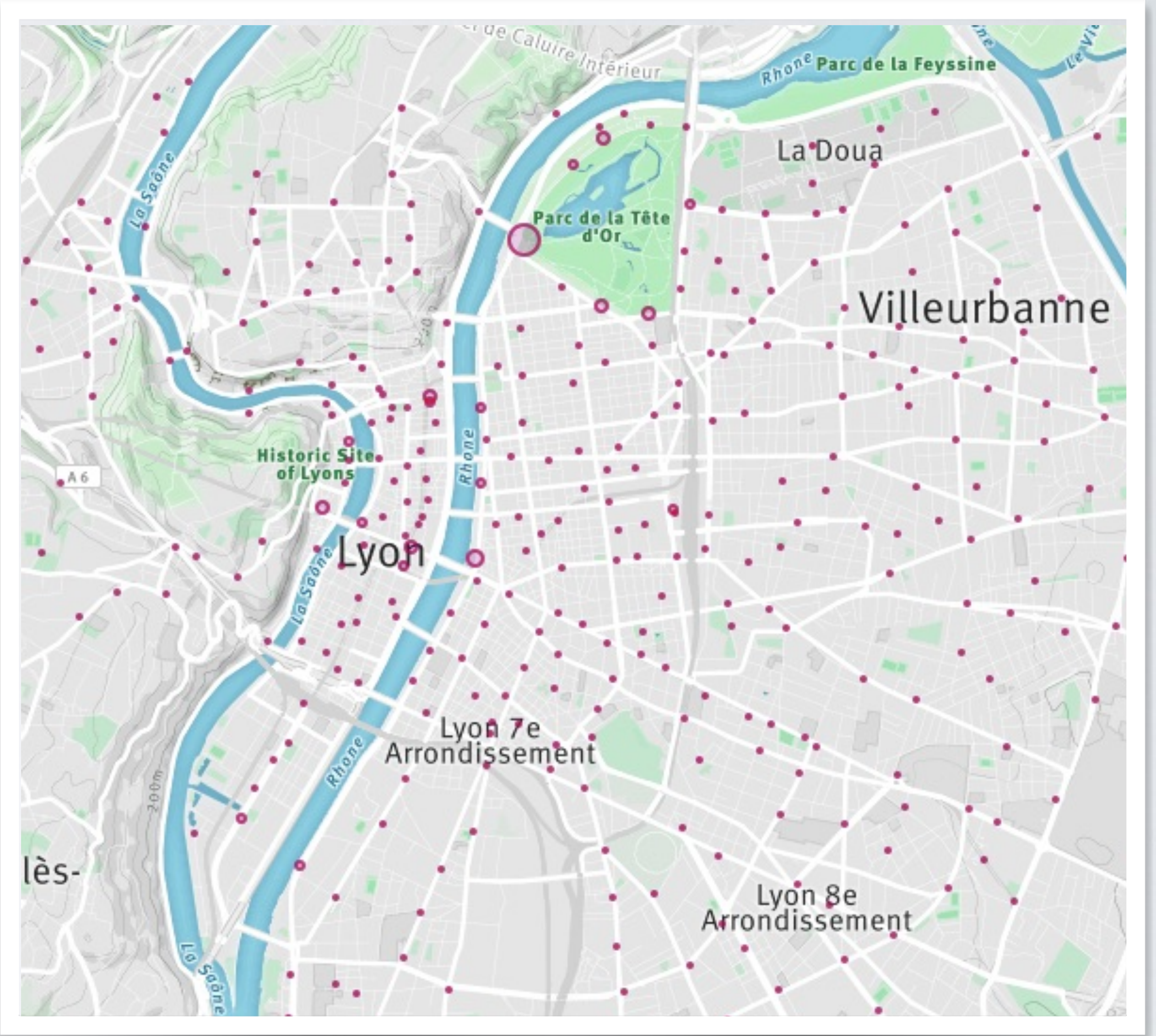
"Leisure" ?

...

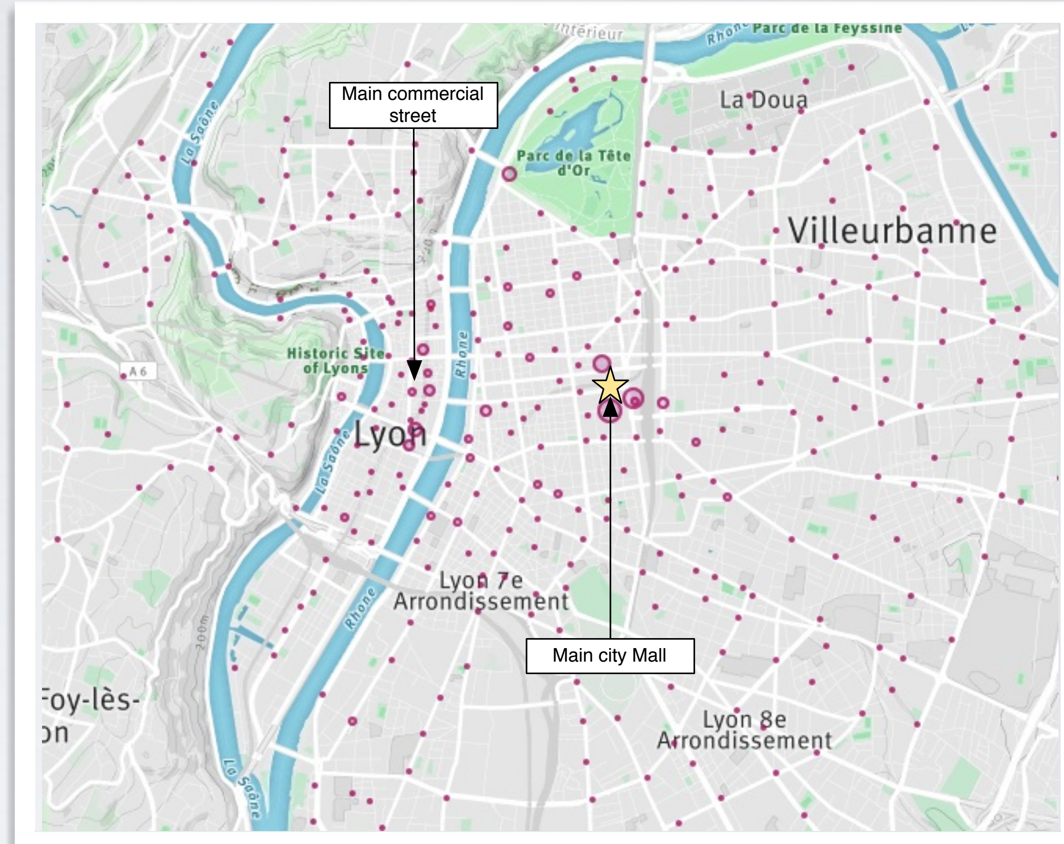
For each pattern, for each station,
we have a value
=>Total trips due to this pattern

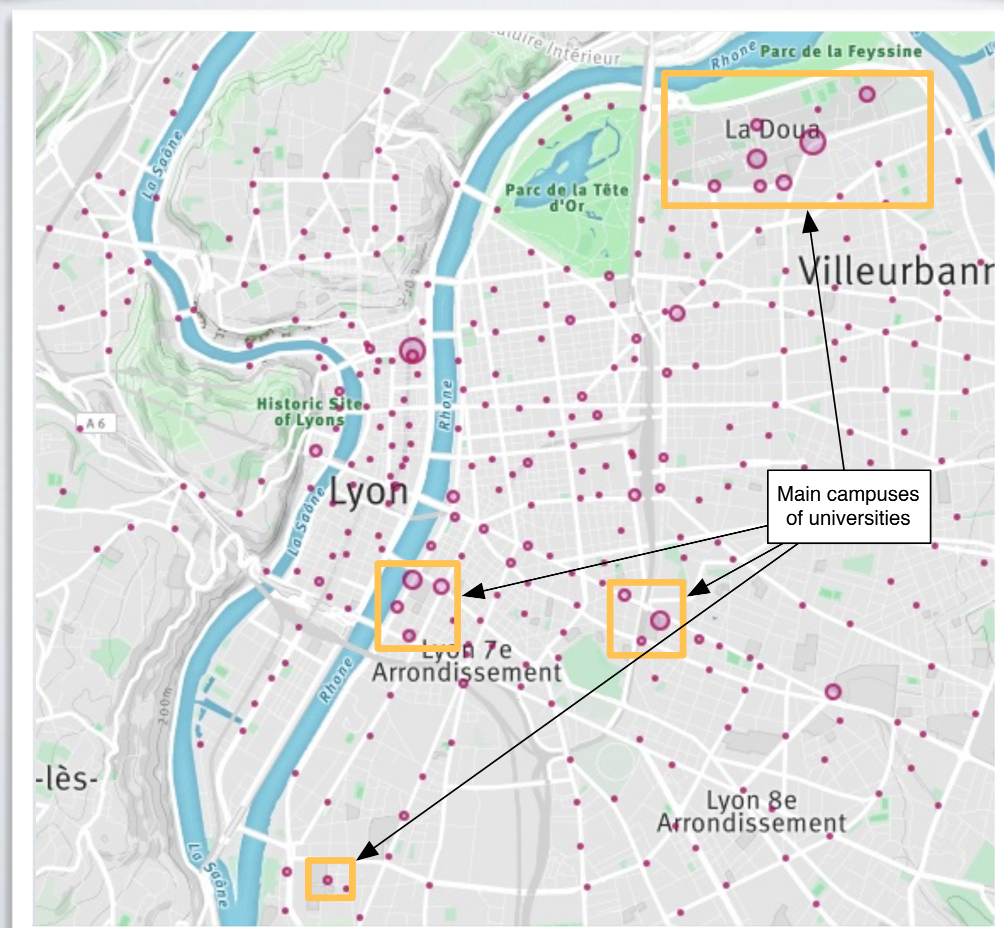


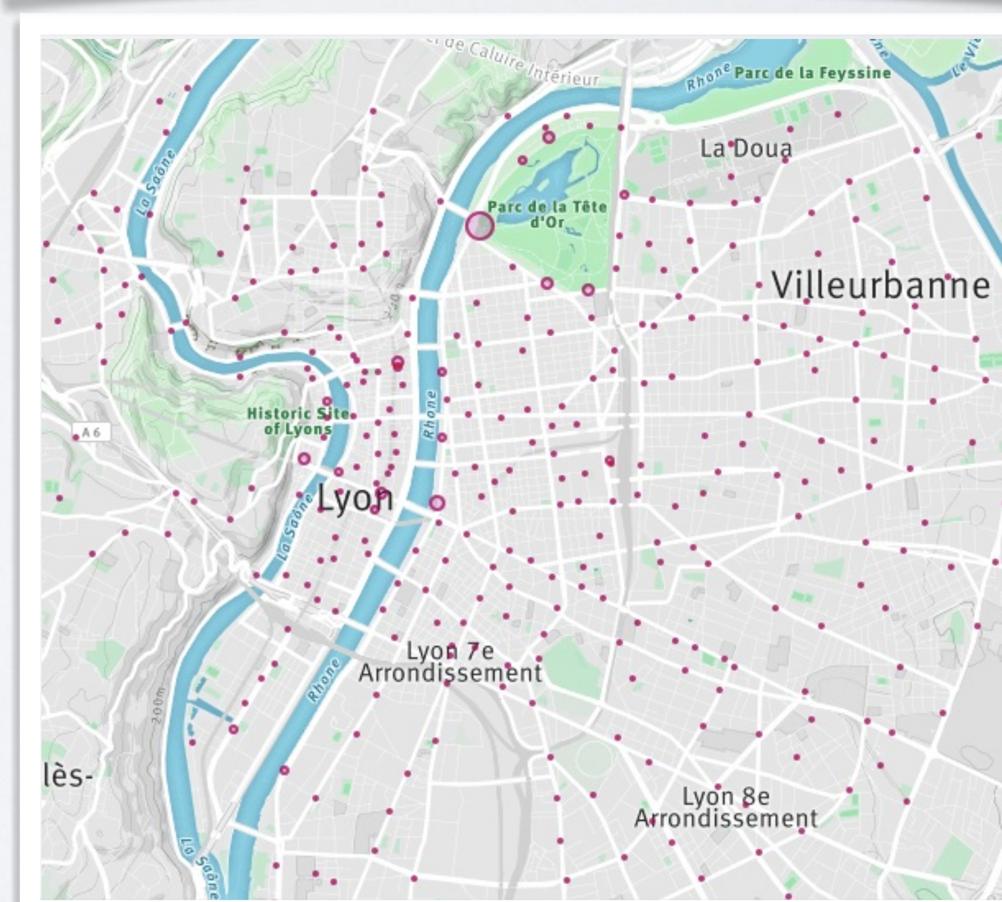
"Leisure" ?




La Doua
Parc de la Tête d'Or
Villeurbanne
Historic Site of Lyons
Lyon
Lyon 7e Arrondissement
Main train station
Lyon 8e Arrondissement
-lès-


Main commercial street
Parc de la Feyssine
La Doua
Parc de la Tête d'Or
Villeurbanne
Historic Site of Lyons
Lyon
Lyon 7e Arrondissement
Main city Mall
Foy-lès-
Lyon 8e Arrondissement


Parc de la Feyssine
La Doua
Parc de la Tête d'Or
Villeurbann
Historic Site of Lyons
Lyon
Main campuses of universities
Lyon 7e Arrondissement
-lès-
Lyon 8e Arrondissement

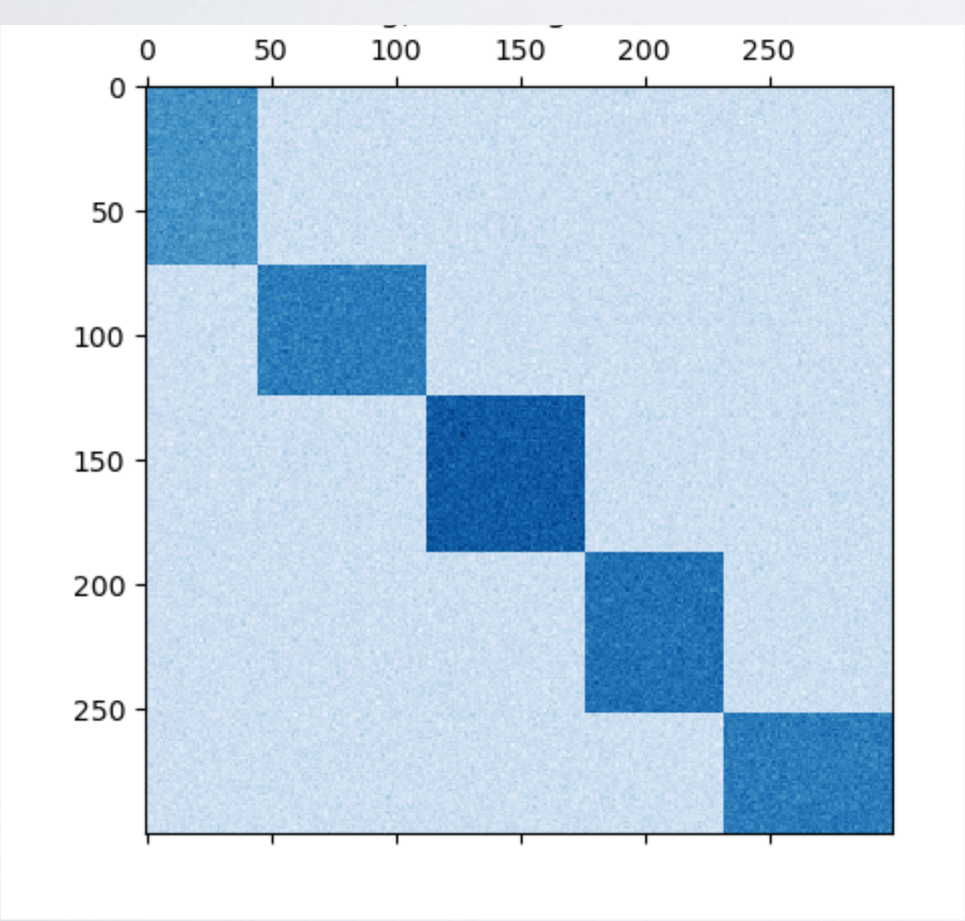

Parc de la Feyssine
La Doua
Parc de la Tête d'Or
Villeurbanne
Historic Site of Lyons
Lyon
Lyon 7e Arrondissement
lès-
Lyon 8e Arrondissement

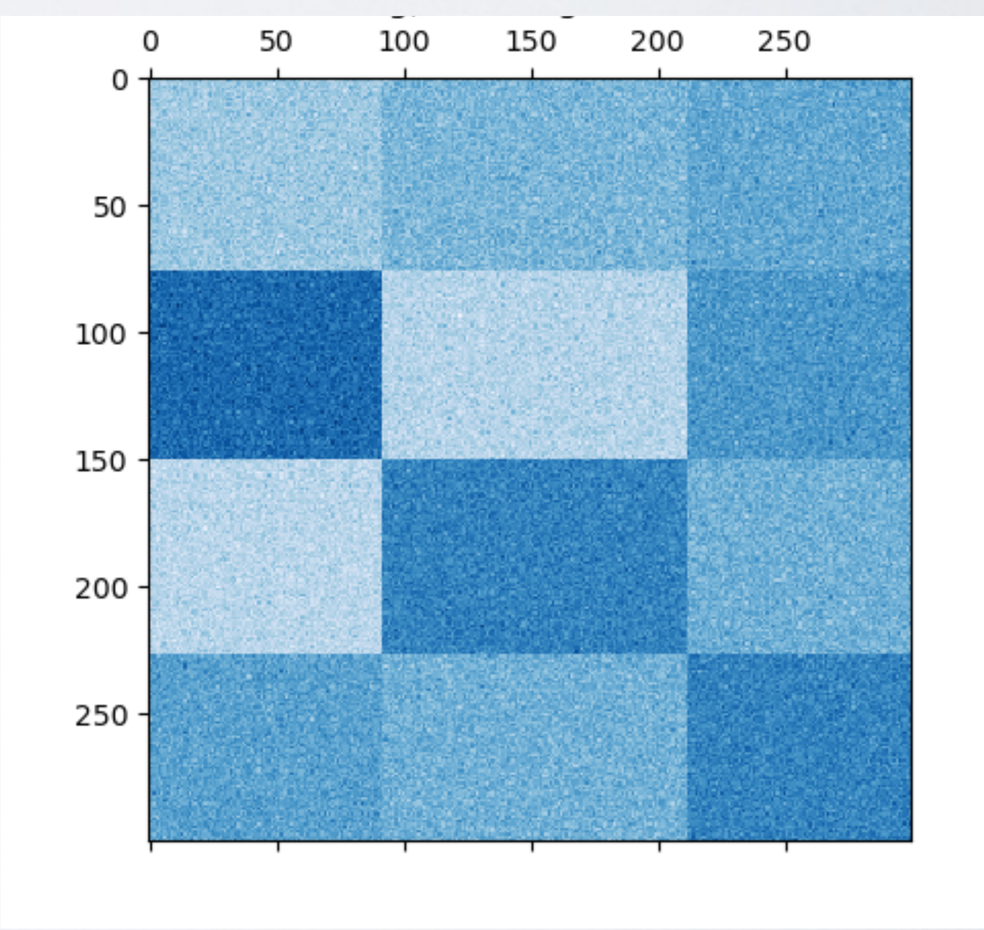
CO-CLUSTERING

Or Bi-clustering, two-mode clustering, block clustering

CO-CLUSTERING

- Objective: Find dense submatrices in a matrix
- Groups of rows that are preferentially related to groups of columns





CO-CLUSTERING

- Various algorithms exist, a simple one for sparse data consists in optimizing a modified version of the modularity on the bipartite graph (user-item)

- $$Q = \sum_{i}^{n} \sum_{j}^{d} A_{ij} - \frac{k_i k_j}{|A|} \delta_{ij}$$
  - With A the matrix to co-cluster, dimension $n \times d$
  - k_i the weighted degree(strength) of i
  - δ_{ij}=1 if i, j belong to the same co-cluster
  - $|A|$ sum of all values in the matrix

https://dl.acm.org/doi/pdf/10.1145/2806416.2806639

CO-CLUSTERING

- Co-cluster make natural sense in user-item matrices
  - Group of people who like the same type of products, and products liked by the same people
- Co-clustering can be used to improve recommender systems
  - To improve scalability, one can compute co-cluster first, and then use only users/items in the same co-cluster for recommendation
  - It can also improve precision: remove the effect of most popular items, that tend to be recommended to everyone