GEPHI - NETWORK SCIENCE

WHO AM I

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  - LIRIS, DM2L Team (Data Mining & Machine Learning)
- Computer Scientist => Network Scientist
- Member of IXXI
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- http://cazabetremy.fr

OUTLINE

- Introduction to network science

- Using Gephi

GEPHI

- A software for network visualisation and basic analysis

- Several other software exists but less options
  - Gephi (UTC Compiègne, science po, ...=> International)
  - Cytoscape (Seattle => International)
  - Tulip (bordeaux)

- Libraries in programming languages
  - Networkx, igraph, graph-tools, etc.

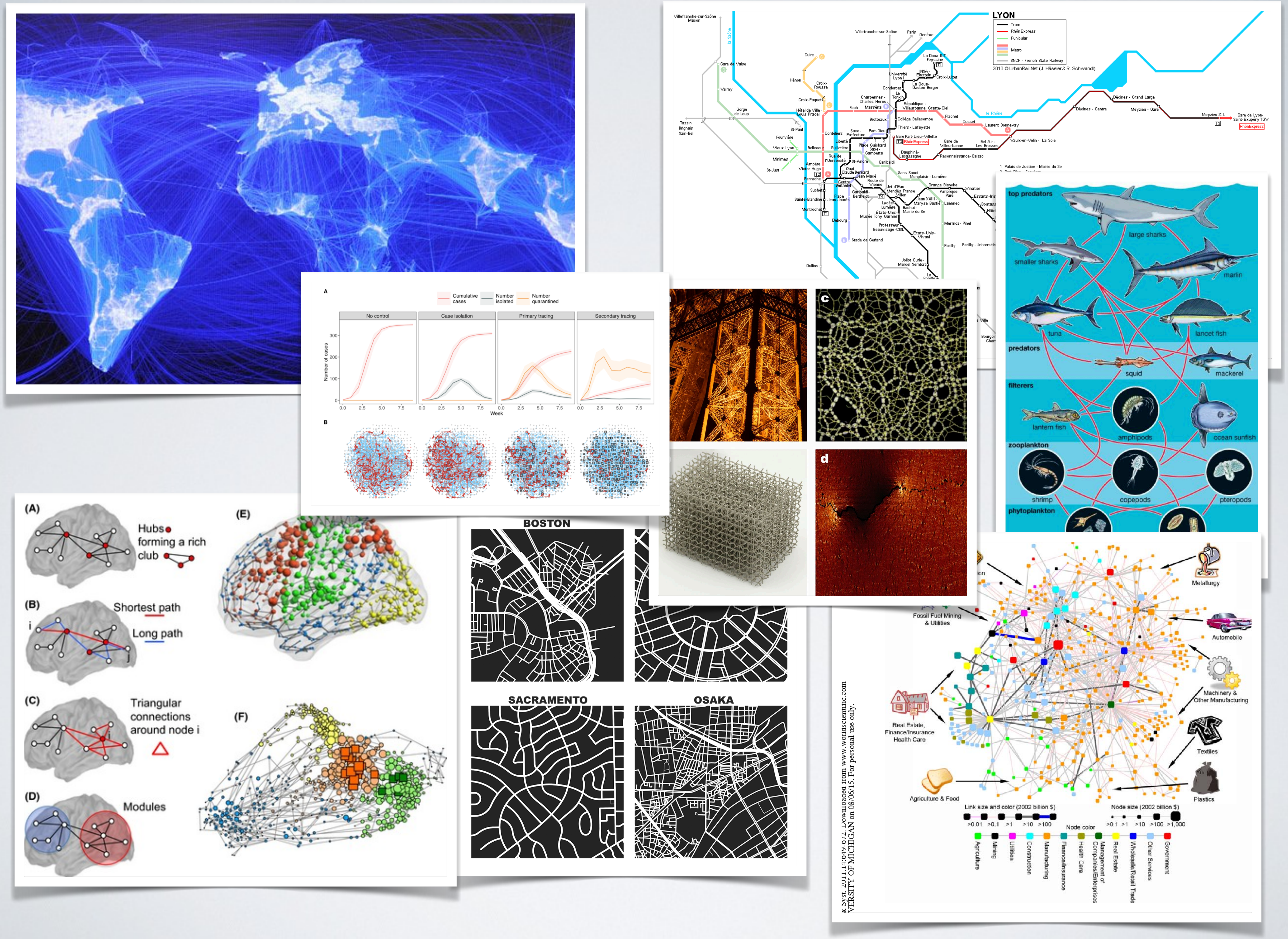
COMPLEX NETWORKS

WHAT?
WHY?
WHY NOW?
WHAT FOR?

COMPLEX SYSTEMS

- **Complex systems**: Systems composed of multiple **parts** in **interactions**

- Complex networks model the interactions between the parts
  - A common framework applicable to many systems
  - =>Many networks share similar characteristics
  - =>Similar processes shape the networks


LYON
Tram
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Villefranche-sur-Saône
Macon
Paris
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Croix-Rousse
Hôtel de Ville - Louis Pradel
Charpennes - Charles Hernu
Part-Dieu
Gare Part-Dieu-Villette
Perrache
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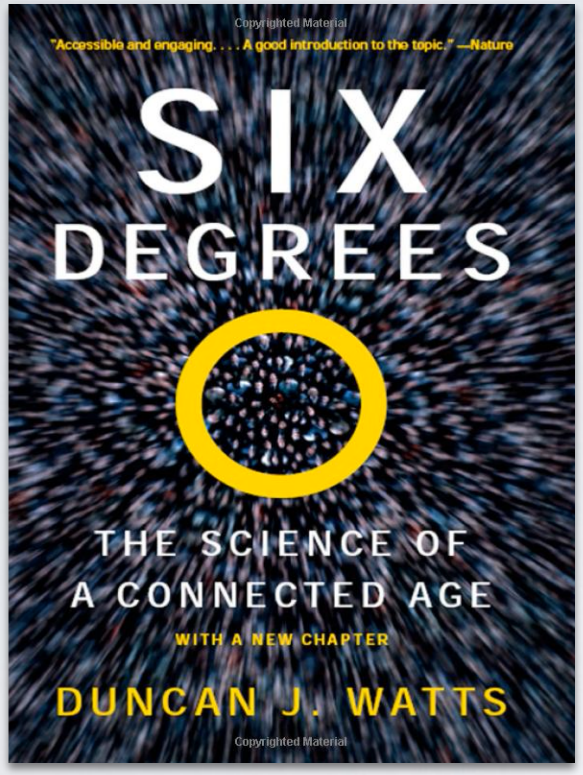
Laurent Bonnevay
Vaulx-en-Velin - La Soie
Décines - Grand Large
Meyzieu Z.I.
Gare de Lyon-Saint-Exupéry TGV
Stade de Gerland
A
Cumulative cases
Number isolated
Number quarantined
No control
Case isolation
Primary tracing
Secondary tracing
Number of cases
0
100
200
300
0.0
2.5
5.0
7.5
Week
B
c
d
top predators
large sharks
smaller sharks
marlin
tuna
lancet fish
predators
squid
mackerel
filterers
lantern fish
amphipods
ocean sunfish
zooplankton
shrimp
copepods
pteropods
phytoplankton
(A)
Hubs forming a rich club
(B)
Shortest path
Long path
i
j
(C)
Triangular connections around node i
(D)
Modules
(E)
(F)
BOSTON
SACRAMENTO
OSAKA
Metallurgy
Fossil Fuel Mining & Utilities
Automobile
Machinery & Other Manufacturing
Real Estate, Finance/Insurance Health Care
Textiles
Agriculture & Food
Plastics
Link size and color (2002 billion $)
>0.01
>0.1
>1
>10
>100
Node size (2002 billion $)
>0.1
>1
>10
>100
>1,000
Node color
Agriculture
Mining
Utilities
Construction
Manufacturing
Finance/Insurance
Health Care
Management of Companies/Enterprises
Real Estate
Wholesale/Retail Trade
Other Services
Government

Materials

Pop-science books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THE SCIENCE OF A CONNECTED AGE
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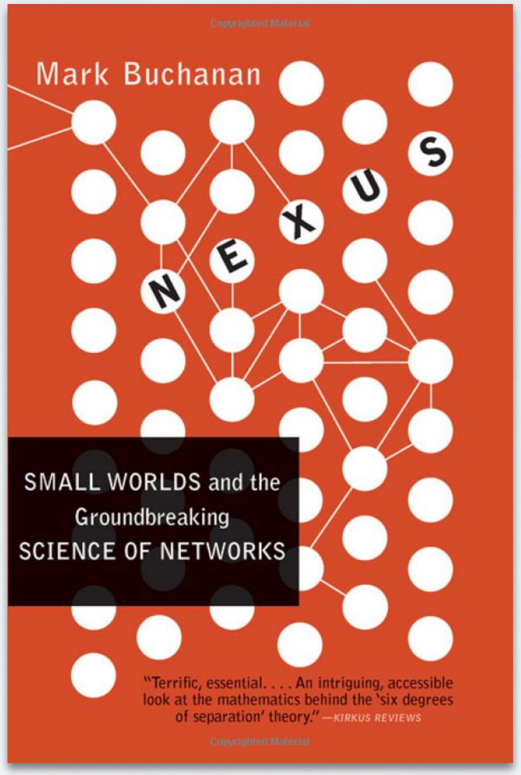
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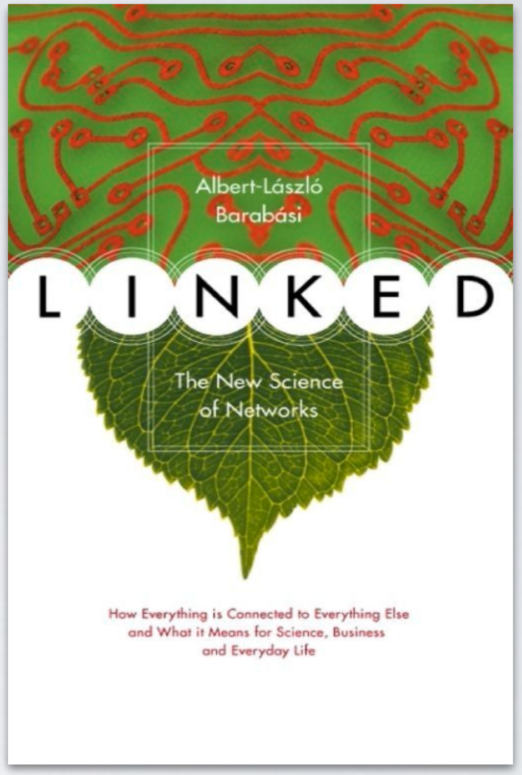
Albert-László Barabási
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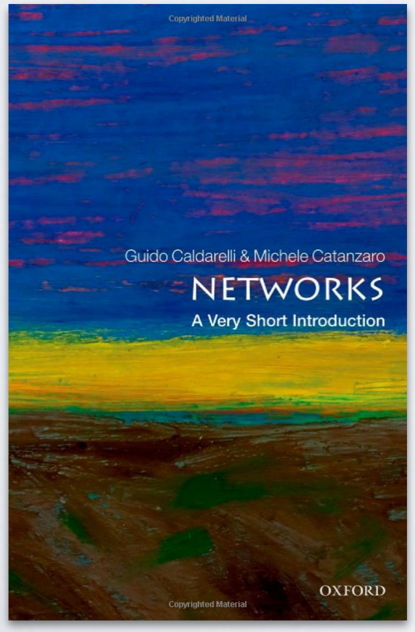
The New Science of Networks
How Everything is Connected to Everything Else and What it Means for Science, Business and Everyday Life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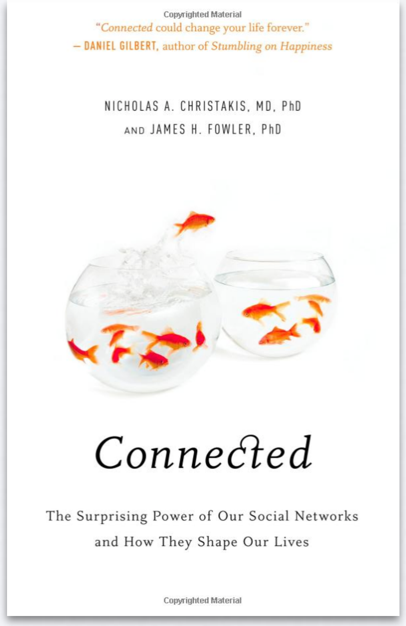
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— DANIEL GILBERT, author of Stumbling on Happiness
NICHOLAS A. CHRISTAKIS, MD, PhD
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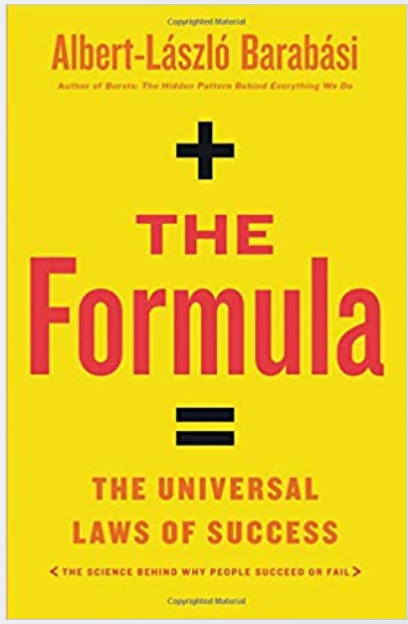
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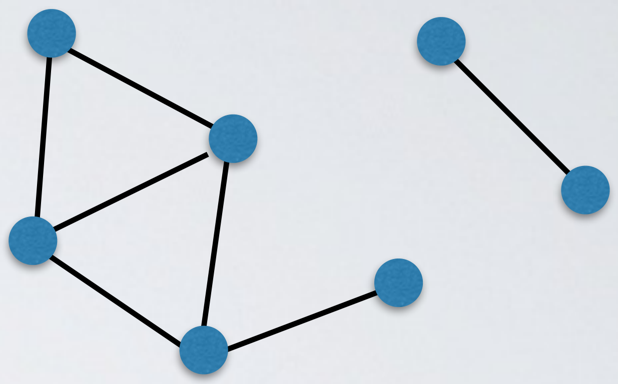
GRAPHS & NETWORKS

GRAPHS & NETWORKS



Networks often refers to real systems

- www,
- social network
- metabolic network.
- Language: (Network, node, link)

Vertex	Edge
person	friendship
neuron	synapse
Website	hyperlink
company	ownership
gene	regulation

Graph is the mathematical representation of a network

- Language: (Graph, vertex, edge)

In most cases we will use the two terms interchangeably.

GRAPH REPRESENTATION

NETWORK REPRESENTATIONS

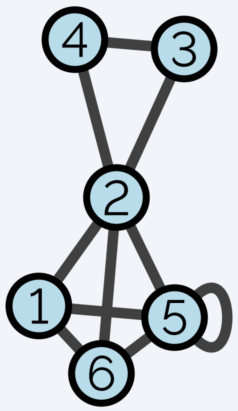
Networks: Graph notation

Graph notation : $G = (V, E)$

V	set of vertices/nodes.
E	set of edges/links.
$u \in V$	a node.
$(u, v) \in E$	an edge.

Network - Graph notation

Graph



Graph notation

$$G = (V, E)$$
$$V = \{1, 2, 3, 4, 5, 6\}$$
$$E = \{(1, 2), (1, 6), (1, 5), (2, 4), (2, 3), (2, 5), (2, 6), (6, 5), (5, 5), (4, 3)\}$$

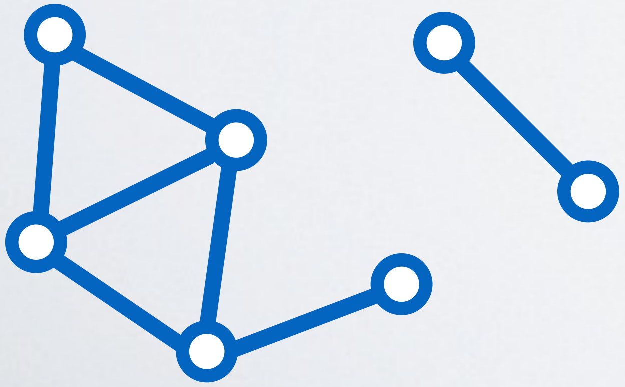
Types of Networks

Undirected networks

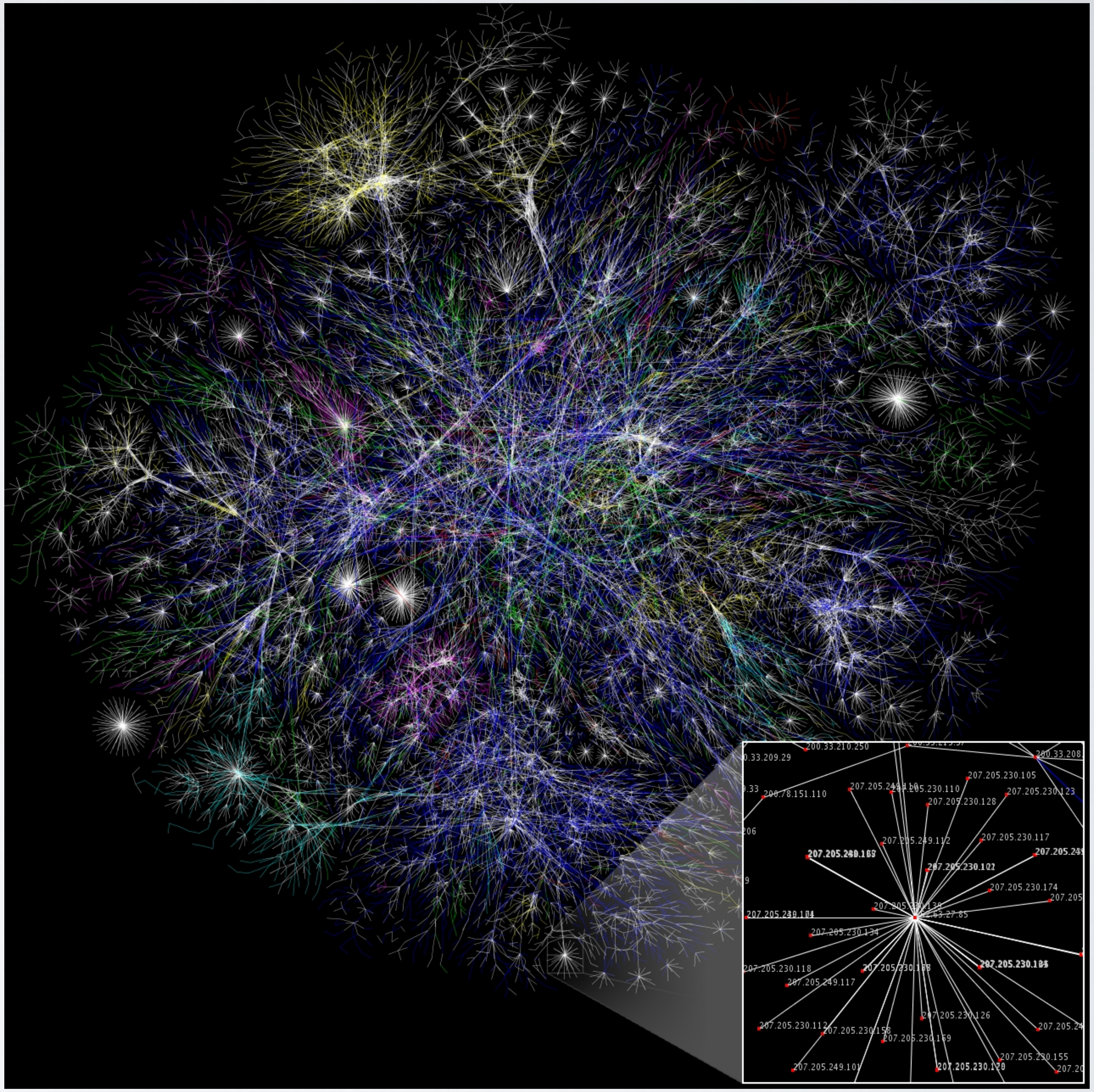
$G=(V, E)$

$(u,v) \in E \equiv (v,u) \in E$

- The directions of edges do not matter
- Interactions are possible between connected entities in both directions



Opte project



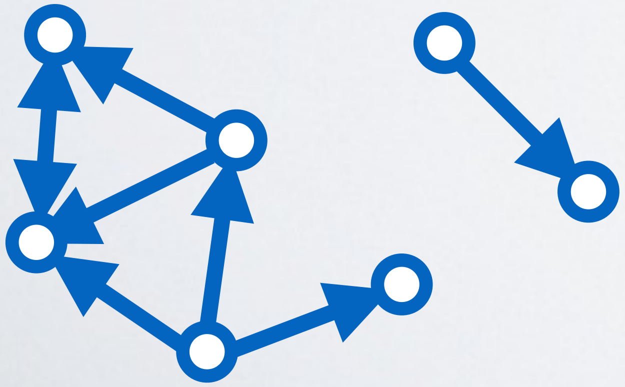
The Internet: Nodes - routers, Links - physical wires

Directed networks

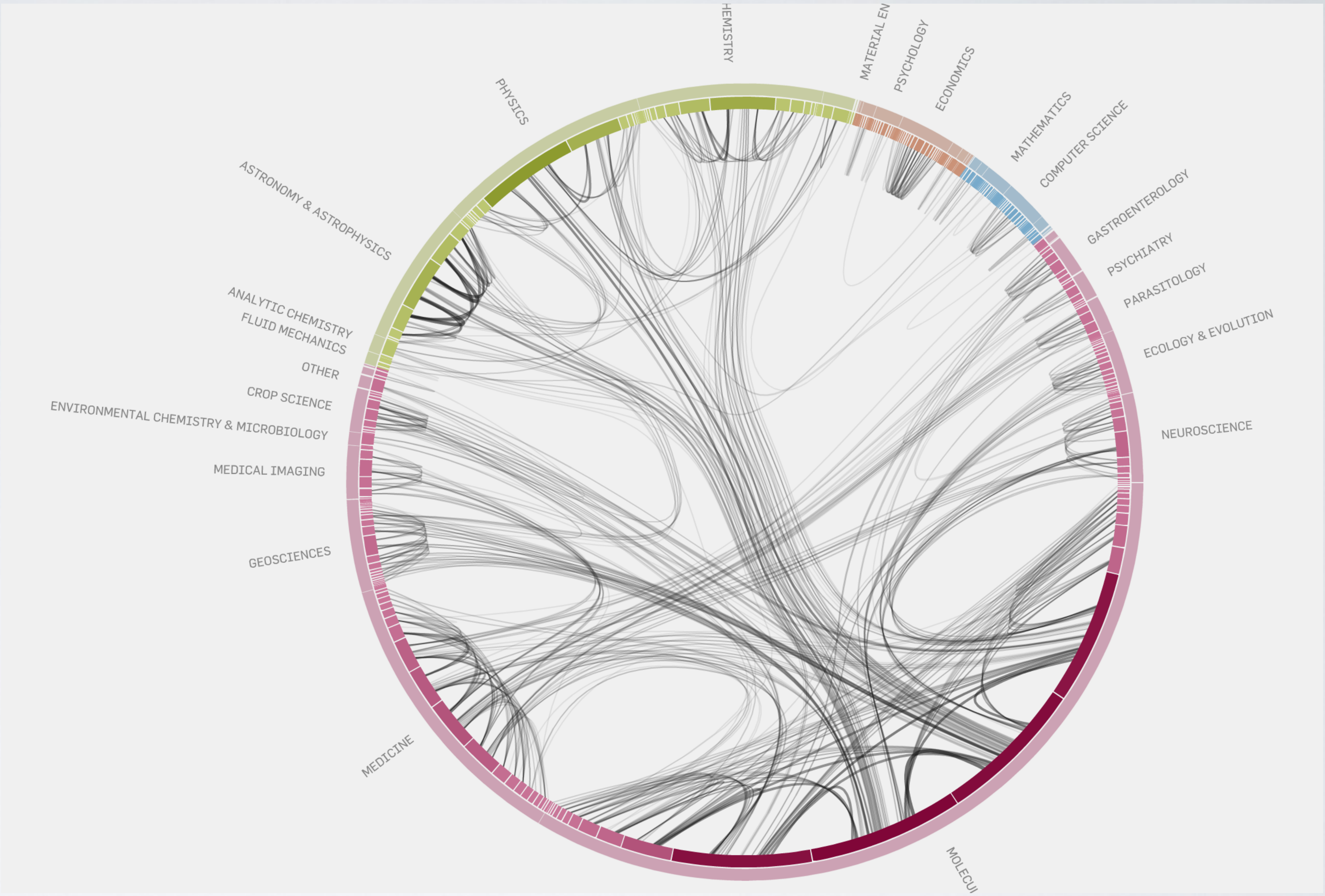
$$G=(V, E)$$

$$(u,v) \in E \not\equiv (v,u) \in E$$

- The directions of edges matter
- Interactions are possible between connected entities only in specified directions



Moritz Stefaner, eigenfactor.com



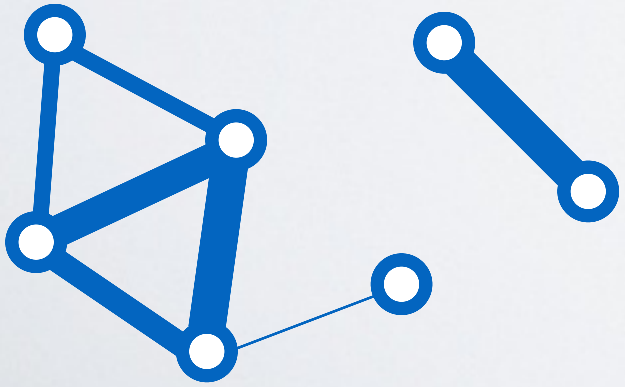
Citation network: Nodes - publications, Links - references

Weighted networks

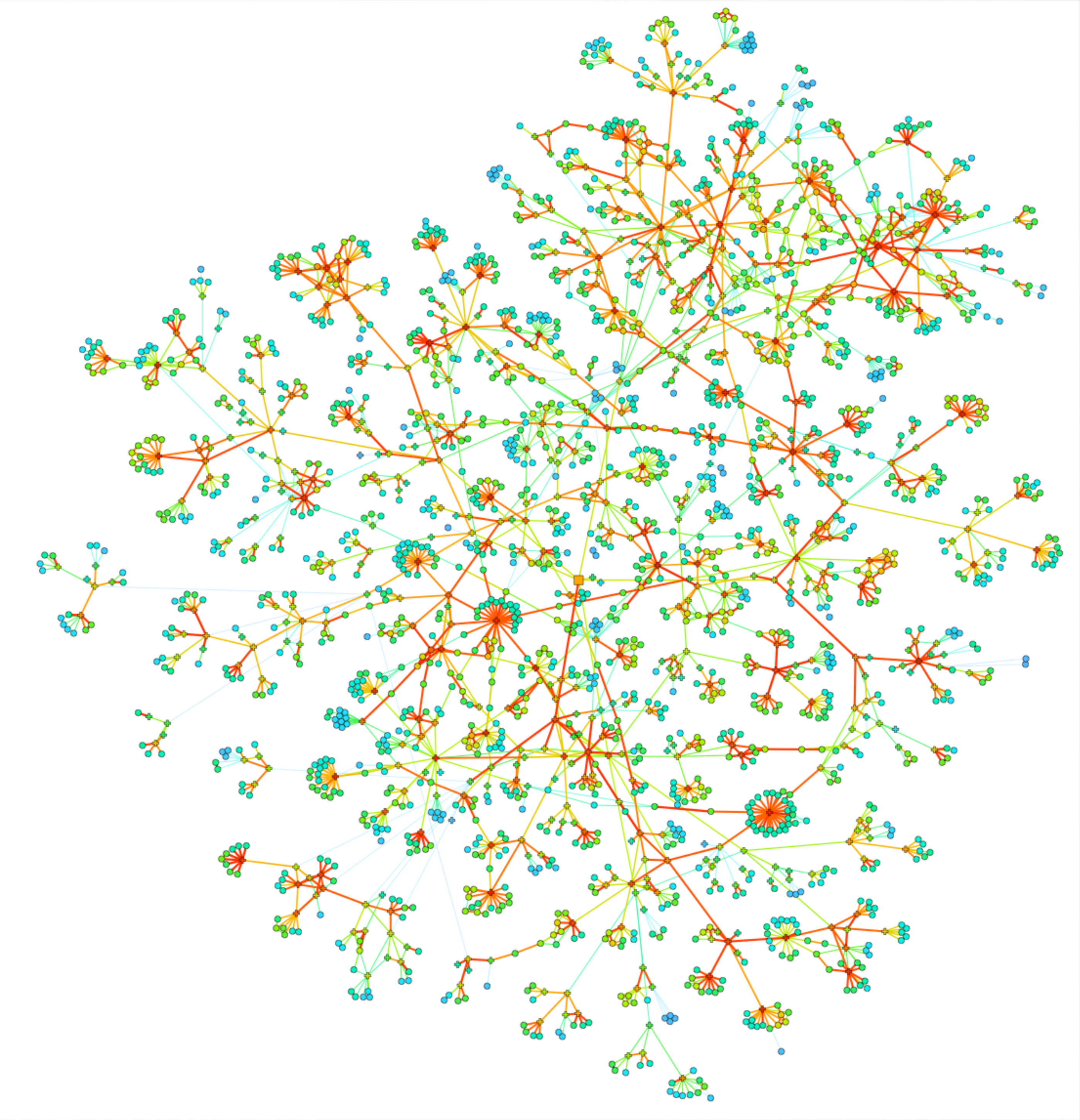
$G=(V, E, w)$

$w: (u,v) \in E \Rightarrow R$

- Strength of interactions are assigned by the weight of links

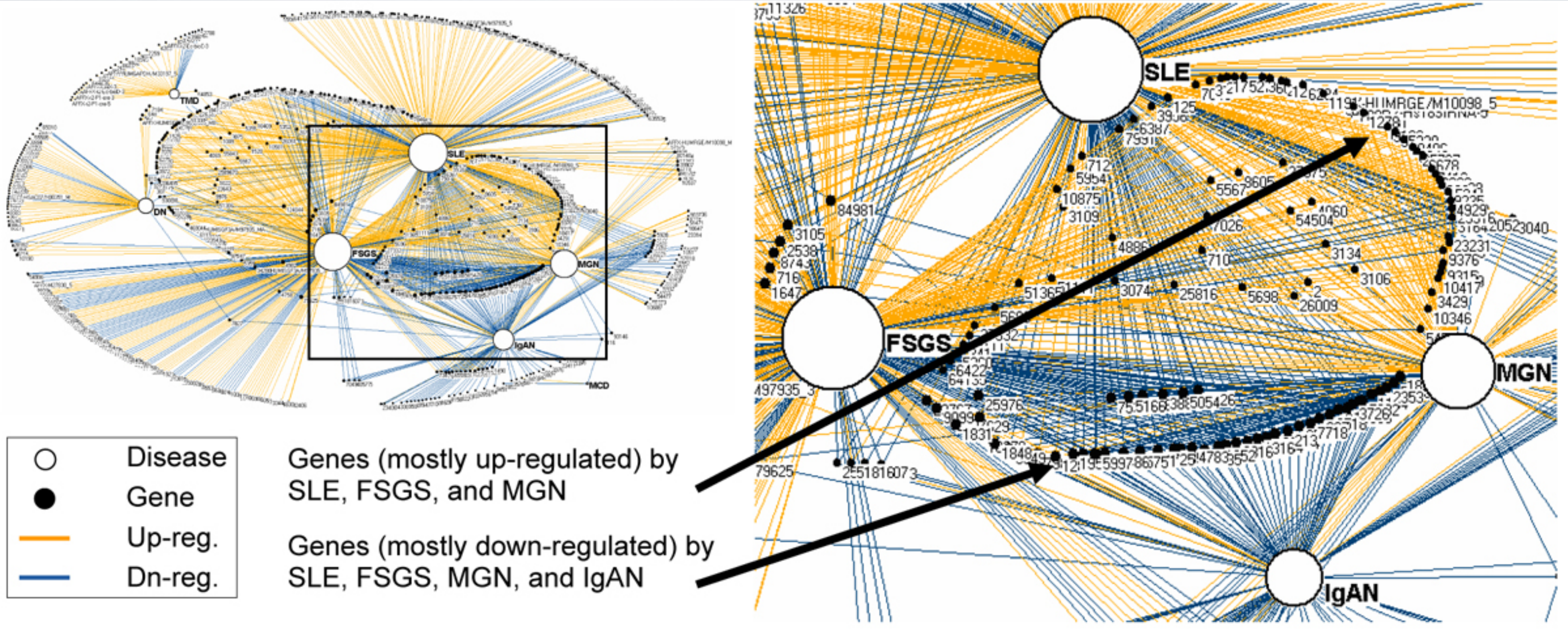


Onnela et.al. New Journal of Physics 9, 179 (2007).



Social interaction network: Nodes - individuals
Links - social interactions

Bipartite network

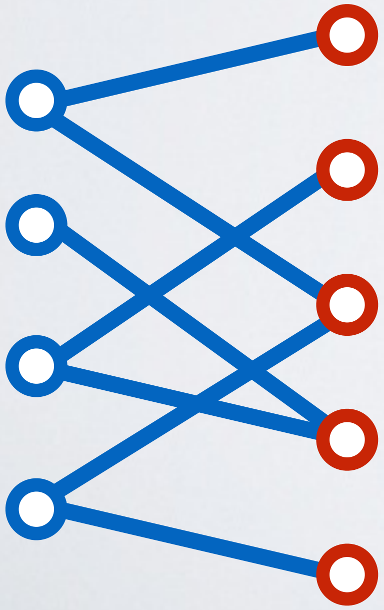

SLE
FSGS
MGN
IgAN
DN
TMD
MCD
○ Disease
● Gene
Up-reg.
Dn-reg.
Genes (mostly up-regulated) by SLE, FSGS, and MGN
Genes (mostly down-regulated) by SLE, FSGS, MGN, and IgAN

Bhavnani et.al. BMC Bioinformatics 2009, **10**(Suppl 9):S3

Gene-desease network:

Nodes - Desease (7)&Genes (747)

Links - gene-desease relationship



$$G=(U, V, E)$$

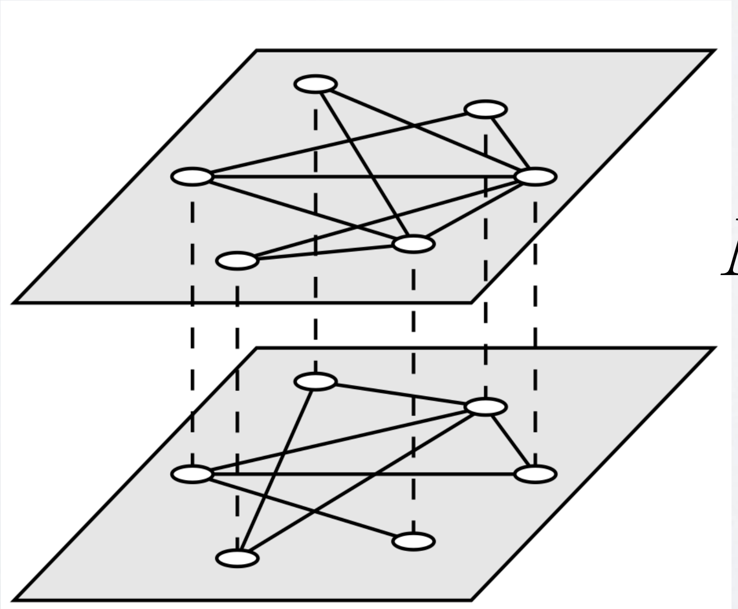
$$U \cap V = \varnothing$$

$$\forall (u,v) \in E,\ u \in U \text{ and } v \in V$$

Multiplex and multilayer networks

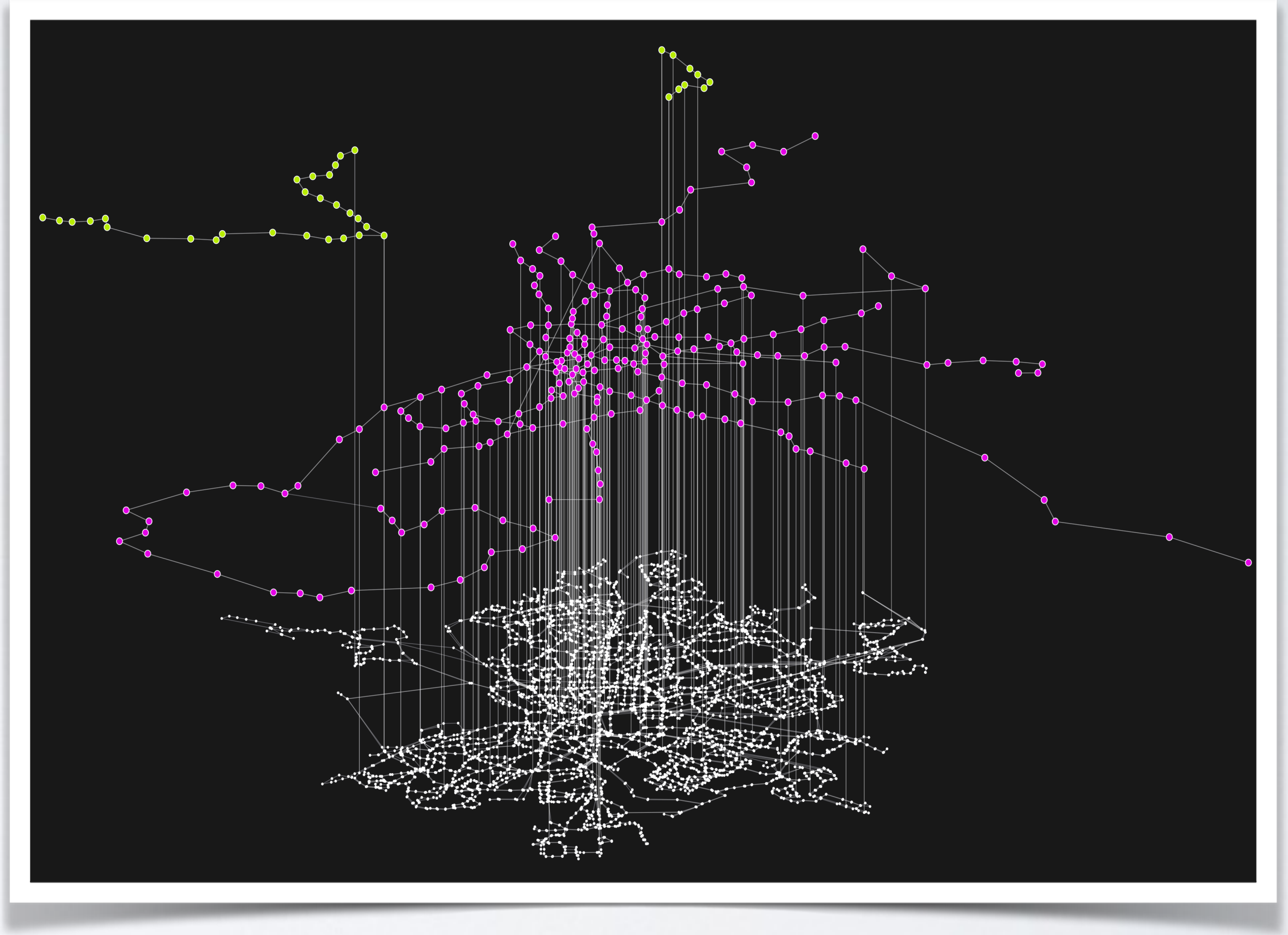
$G=(V, E_i), i=1\ldots M$

- Nodes can be present in multiple networks simultaneously
- These networks are connected (can influence each other) via the common nodes



Gomes et.al. Phys. Rev. Lett. 110, 028701 (2013)

$M=2$



[Mendez-Bermudez et al. 2017]

Temporal and evolving networks

$G=(V, E_t),\ (u,v,t,d) \in E_t$

t - time of interaction (u,v)
d - duration of interaction (u,v,t)

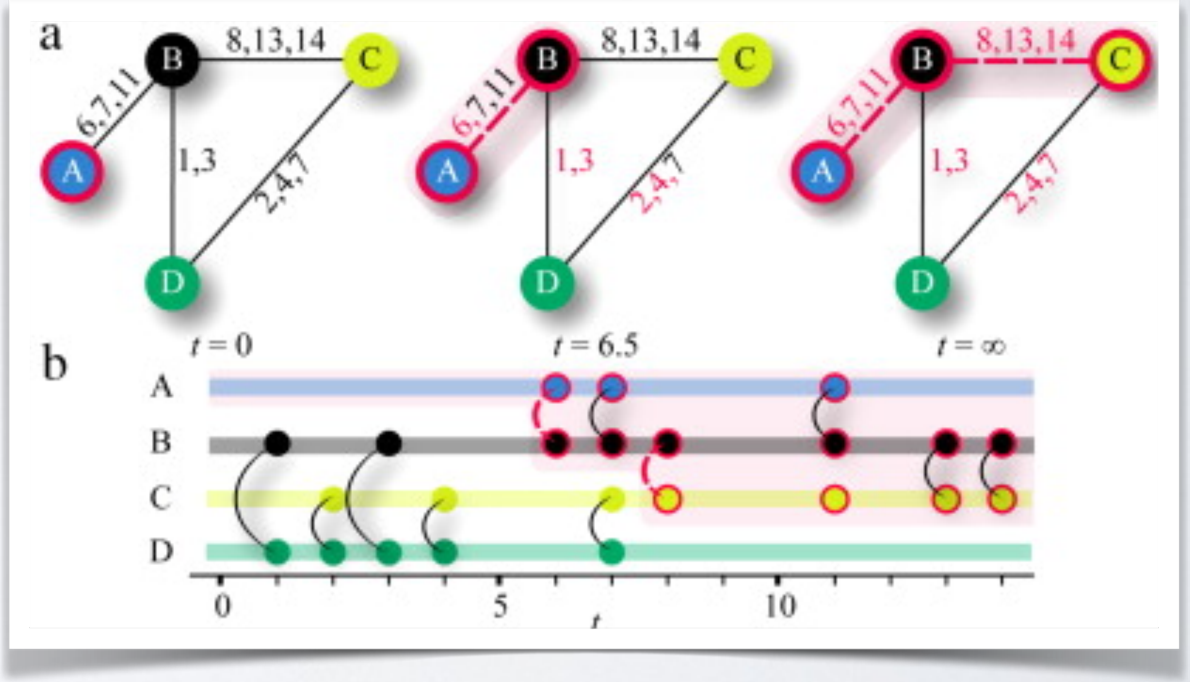
- Temporal links encode time varying interactions

$G=(V_{t'}, E_{t'})$

$v(t) \in V_{t'}$

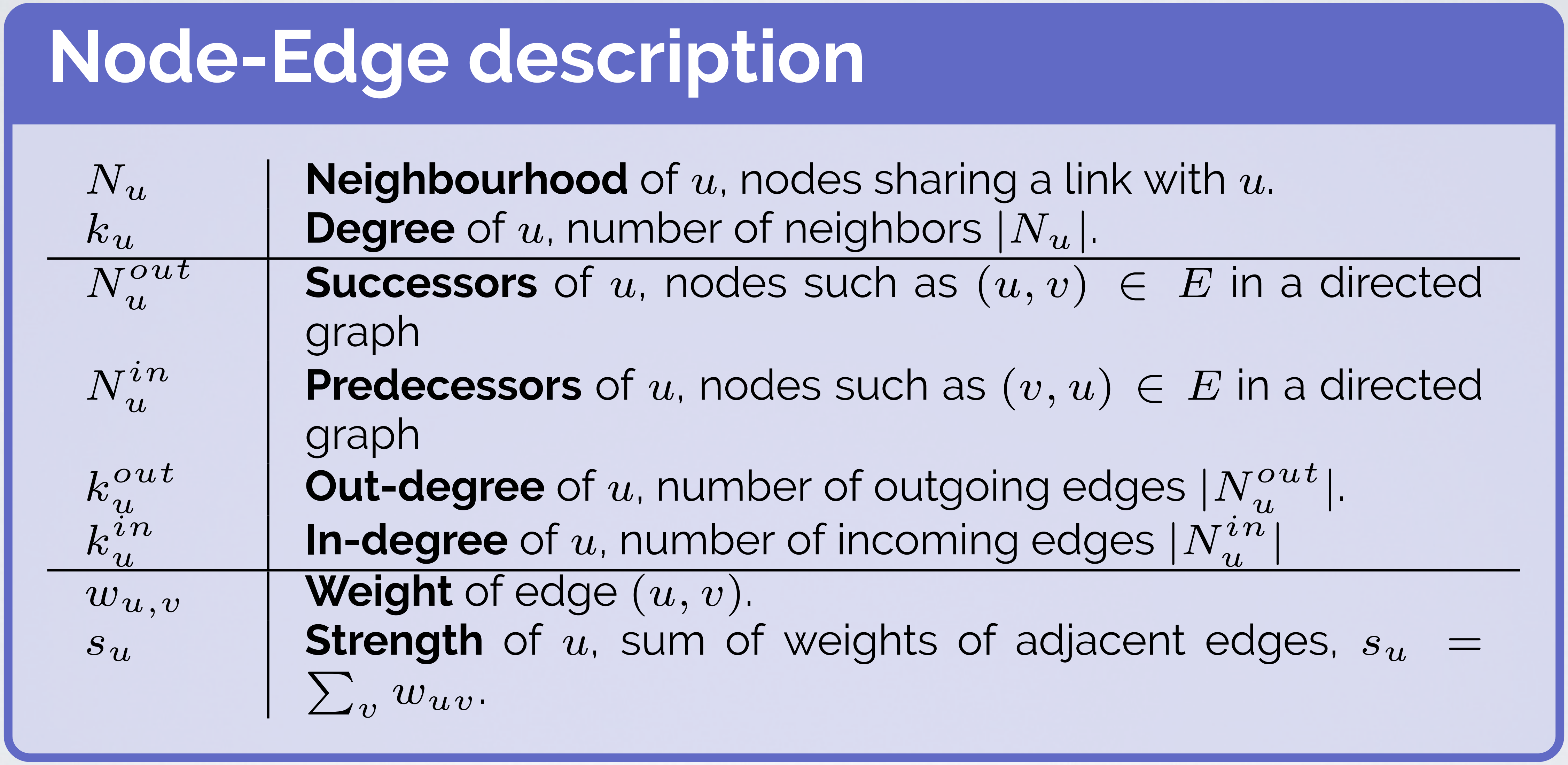
$(u,v,t) \in E_{t'}$

- Dynamical nodes and links encode the evolution of the network



Mobile communication network
Nodes - individuals
Links - calls and SMS

NETWORK REPRESENTATIONS



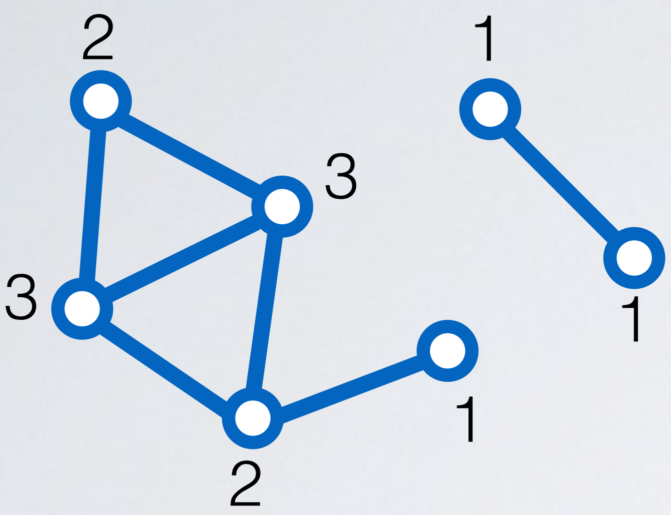
Node-Edge description

N_u	**Neighbourhood** of u, nodes sharing a link with u.
k_u	**Degree** of u, number of neighbors $\|N_u\|$.
N_u^{out}	**Successors** of u, nodes such as $(u, v) \in E$ in a directed graph
N_u^{in}	**Predecessors** of u, nodes such as $(v, u) \in E$ in a directed graph
k_u^{out}	**Out-degree** of u, number of outgoing edges $\|N_u^{out}\|$.
k_u^{in}	**In-degree** of u, number of incoming edges $\|N_u^{in}\|$
$w_{u,v}$	**Weight** of edge (u, v).
s_u	**Strength** of u, sum of weights of adjacent edges, $s_u = \sum_v w_{uv}$.

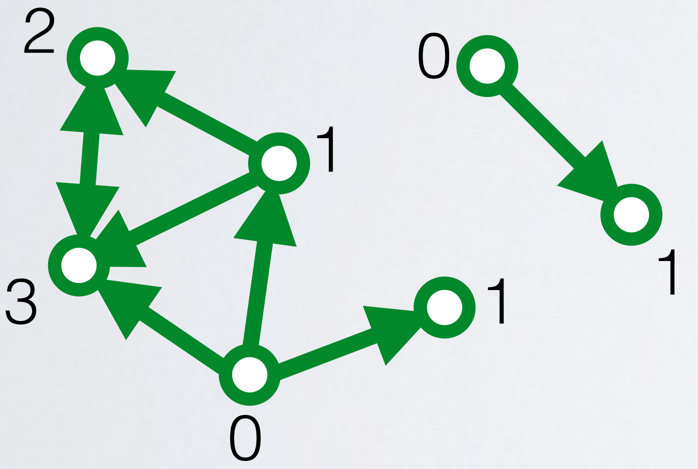
Node degree

Number of connections of a node

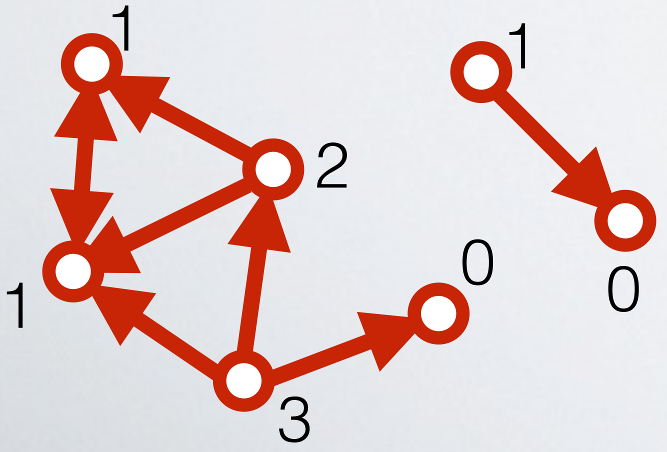
- Undirected network



- Directed network

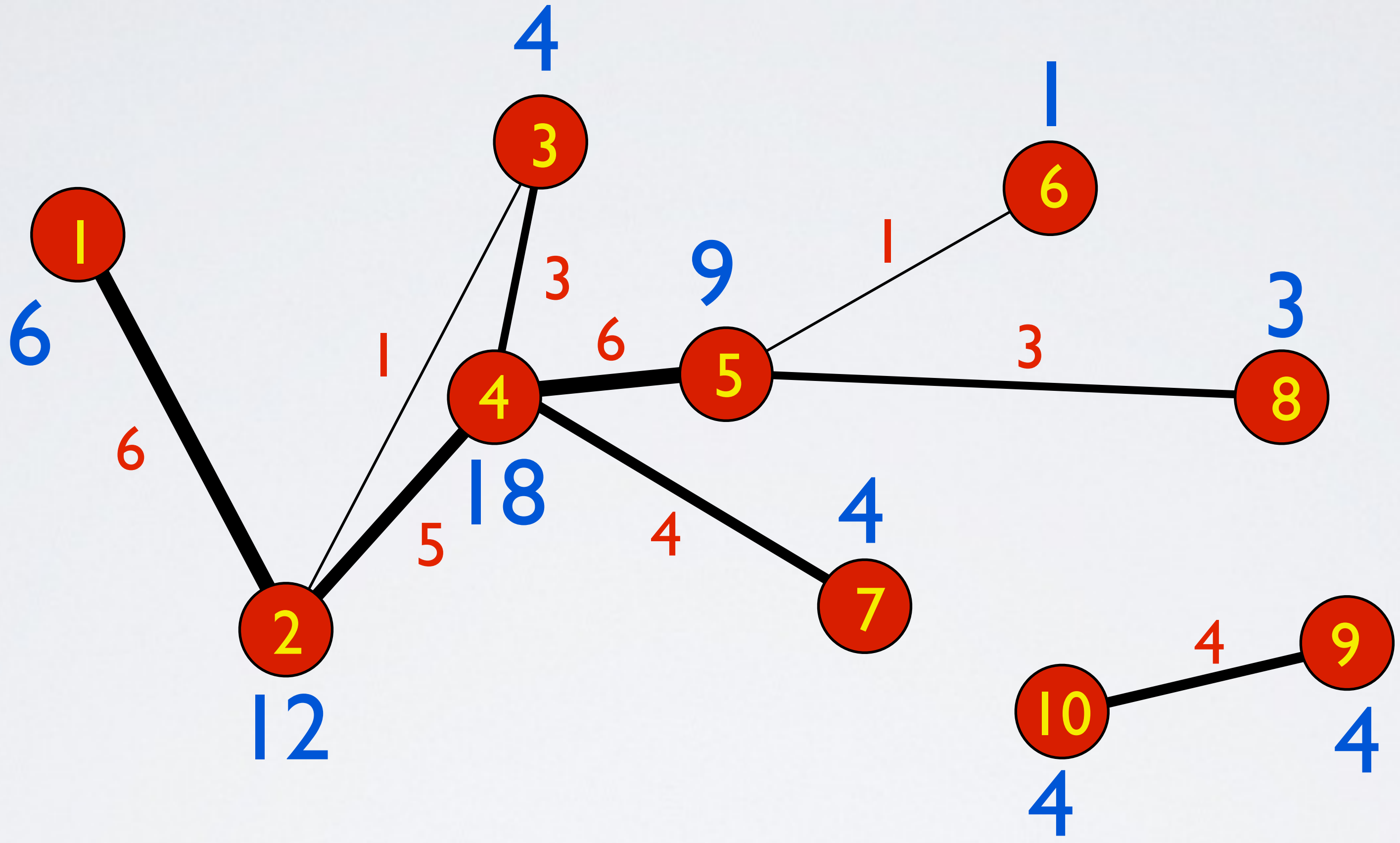


In degree



Out degree

Weighted degree: strength



DESCRIPTION OF GRAPHS

DESCRIPTION OF GRAPHS

- When confronted with a graph, how to describe it?
- How to compare graphs?
- What can we say about a graph?

SIZE

Counting nodes and edges

N/n	**size**: number of nodes $\|V\|$.
L/m	number of edges $\|E\|$
L_{max}	Maximum number of links

Undirected network: $\binom{N}{2} = N(N-1)/2$

Directed network: $\binom{N}{2} = N(N-1)$

SIZE

	#nodes (n)	#edges (m)
Wikipedia HL	2M	30M
Twitter 2015	288M	60B
Facebook 2015	1.4B	400B
Brain c. Elegans	280	6393
Roads US	2M	2.7M
Airport traffic	3k	31k

DENSITY

Network descriptors 1 - Nodes/Edges

$\langle k \rangle$ **Average degree**: Real networks are sparse, i.e., typically $\langle k \rangle \ll n$. Increases slowly with network size, e.g., $d \sim \log(m)$

$$\langle k \rangle = \frac{2m}{n}$$

$d/d(G)$ **Density**: Fraction of pairs of nodes connected by an edge in G.

$$d = L/L_{\max}$$

DENSITY

	#nodes	#edges	Density	avg. deg
Wikipedia	2M	30M	1.5x10^{-5}	30
Twitter 2015	288M	60B	1.4x10^{-6}	416
Facebook	1.4B	400B	4x10^{-9}	570
Brain c.	280	6393	0,16	46
Roads Calif.	2M	2.7M	6x10^{-7}	2,7
Airport	3k	31k	0,007	21

Beware: density hard to compare between graphs of different sizes

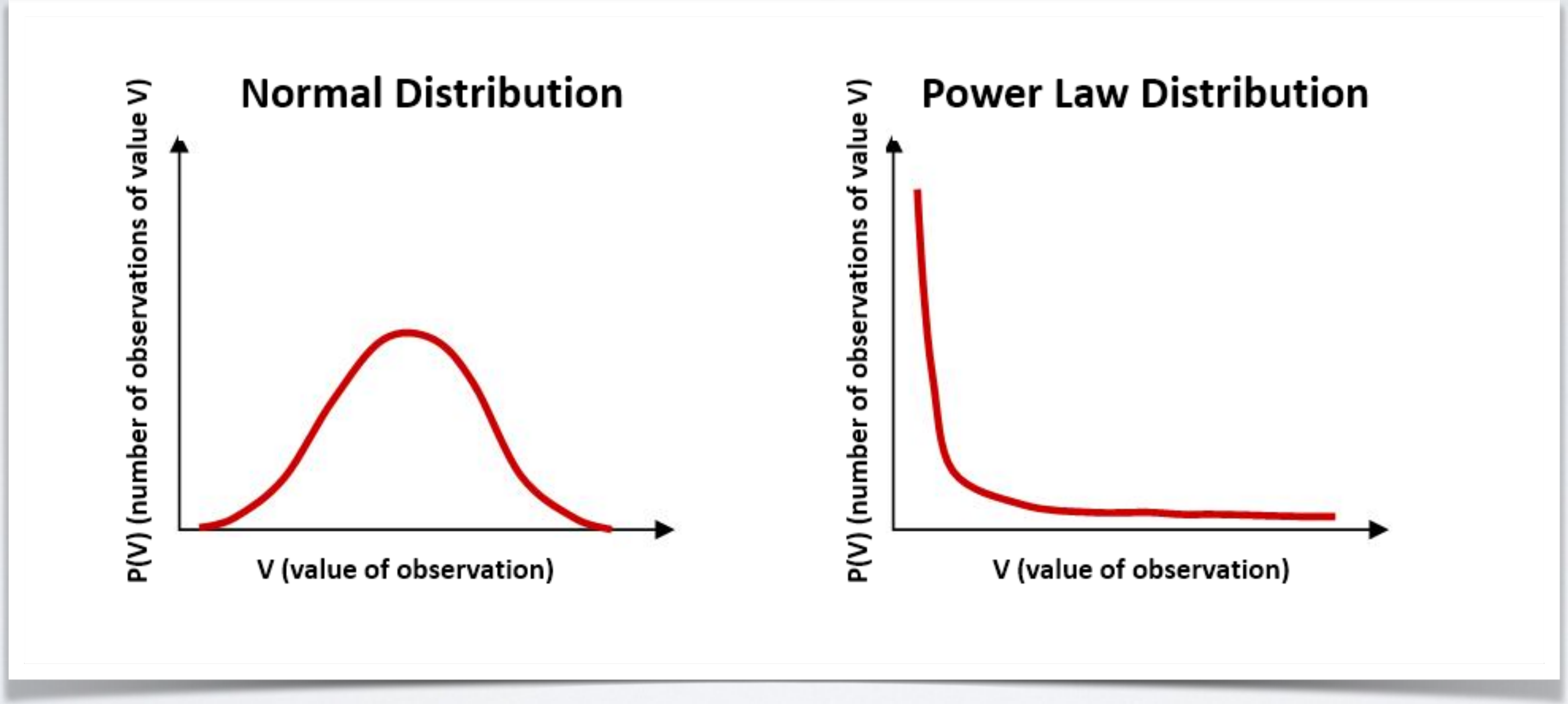
DENSITY

- It has been observed that: [Leskovec. 2006]
    - When graphs increase in size, the average degree increases
        - (Density on the contrary, decreases)
    - This increase is very slow

- Think of friends in a social network

Leskovec, Jure, Jon Kleinberg, and Christos Faloutsos. "Graphs over time: densification laws, shrinking diameters and possible explanations." *Proceedings of the eleventh ACM SIGKDD international conference on Knowledge discovery in data mining*. 2005.

DEGREE DISTRIBUTION



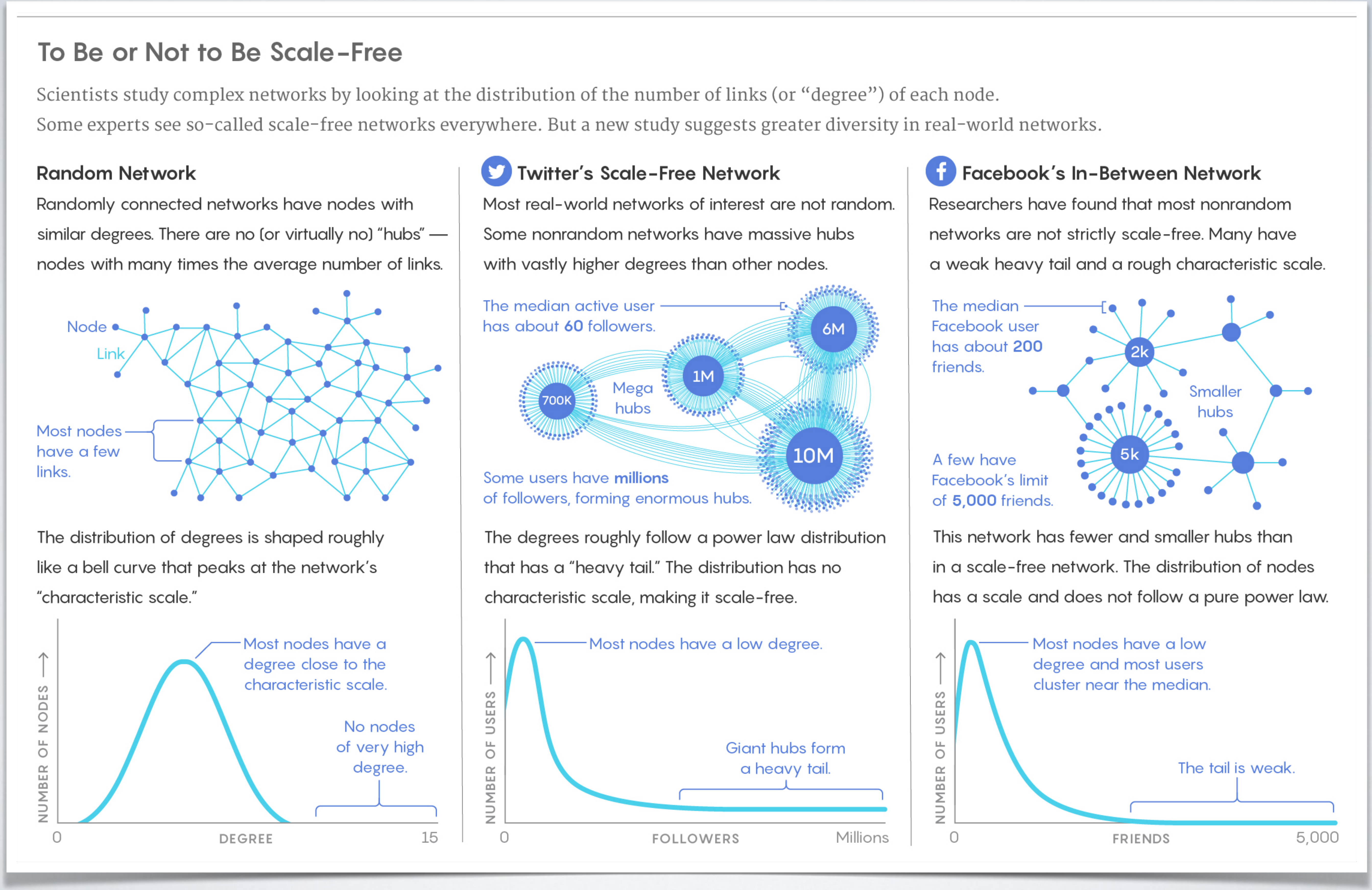
PDF (Probability Distribution Function)

DEGREE DISTRIBUTION

- In a fully random graph (Erdos-Renyi), degree distribution is (close to) a normal distribution centered on the average degree

- In real graphs, in general, it is not the case:
  - A high majority of small degree nodes
  - A small minority of nodes with very high degree (Hubs)

- Often modeled by a **power law**
  - More details later in the course



To Be or Not to Be Scale-Free

Scientists study complex networks by looking at the distribution of the number of links (or "degree") of each node.
Some experts see so-called scale-free networks everywhere. But a new study suggests greater diversity in real-world networks.

Random Network

Randomly connected networks have nodes with similar degrees. There are no (or virtually no) "hubs" — nodes with many times the average number of links.

The distribution of degrees is shaped roughly like a bell curve that peaks at the network's "characteristic scale."

Most nodes have a degree close to the characteristic scale.
No nodes of very high degree.
NUMBER OF NODES →
0
DEGREE
15

Twitter's Scale-Free Network

Most real-world networks of interest are not random. Some nonrandom networks have massive hubs with vastly higher degrees than other nodes.

The median active user has about **60** followers.
6M
1M
700K
Mega hubs
10M
Some users have **millions** of followers, forming enormous hubs.

The degrees roughly follow a power law distribution that has a "heavy tail." The distribution has no characteristic scale, making it scale-free.

Facebook's In-Between Network

Researchers have found that most nonrandom networks are not strictly scale-free. Many have a weak heavy tail and a rough characteristic scale.

The median Facebook user has about **200** friends.
2k
Smaller hubs
5k
A few have Facebook's limit of **5,000** friends.

This network has fewer and smaller hubs than in a scale-free network. The distribution of nodes has a scale and does not follow a pure power law.

Most nodes have a low degree and most users cluster near the median.
The tail is weak.
NUMBER OF USERS →
0
FRIENDS
5,000

[Quanta magazine 2018]

DEGREE DISTRIBUTION

- This has important implications:
  - There is no “scale” in the degree: the average degree is not representative
  - It is not realistic to use “random graphs” (ER) for evaluating algorithms performance

SUBGRAPHS

Subgraphs

Subgraph $H(W)$ (induced subgraph): subset of nodes W of a graph $G = (V, E)$ and edges connecting them in G, i.e., subgraph $H(W) = (W, E'), W \subset V, (u, v) \in E' \iff u, v \in W \wedge (u, v) \in E$
Clique: subgraph with $d = 1$
Triangle: clique of size 3
Connected component: a subgraph in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the supergraph
Strongly Connected component: In directed networks, a subgraph in which any two vertices are connected to each other by paths
Weakly Connected component: In directed networks, a subgraph in which any two vertices are connected to each other by paths if we disregard directions

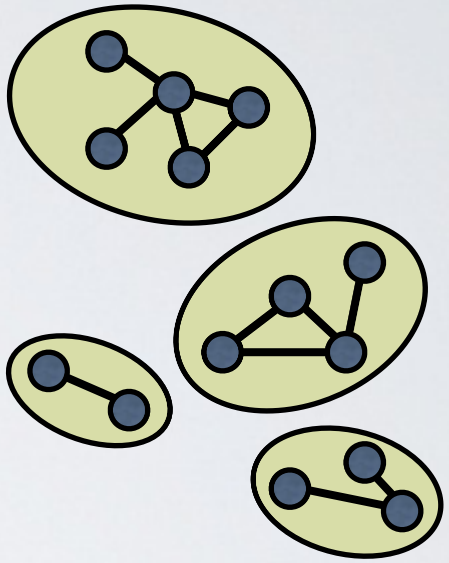
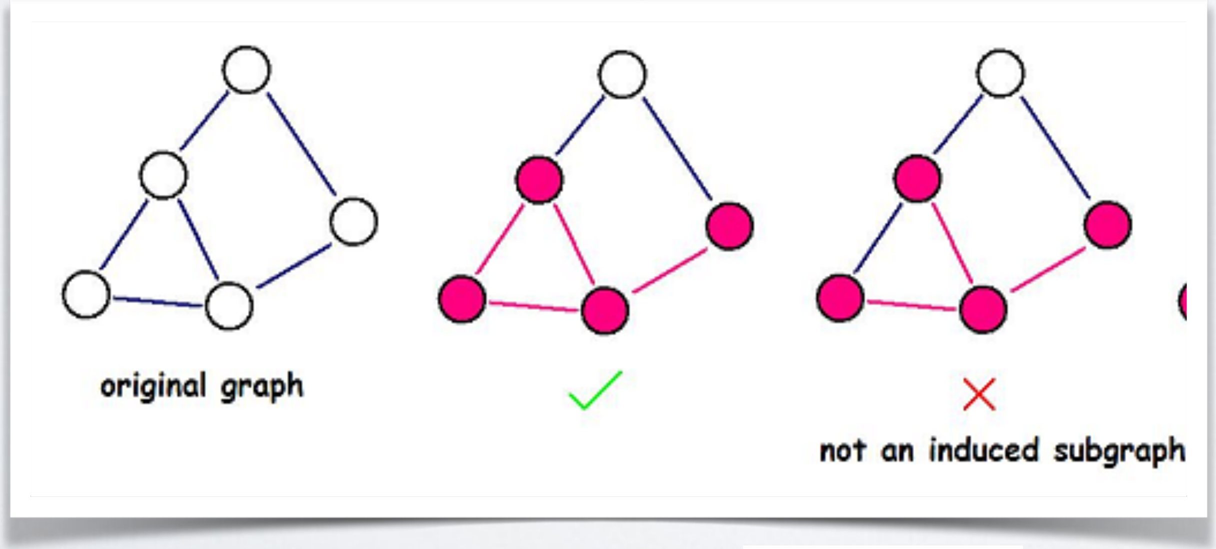


Figure after Newman, 2010



After "A. DZY Loves Physics"

CLUSTERING COEFFICIENT

- **Clustering coefficient** or **triadic closure**
- Triangles are considered important in real networks
  - Think of social networks: *friends of friends are my friends*
  - # triangles is a big difference between real and random networks

CLUSTERING COEFFICIENT

Triangles counting

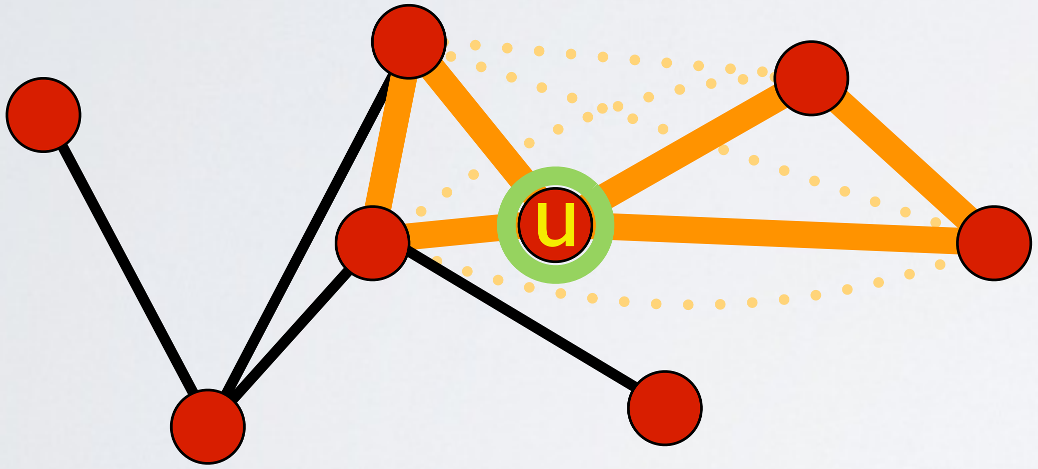
δ_u - **triads of u:** number of triangles containing node u
Δ - **number of triangles in the graph** total number of triangles in the graph, $\Delta = \frac{1}{3}\sum_{u \in V} \delta_u$.

Each triangle in the graph is counted as a triad once by each of its nodes.

$\delta_u^{\max}$ - **triads potential of u:** maximum number of triangles that could exist around node u, given its degree: $\delta_u^{\max} = \tau(u) = \binom{k_i}{2}$
$\Delta^{\max}$ - **triangles potential of G:** maximum number of triangles that could exist in the graph, given its degree distribution: $\Delta^{\max} = \frac{1}{3}\sum_{u \in V} \delta^{\max}(u)$

CLUSTERING COEFFICIENT

C_u - **Node clustering coefficient:** density of the subgraph induced by the neighborhood of u, $C_u = d(H(N_u))$. Also interpreted as the fraction of all possible triangles in N_u that exist, $\frac{\delta_u}{\delta_u^{\max}}$



Edges: 2

Max edges: 4*3/2=6

C_u=2/6=1/3

Triangles=2

Possible triangles=$\binom{4}{2}$=6

C_u=2/6=1/3

CLUSTERING COEFFICIENT

$\langle C \rangle$ - **Average clustering coefficient:** Average clustering coefficient of all nodes in the graph, $\bar{C} = \frac{1}{N} \sum_{u \in V} C_u$.

Be careful when interpreting this value, since all nodes contributes equally, irrespectively of their degree, and that low degree nodes tend to be much more frequent than hubs, and their C value is very sensitive, i.e., for a node u of degree 2, $C_u \in 0, 1$, while nodes of higher degrees tend to have more contrasted scores.

C^g - **Global clustering coefficient:** Fraction of all possible triangles in the graph that do exist, $C^g = \frac{3\Delta}{\Delta^{\max}}$

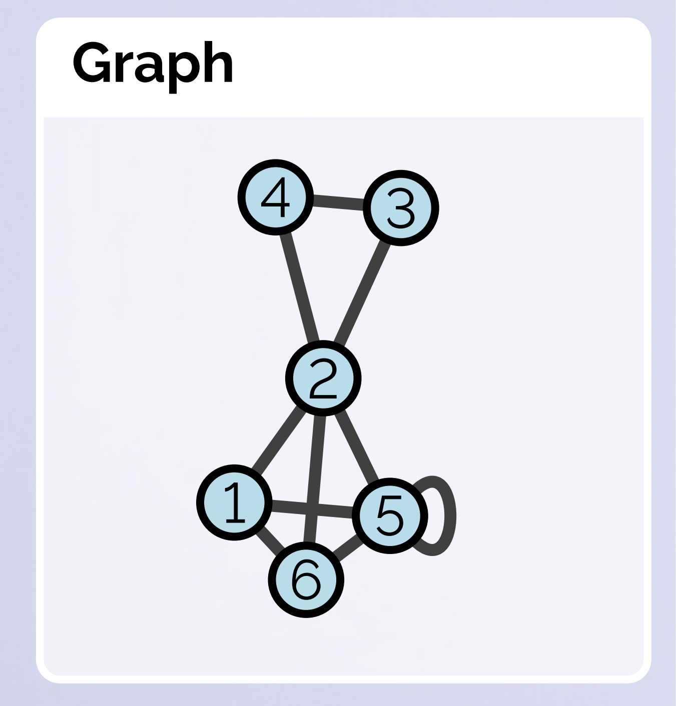
CLUSTERING COEFFICIENT

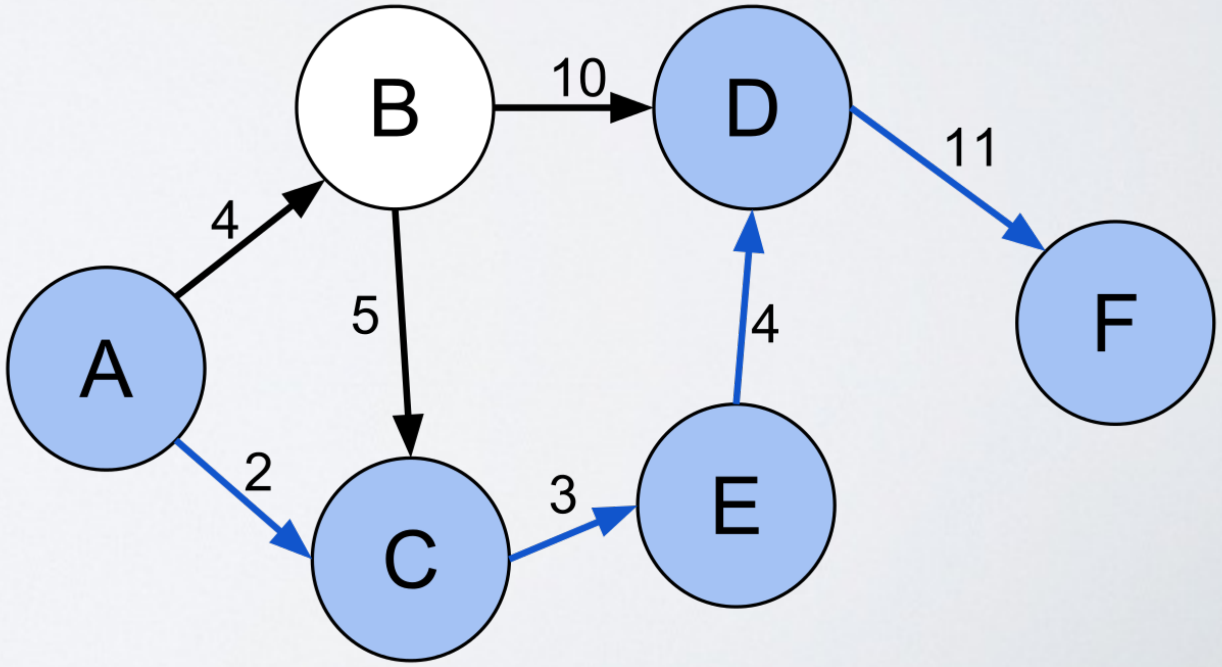
- Global CC:
  - In random networks, GCC = density
    - =>very small for large graphs
  - Facebook ego-networks: 0.6
  - Twitter lists: 0.56
  - California Road networks: 0.04

PATH RELATED SCORES

Paths - Walks - Distance

Walk: Sequences of adjacent edges or nodes (e.g., **1.2.1.6.5** is a valid walk)
Path: a walk in which each node is distinct.
Path length: number of edges encountered in a path
Weighted Path length: Sum of the weights of edges on a path
Shortest path: The shortest path between nodes u, v is a path of minimal *path length*. Often it is not unique.
Weighted Shortest path: path of minimal *weighted path length*.
$\ell_{u,v}$**: Distance**: The distance between nodes u, v is the length of the shortest path


Graph



PATH RELATED SCORES

Network descriptors 2 - Paths

$\ell_{\max}$
$\langle \ell \rangle$

Diameter: maximum *distance* between any pair of nodes.
Average distance:

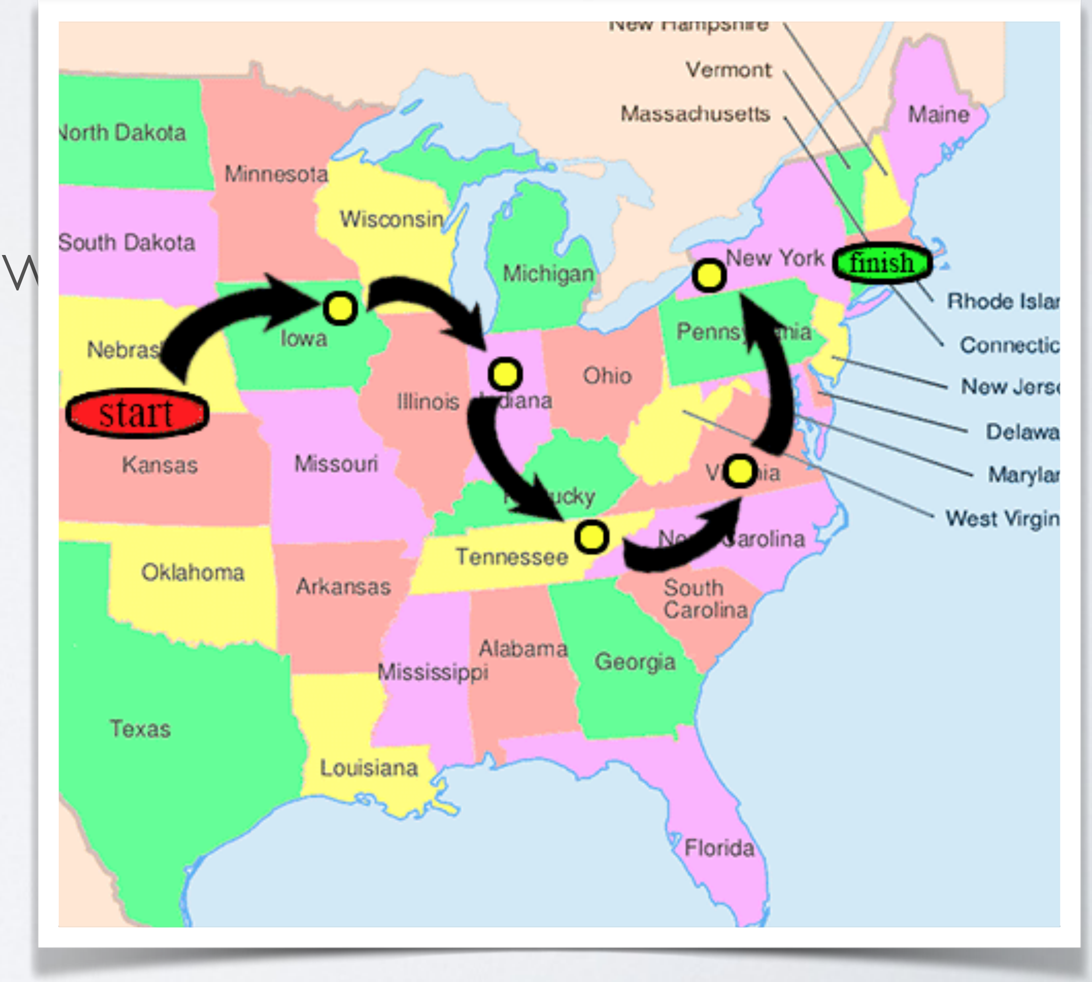
$$\langle \ell \rangle = \frac{1}{n(n-1)} \sum_{i \neq j} d_{ij}$$

AVERAGE PATH LENGTH

- The famous 6 degrees of separation (Milgram experiment)
  - (More on that next slide)
- Not too sensible to noise
- Tells you if the network is “stretched” or “hairball” like

SIDE-STORY: MILGRAM EXPERIMENT

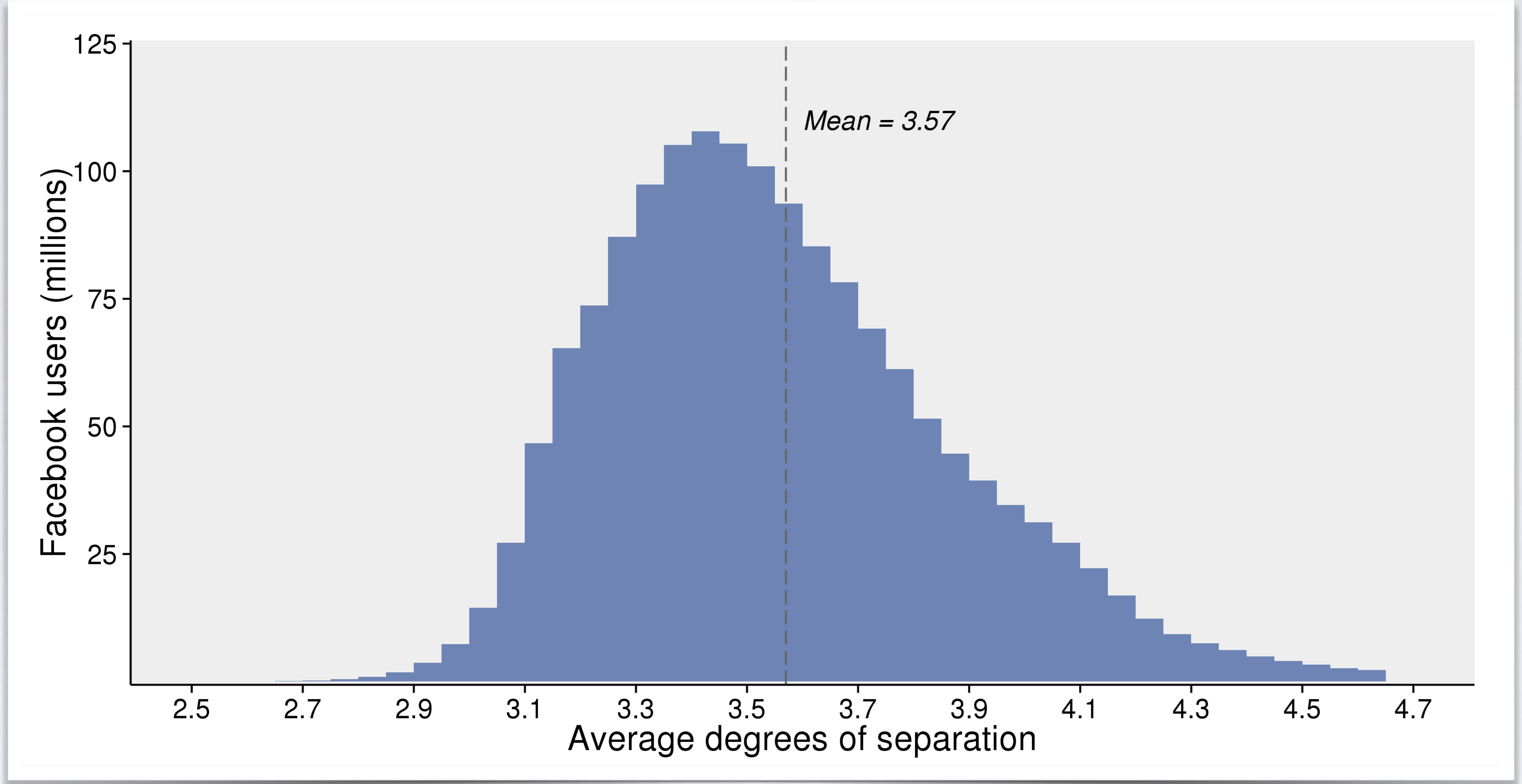
- Small world experiment (60's)
  - Give a (physical) mail to random people
  - Ask them to send to someone they don't know
    - They know his city, job
  - They send to their most relevant contact
- Results: In average, 6 hops to arrive



SIDE-STORY: MILGRAM EXPERIMENT

- Many criticism on the experiment itself:
  - Some mails did not arrive
  - Small sample
  - …
- Checked on "real" complete graphs (giant component):
  - MSN messenger
  - Facebook
  - The world wide web
  - …

SIDE-STORY: MILGRAM EXPERIMENT



Facebook

SMALL WORLD

Small World Network

A network is said to have the **small world** property when it has some structural properties. The notion is not quantitatively defined, but two properties are required:

- Average distance must be short, i.e., $\langle \ell \rangle \approx \log(N)$
- Clustering coefficient must be high, i.e., much larger than in a random network , e.g., $C^g \gg d$, with d the network density

More on this during the random network class

GRAPHS AS MATRICES

Matrices in short

Matrices are mathematical objects that can be thought as *tables* of numbers. The size of a matrix is expressed as $m \times n$, for a matrix with m rows and n columns. **The order (row/column) is important**.
M_{ij} is a notation representing the element on **row** m and **column** j.

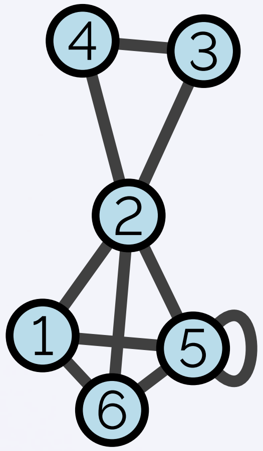
ADJACENCY MATRIX

A - Adjacency matrix

The most natural way to represent a graph as a matrix is called the Adjacency matrix A. It is defined as a square matrix, such as the number of rows (and the number of columns) is equal to the number of nodes N in the graph. Nodes of the graph are numbered from 1 to N, and there is an edge between nodes i and j if the corresponding position of the matrix A_{ij} is not 0.

- A value on the diagonal means that the corresponding node has a **self-loop**
- the graph is **undirected**, the matrix is **symmetric**: $A_{ij} = A_{ji}$ for any i, j.
- In an **unweighted** network, and edge is represented by the value 1.
- In a **weighted** network, the value A_{ij} represents the **weight** of the edge (i, j)

Graph



A - Adjacency Mat.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

ADJACENCY MATRIX

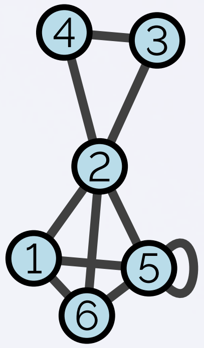
Typical operations on A

Some operations on Adjacency matrices have straightforward interpretations and are frequently used

Multiplying A by **itself** allows to know the number of walks of a given length that exist between any pair of nodes: A^2_{ij} corresponds to the number of walks of length 2 from node i to node j, A^3_{ij} to the number of walks of length 3, etc.

Multiplying A by a **column vector** W of length $1 \times N$ can be thought as setting the i th value of the vector to the ith node, and each node *sending* its value to its neighbors (for undirected graphs). The result is a column vector with N elements, the ith element corresponding to the sum of the values of its neighbors in W. This is convenient when working with **random walks** or **diffusion** phenomenon.

Graph



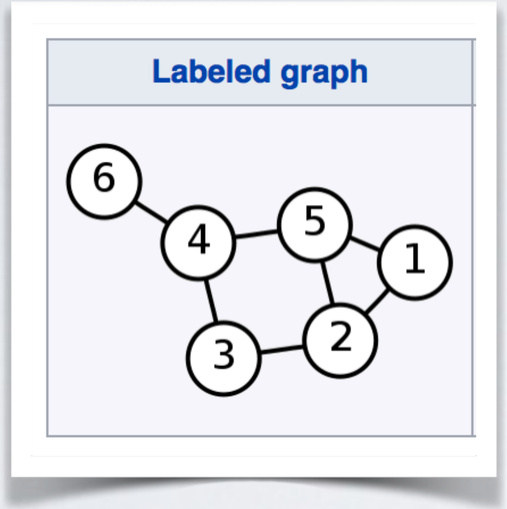
A - Adjacency Mat.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

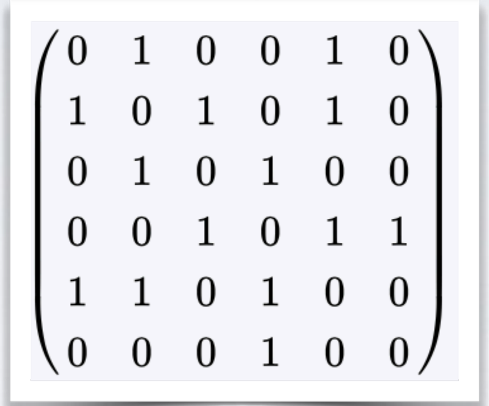
A^2

$$\begin{pmatrix} 3 & 2 & 1 & 1 & 3 & 2 \\ 2 & 5 & 1 & 1 & 3 & 2 \\ 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 \\ 3 & 3 & 1 & 1 & 4 & 3 \\ 2 & 2 & 1 & 1 & 3 & 3 \end{pmatrix}$$

ADJACENCY MATRIX



A=

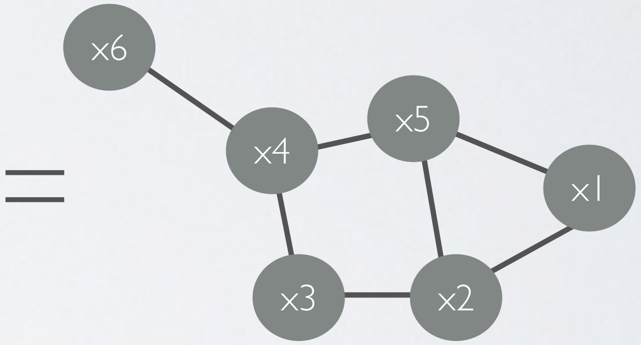


$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

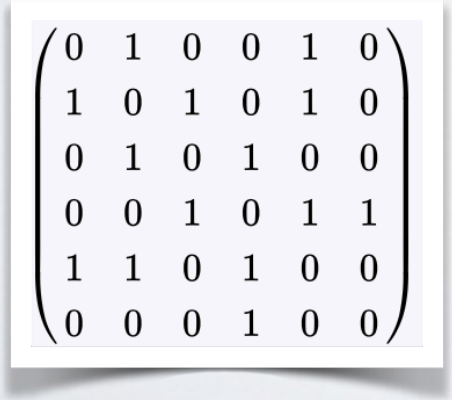
$$x= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_5 \\ x_5 \\ x_6 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_5 \\ x_5 \end{pmatrix}$$

A, x =



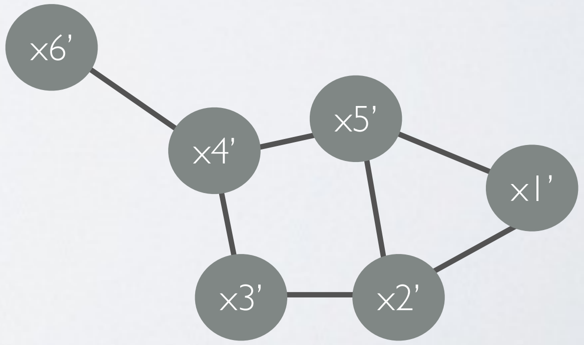
Ax=



$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} x'_1 = x_2 + x_4 \\ x'_2 = x_1 + x_3 + x_5 \\ x'_3 = x_2 + x_4 \\ x'_4 = x_3 + x_5 + x_6 \\ x'_5 = x_1 + x_2 + x_4 \\ x'_6 = x_4 \end{pmatrix}$$

A, Ax=



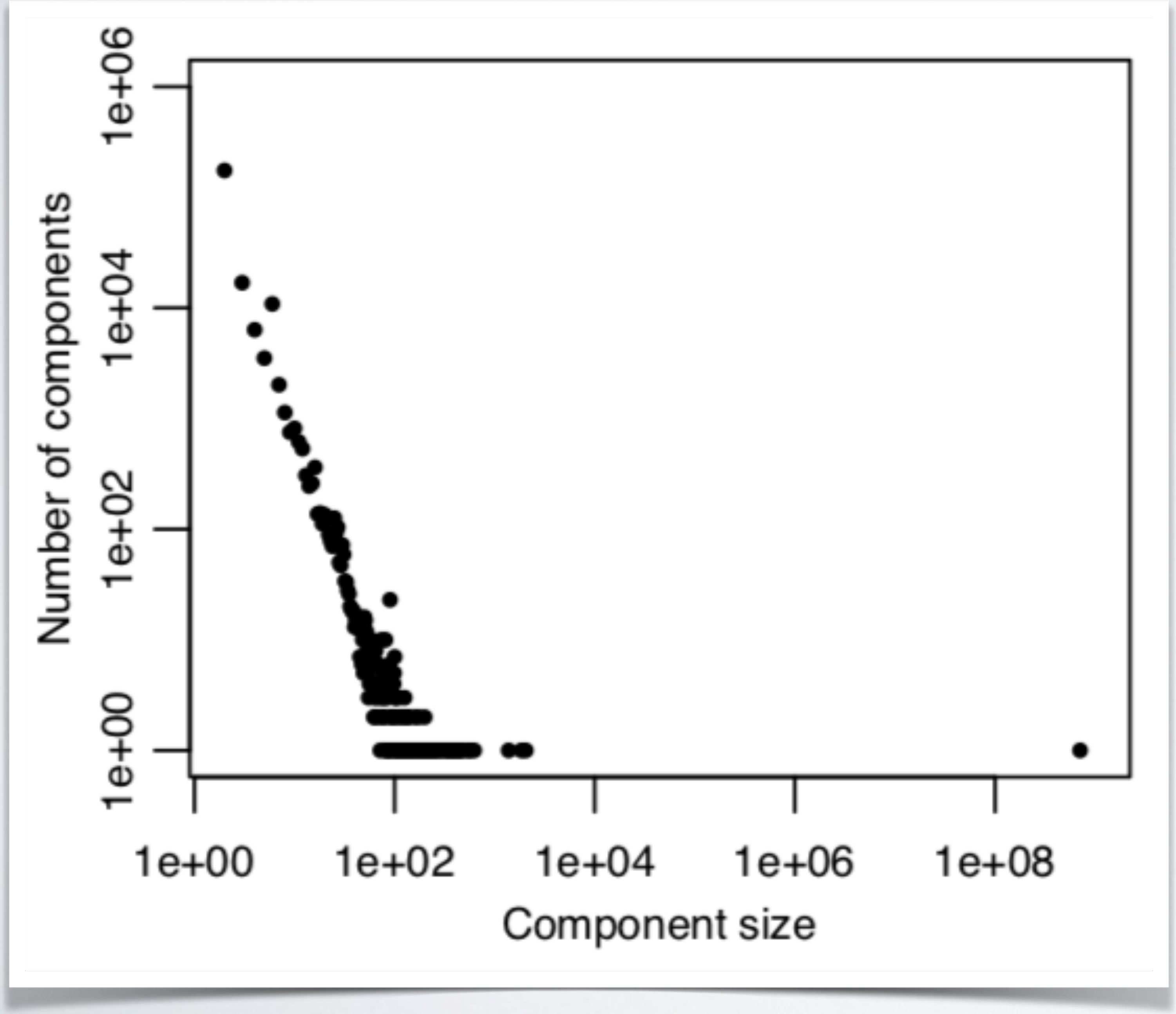
EXEMPLE OF GRAPH ANALYSIS

- Source: [The Anatomy of the Facebook Social Graph, Ugander et al. 2011]
- The Facebook friendship network in 2011

EXEMPLE OF GRAPH ANALYSIS

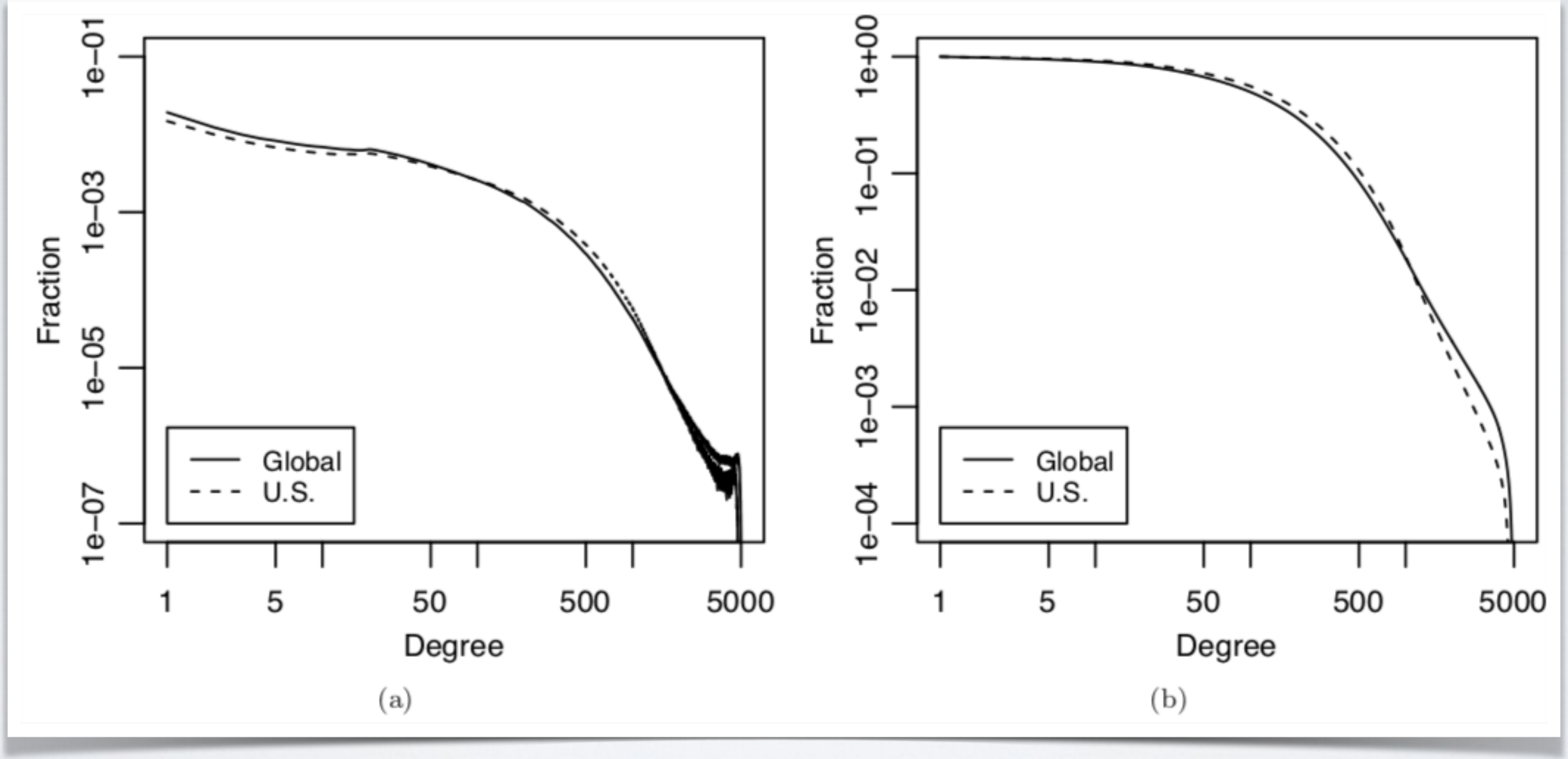
- 721M users (nodes) (active in the last 28 days)
- 68B edges
- Average degree: 190 (average # friends)
- Median degree: 99
- Connected component: 99.91%

EXEMPLE OF GRAPH ANALYSIS



Component size
Distribution

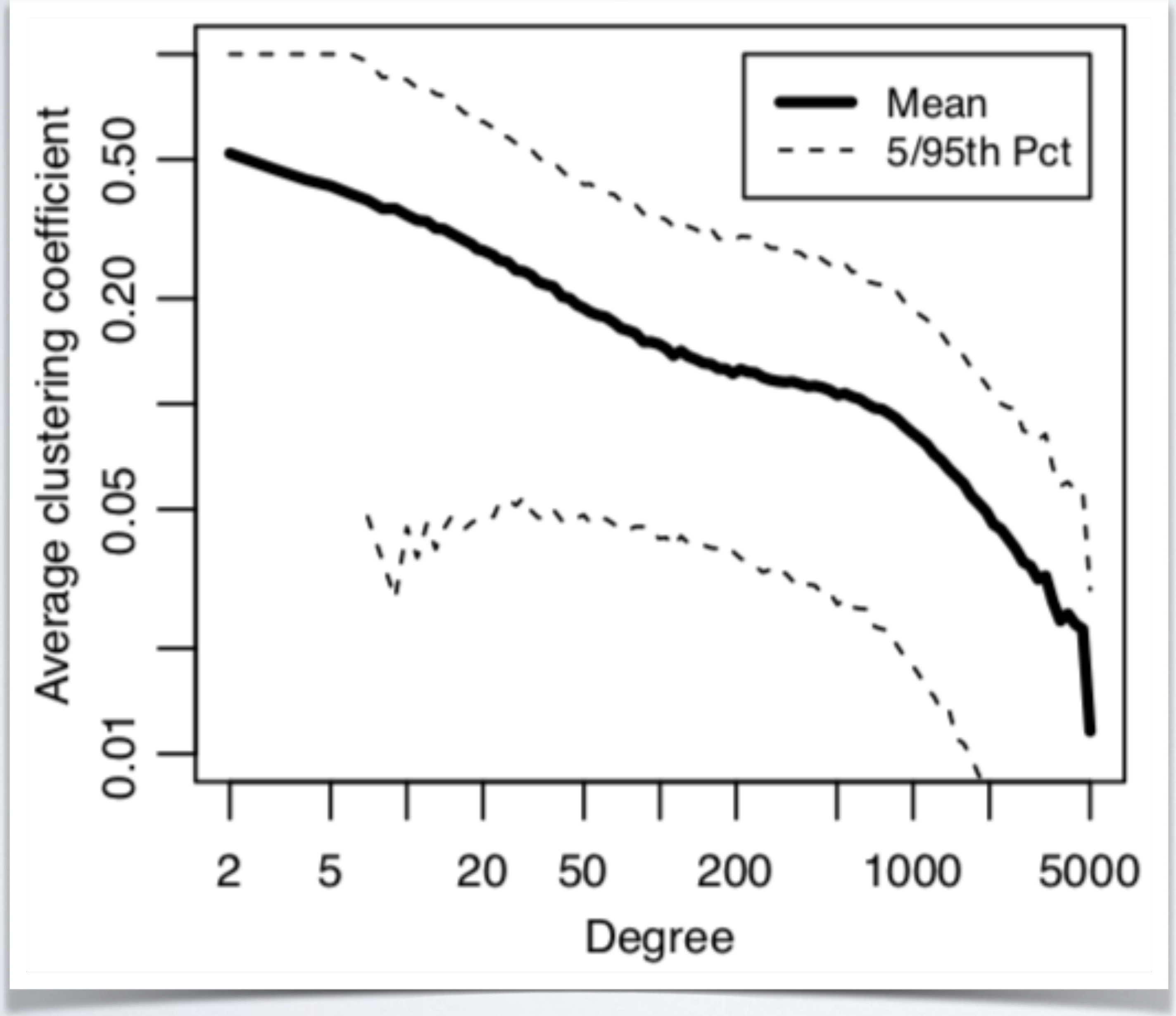
EXEMPLE OF GRAPH ANALYSIS



Cumulative

Degree distribution

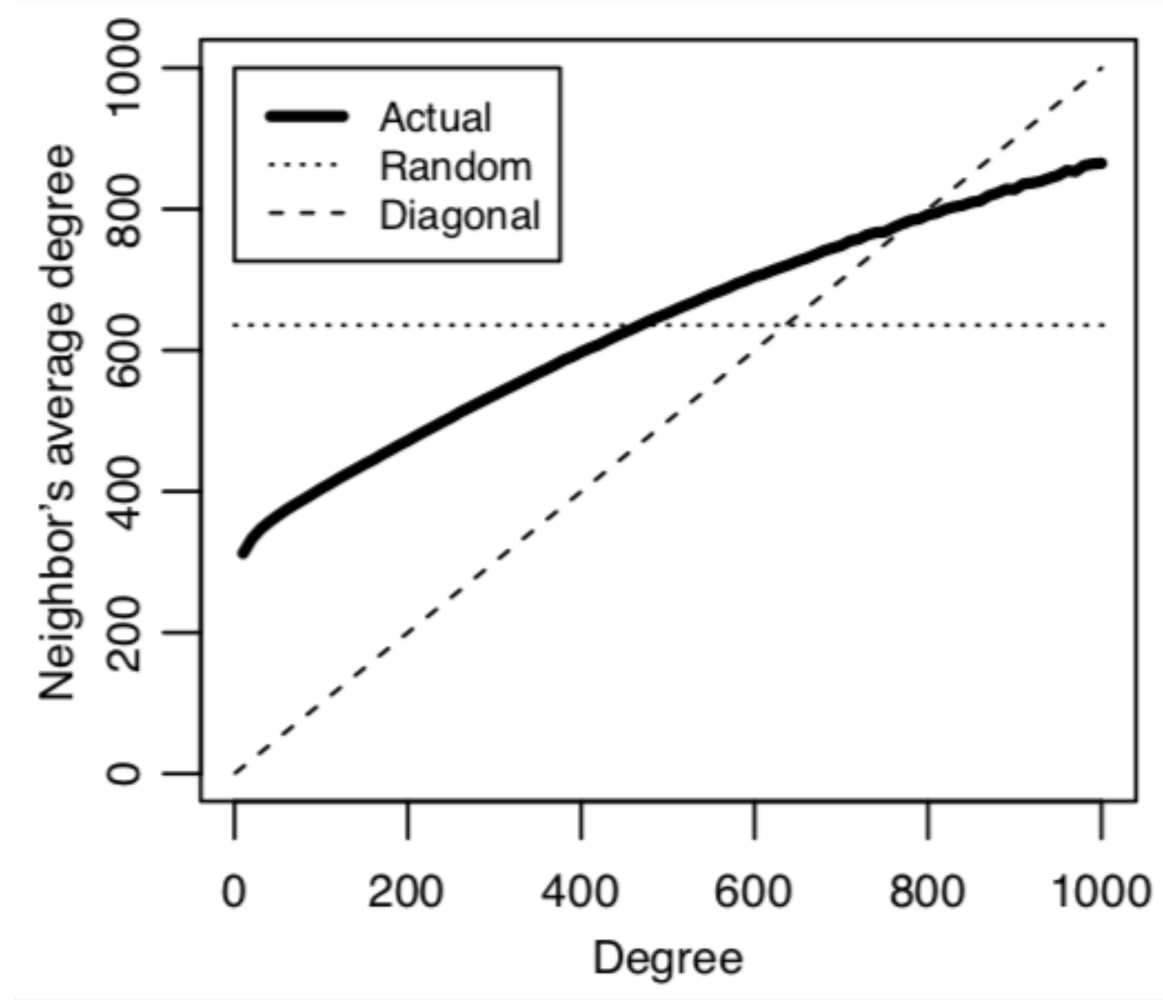
EXEMPLE OF GRAPH ANALYSIS

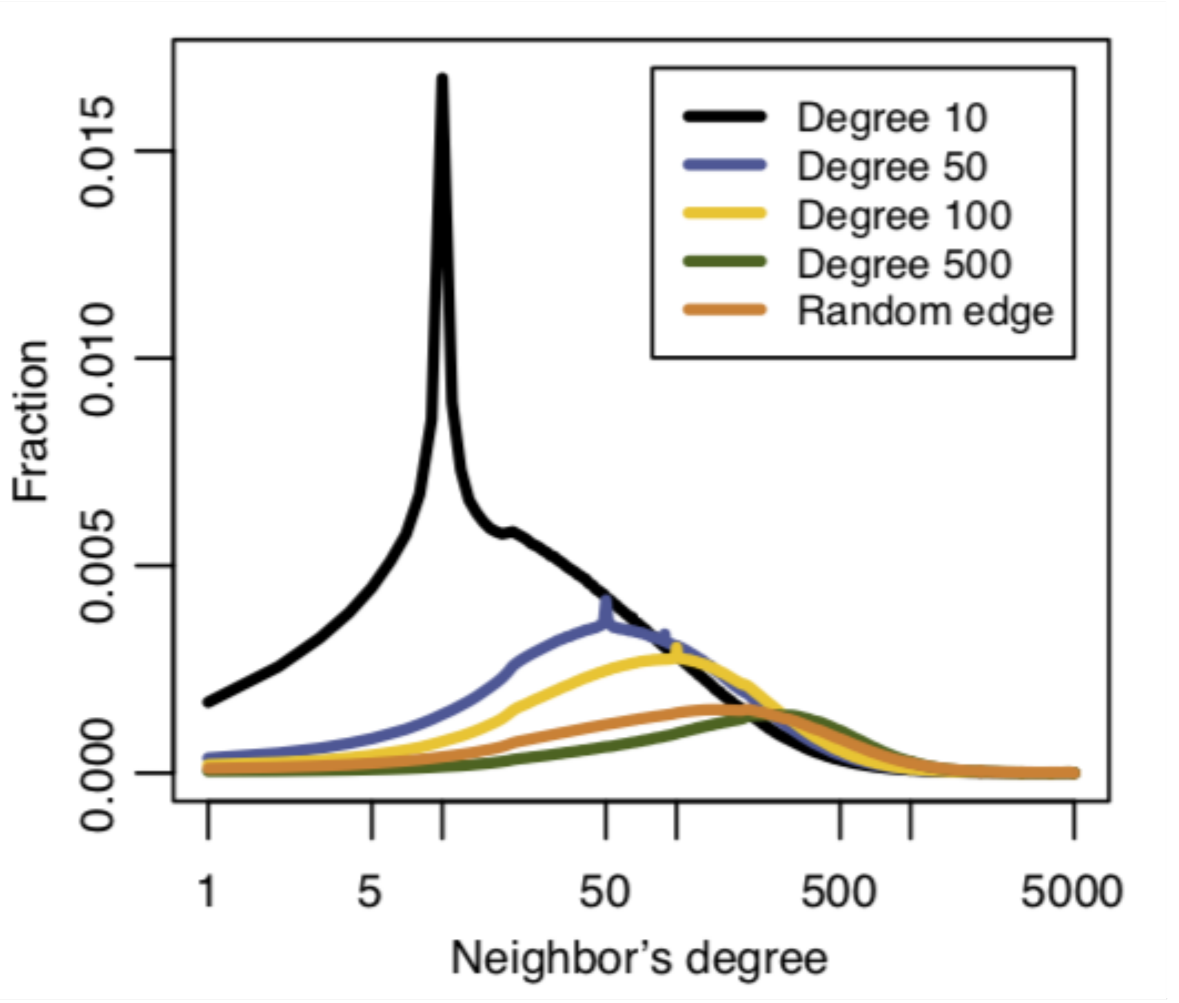


Clustering coefficient
By degree

Median user: 0.14:
14% of users with a common friend are friends

EXEMPLE OF GRAPH ANALYSIS

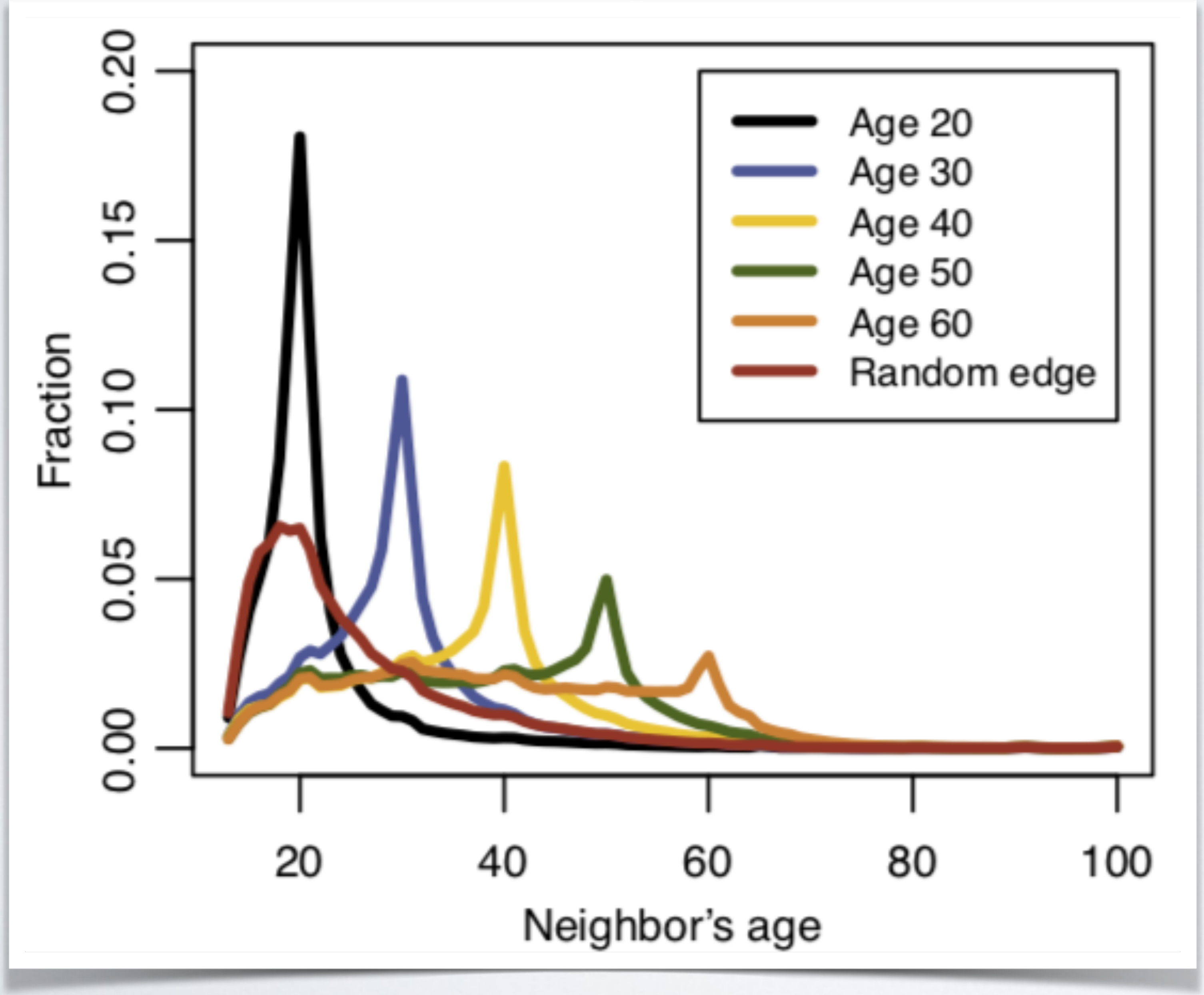




My friends have more
Friends than me!

Many of my friends have the
Same # of friends than me!

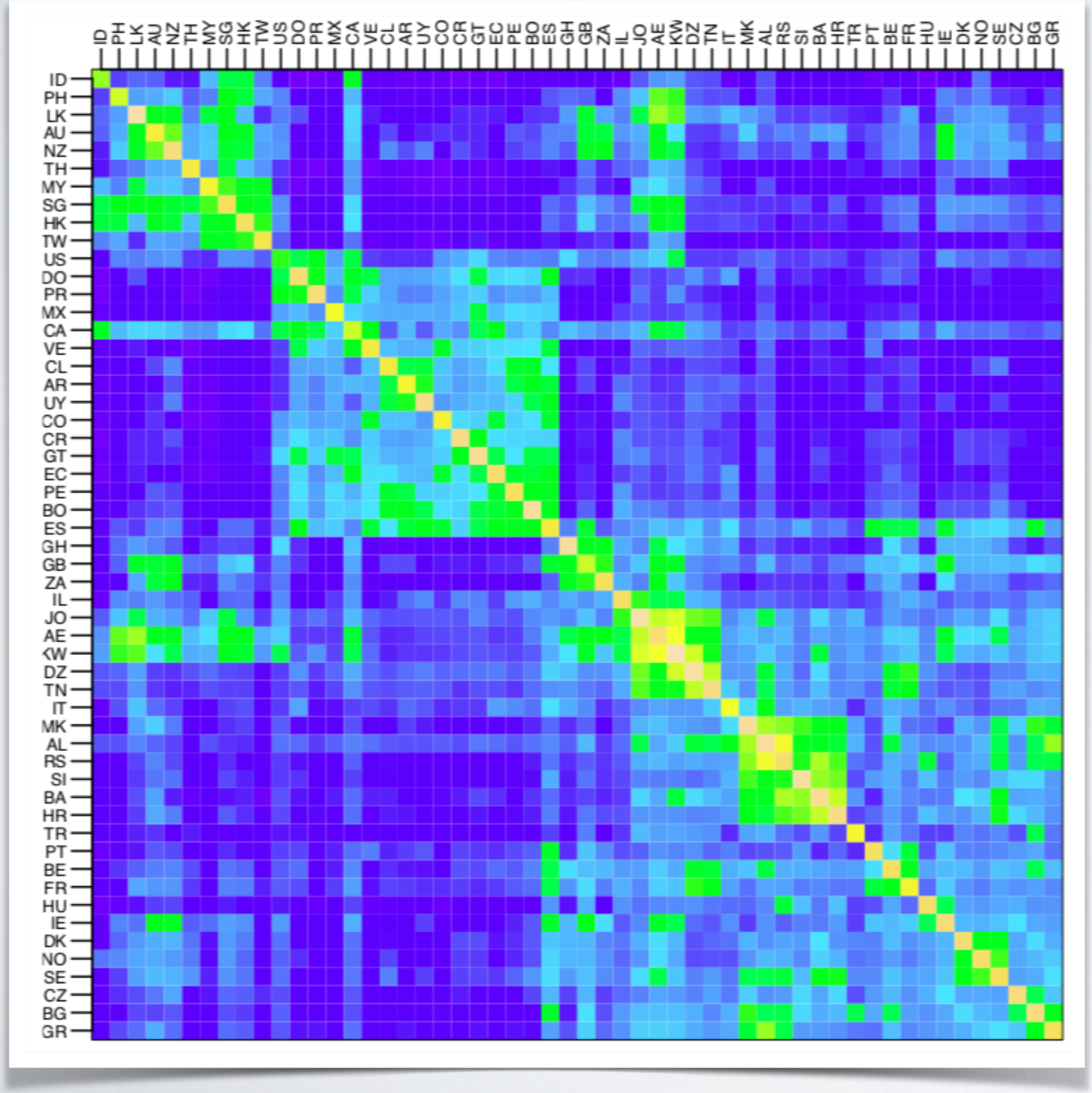
EXEMPLE OF GRAPH ANALYSIS



Age homophily

(More next class)

EXEMPLE OF GRAPH ANALYSIS



Country similarity

84.2% percent of edges are within countries

(More in the community detection class)

WEBSITE

http://cazabetremy.fr

Centrality measures

NODE

- We can measure nodes importance using so-called **centrality**.
- Poor terminology: nothing to do with being central in general
- Usage:
  - Some centralities have straightforward interpretation
  - Centralities can be used as *node features* for machine learning on graph
    - (Classification, link prediction, …)

NODE DEGREE

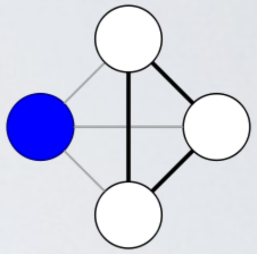
- **Degree**: how many neighbors

- Often enough to find important nodes
  - Main characters of a series talk with the more people
  - Largest airports have the most connections
  - …

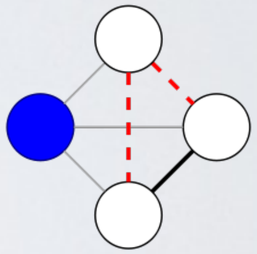
- But not always
  - Facebook users with the most friends are spam
  - Webpages/wikipedia pages with most links are simple lists of references
  - …

NODE CLUSTERING COEFFICIENT

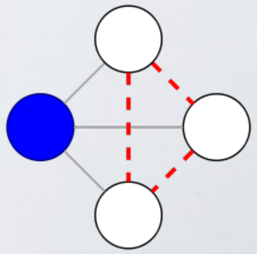
- **Clustering coefficient**: density of neighborhood
- Tells you if the neighbors of the node are connected
- Be careful!
    - Degree 2: value 0 or 1
    - Degree 1000: Not 0 or 1 (usually)
    - Ranking them is not meaningful
- Can be used as a proxy for "communities" belonging:
    - If node belong to single group: high CC
    - If node belong to several groups: lower CC



c = 1



c = 1/3

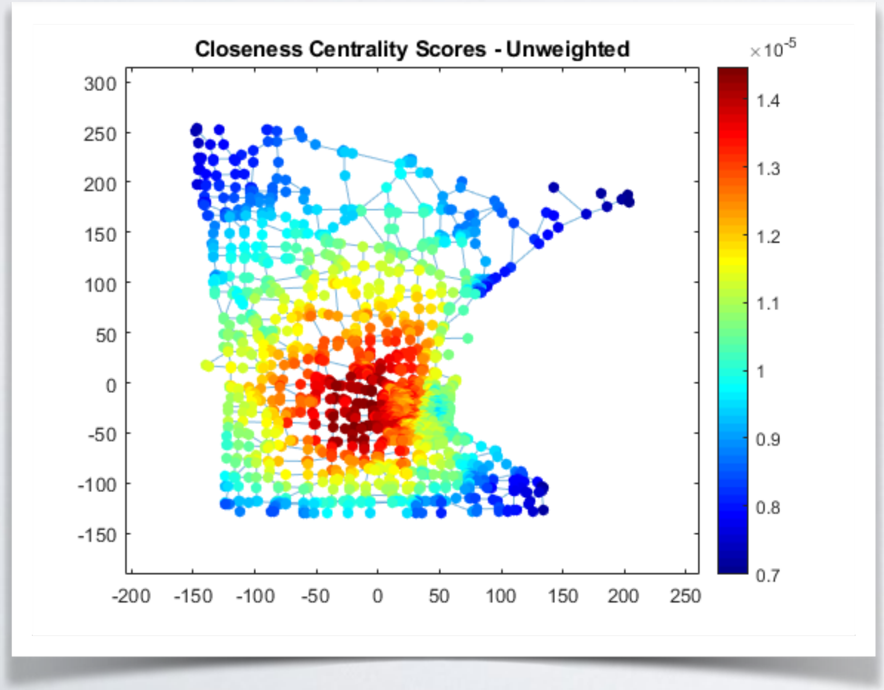


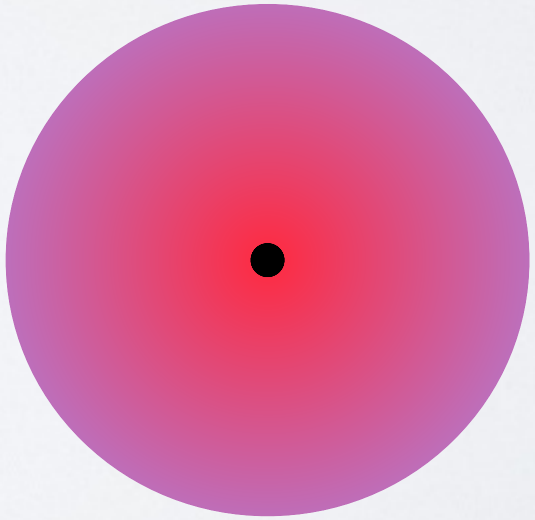
c = 0

FARNESS, CLOSENESS HARMONIC CENTRALITY

FARNESS, CLOSENESS

- How close the node is to all other nodes
- Parallel with the center of a figure:
  - Center of a circle is the point of shorter average distance to any points in the circle





FARNESS, CLOSENESS

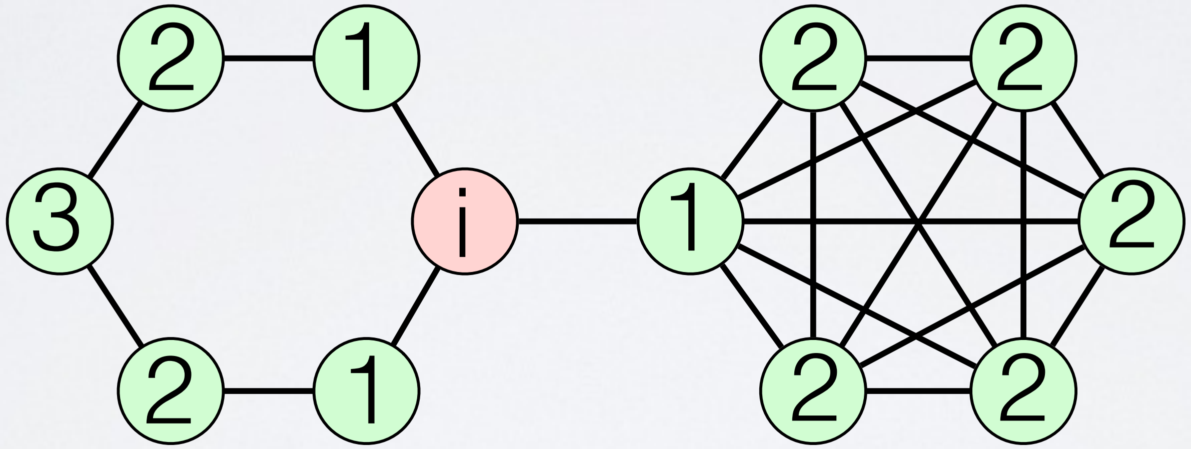
Farness: Average distance to all other nodes in the graph

$$\text{Farness}(u) = \frac{1}{N-1} \sum_{v \in V \setminus u} \ell_{u,v}$$

CLOSENESS CENTRALITY

Closeness: Inverse of the farness, i.e., how close the node is to all other nodes in term of shortest paths.

$$\text{Closeness}(u) = \frac{N-1}{\sum_{v \in V \setminus u} \ell_{u,v}}$$



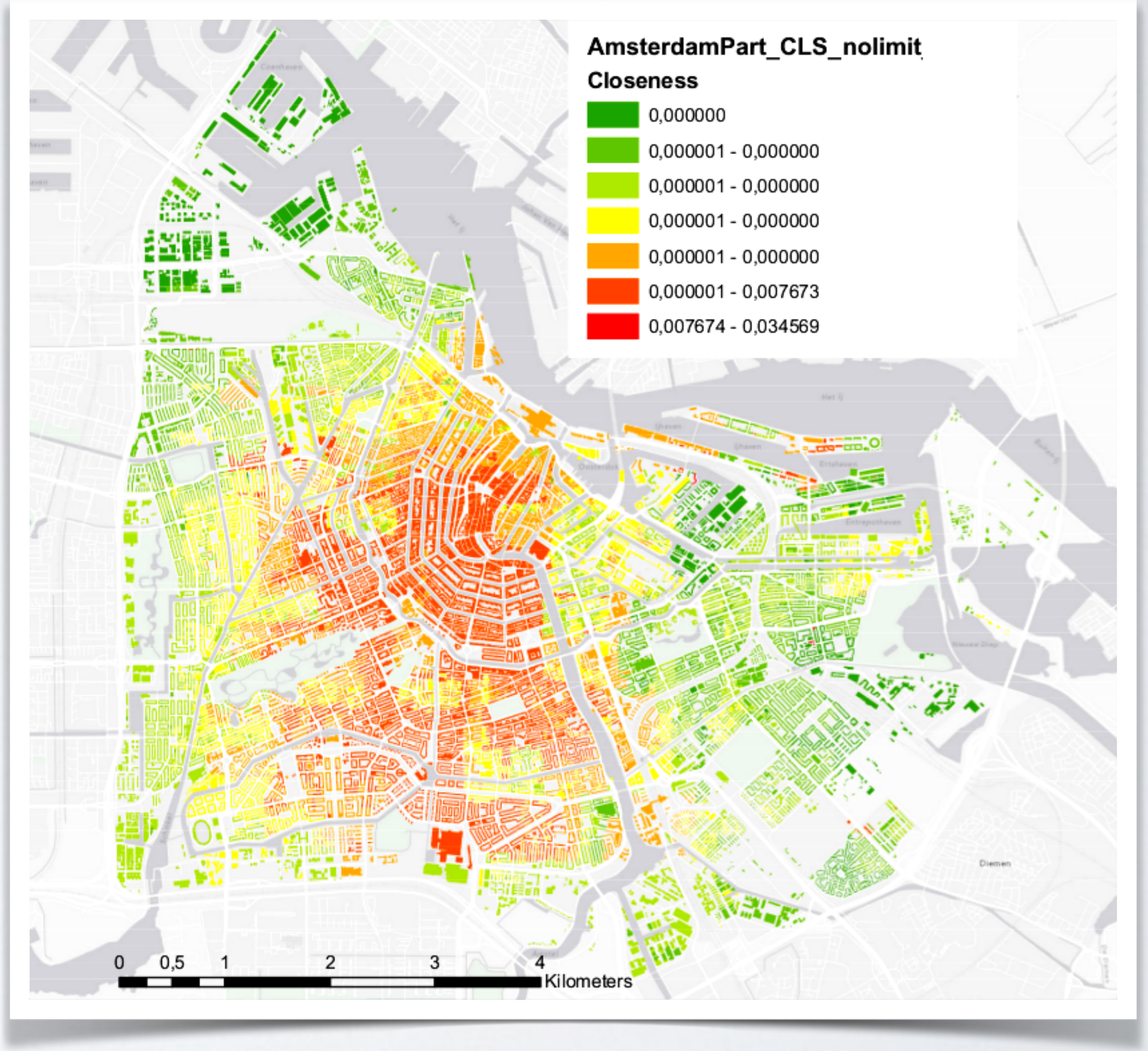
$$C_{cl}(i) = \frac{12-1}{(3 \times 1 + 7 \times 2 + 1 \times 3)} = \frac{11}{20} = 0.55$$

CLOSENESS CENTRALITY

Closeness: Inverse of the farness, i.e., how close the node is to all other nodes in term of shortest paths.

$$\text{Closeness}(u) = \frac{N-1}{\sum_{v \in V \setminus u} \ell_{u,v}}$$

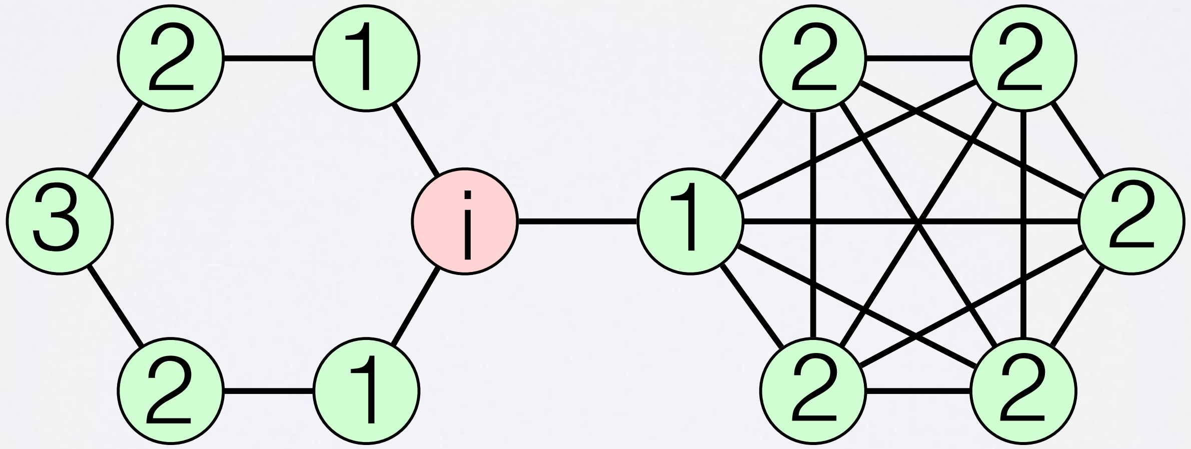
1=all nodes are at distance one



Harmonic Centrality

Harmonic centrality: A variant of the closeness defined as the average of the inverse of distance to all other nodes (Harmonic mean). Well defined on disconnected network with $\frac{1}{\infty} = 0$. Its interpretation is the same as the closeness.

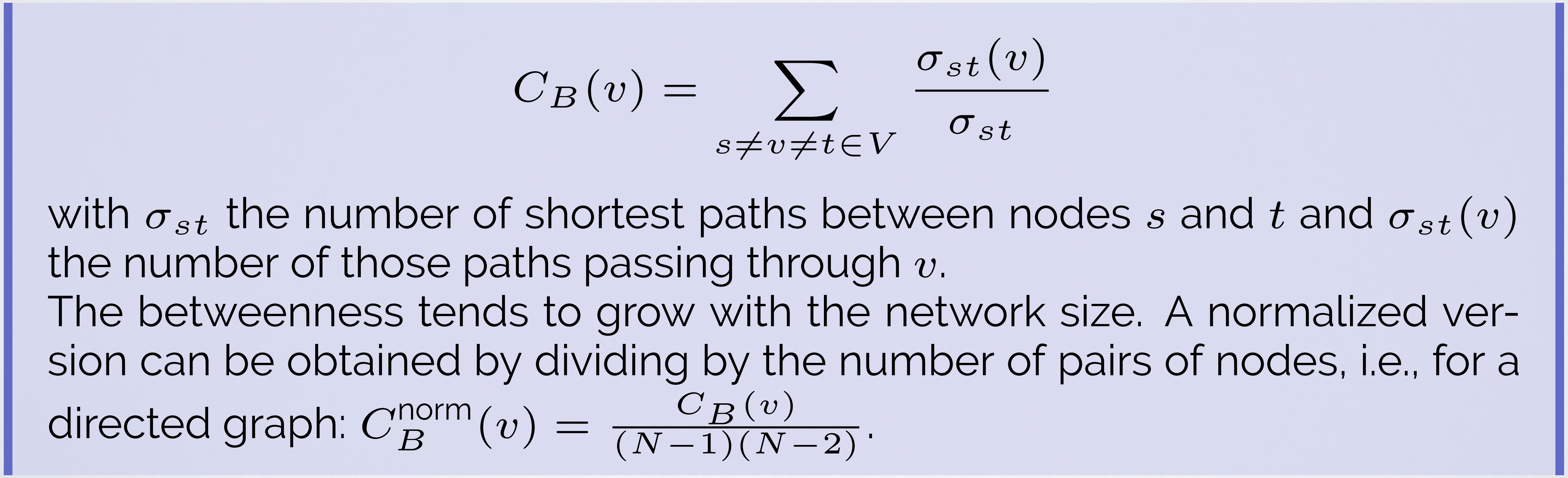
$$\text{Harmonic}(u) = \frac{1}{N-1} \sum_{v \in V \setminus u} \frac{1}{\ell_{u,v}}$$



$$C_h(i) = \frac{1}{12-1}\left(3 \times \frac{1}{1} + 7 \times \frac{1}{2} + 1 \times \frac{1}{3}\right) = \frac{41}{66} = 0.6212$$

BETWEENNESS CENTRALITY

- Measure how much the node plays the role of a bridge
- Betweenness of *u: f*raction of all the shortest paths between all the pairs of nodes going through u.



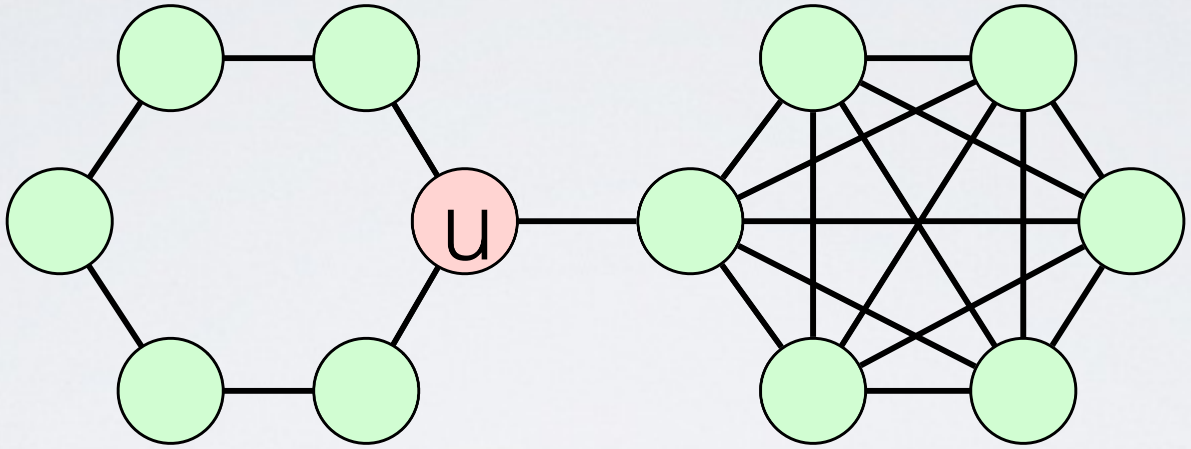
$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

with σ_{st} the number of shortest paths between nodes s and t and $\sigma_{st}(v)$ the number of those paths passing through v.
The betweenness tends to grow with the network size. A normalized version can be obtained by dividing by the number of pairs of nodes, i.e., for a directed graph: $C_B^{\text{norm}}(v) = \frac{C_B(v)}{(N-1)(N-2)}$.

Betweenness Centrality

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

directed graph: $C_B^{\text{norm}}(v) = \frac{C_B(v)}{(N-1)(N-2)}$.



$$C_B(u) = 2\frac{5 * 6 + 1 + \frac{1}{2} + \frac{1}{2}}{11 * 10} = \frac{64}{110}$$

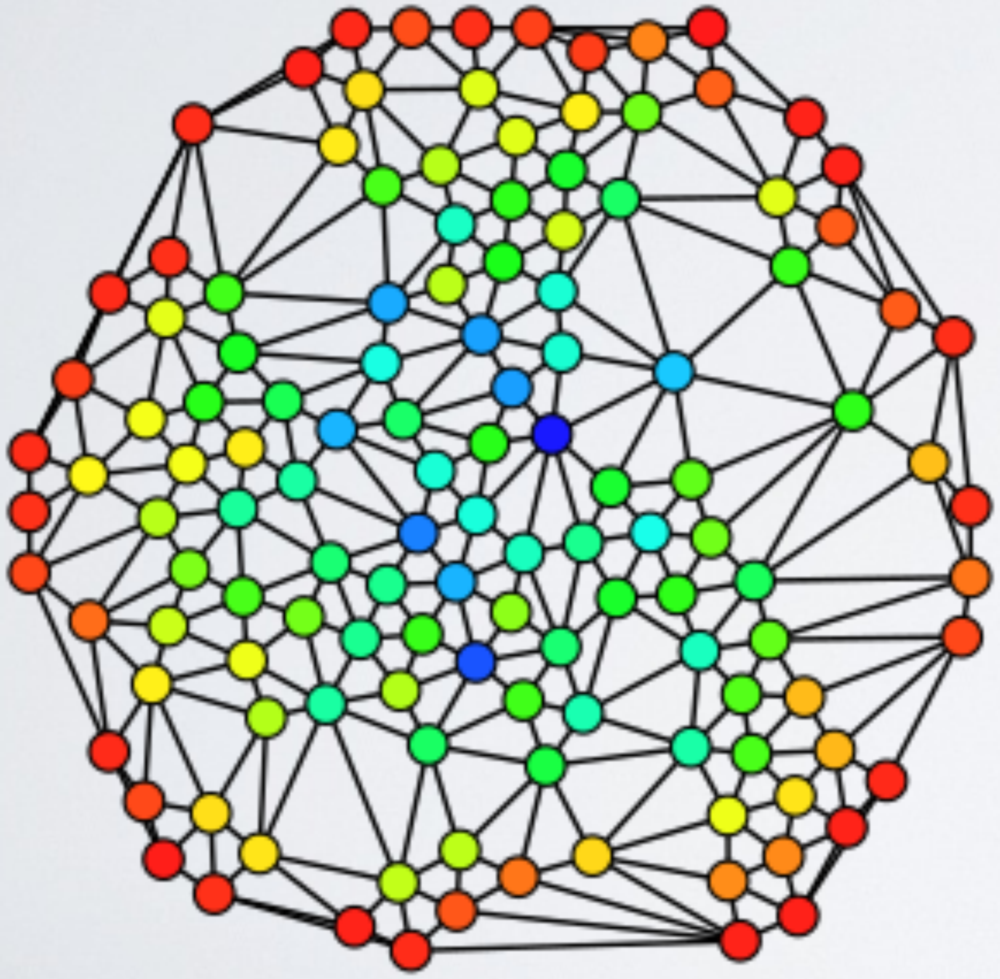
Exact computation:

Floyd-Warshall: *$O(n^3)$ time complexity*
$O(n^2)$ space complexity

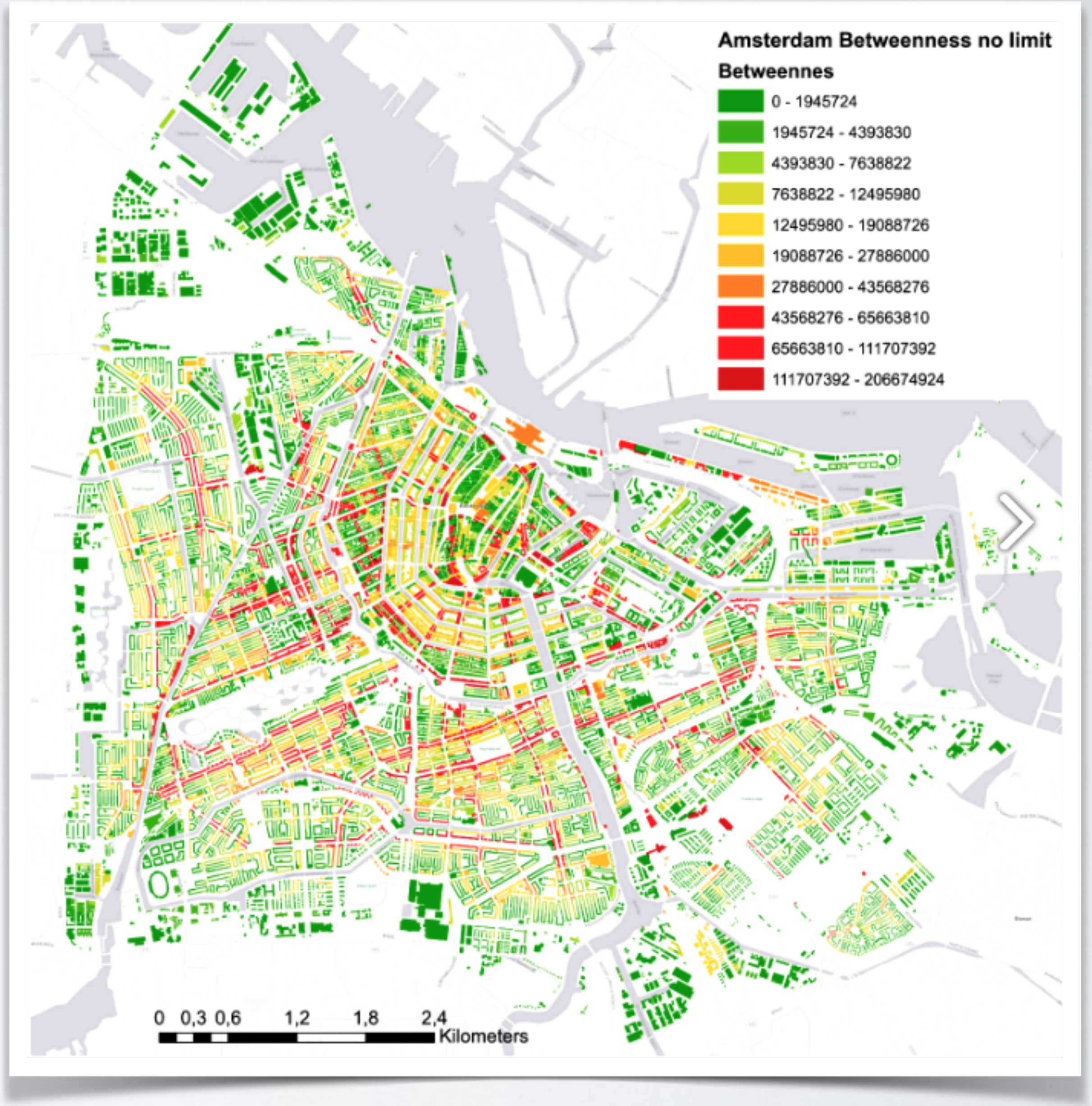
Approximate computation

Dijskstra: *O(n(m+n log n)) time complexity*

BETWEENNESS CENTRALITY



(blue higher)

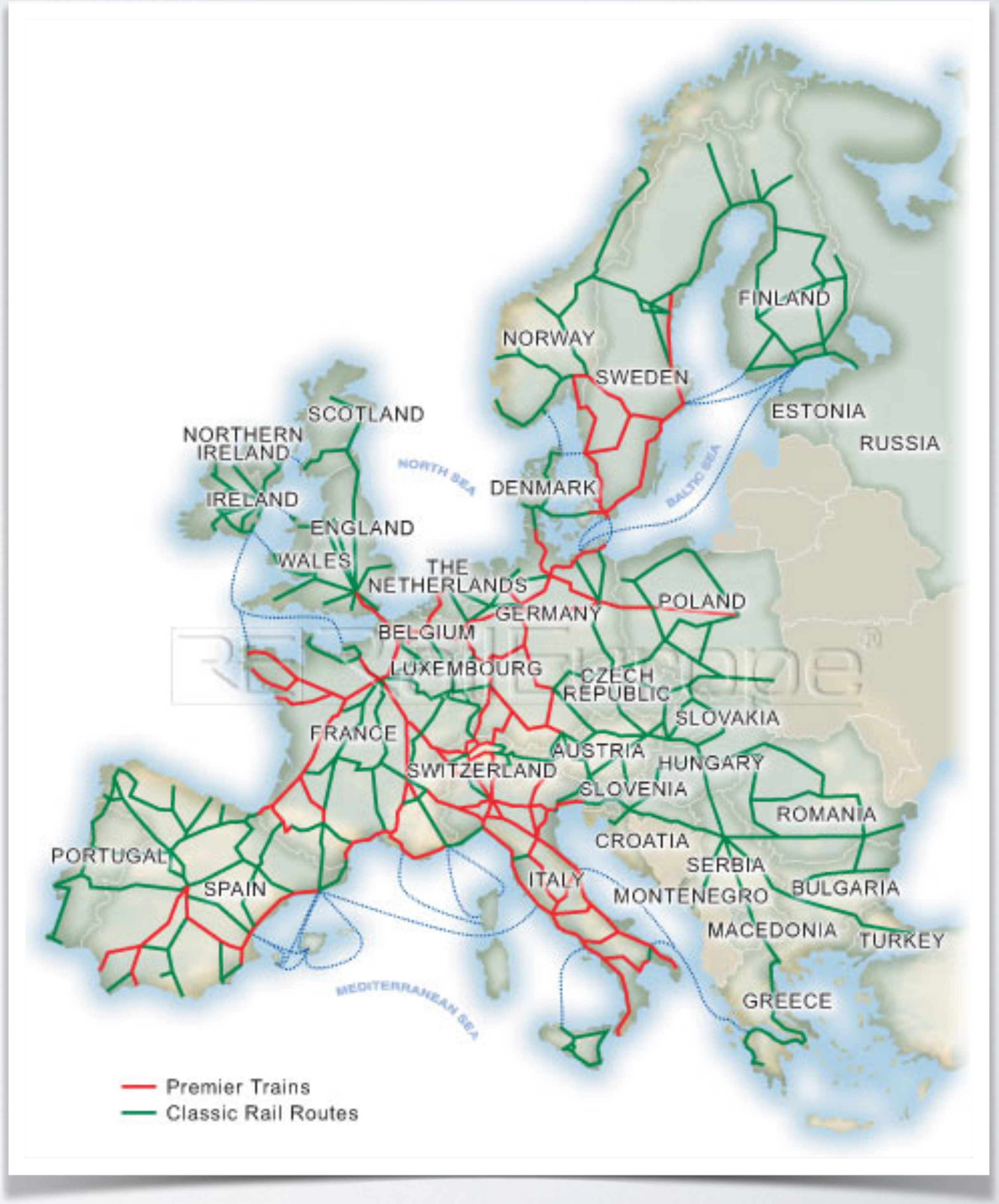


(red higher)

EDGE - BETWEENNESS

Same definition as for nodes

Can you guess the edge of highest betweenness in the European rail network ?



RECURSIVE DEFINITIONS

RECURSIVE DEFINITIONS

- Recursive importance:
  - **Important nodes** are those connected ***to important nodes***
- Several centralities based on this idea:
  - Eigenvector centrality
  - PageRank
  - …

RECURSIVE DEFINITION

- We would like scores such as :
  - Each node has a score (centrality),
  - If every node “sends” its score to its neighbors, the sum of all scores received by each node will be equal to its original score

$$C_u^{t+1} = \frac{1}{\lambda} \sum_{v \in N_u^{in}} C_v^t \tag{1}$$

- With λ a normalisation constant

RECURSIVE DEFINITION

- This problem can be solved by what is called the *power method:*
  - 1) We initialize all scores to random values
  - 2)Each score is updated according to the desired rule, until reaching a stable point (after normalization)

- Why does it converge?
  - Perron-Frobenius theorem (see next slide)
  - =>True for undirected graphs with a single connected component

EIGENVECTOR CENTRALITY

- What we just described is called the Eigenvector centrality

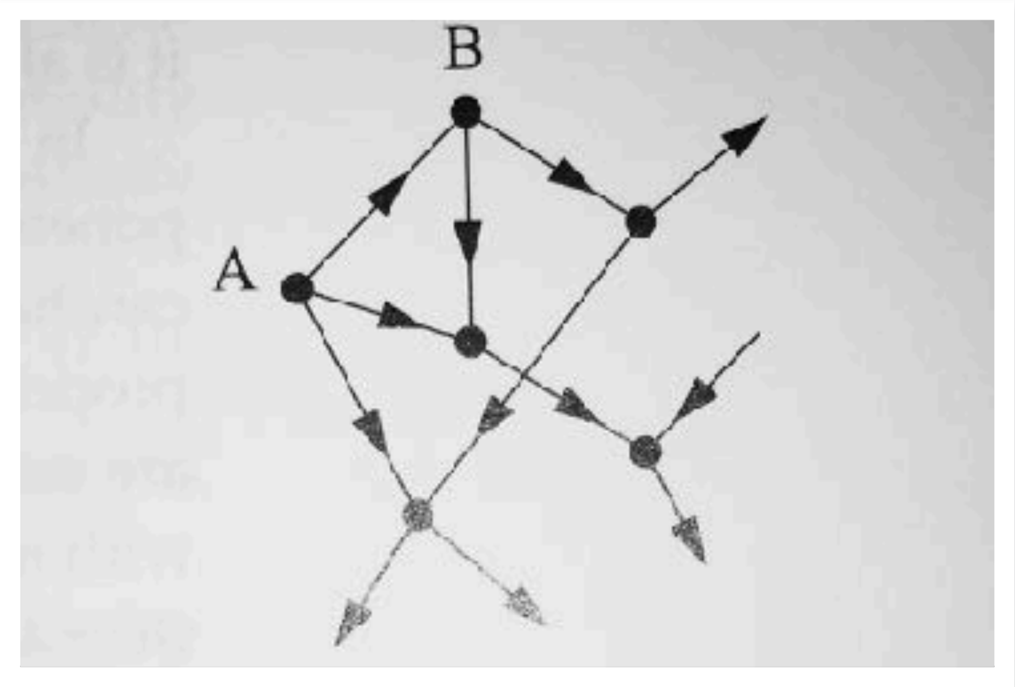
- A couple eigenvector (x) and eigenvalue (λ) is defined by the following relation: $Ax = \lambda x$
  - x is a column vector of size *n,* which can be interpreted as the scores of nodes

- What Perron-Frobenius algorithm says is that the power method will always converge to the *leading eigenvector*, i.e., the eigenvector associated with the highest eigenvalue

Eigenvector Centrality

Some problems in case of directed network:

- Adjacency matrix is asymmetric
- 2 sets of eigenvectors (Left & Right)
- 2 leading eigenvectors
    - Use right eigenvectors : consider nodes that are pointing towards you



But problem with source nodes (0 in-degree)

-Vertex A is connected but has only outgoing link = Its centrality will be 0

-Vertex B has outgoing and an incoming link, but incoming link comes from A = Its centrality will be 0

-etc.

Solution: Only in strongly connected component

Note: Acyclic networks (citation network) do not have strongly connected component

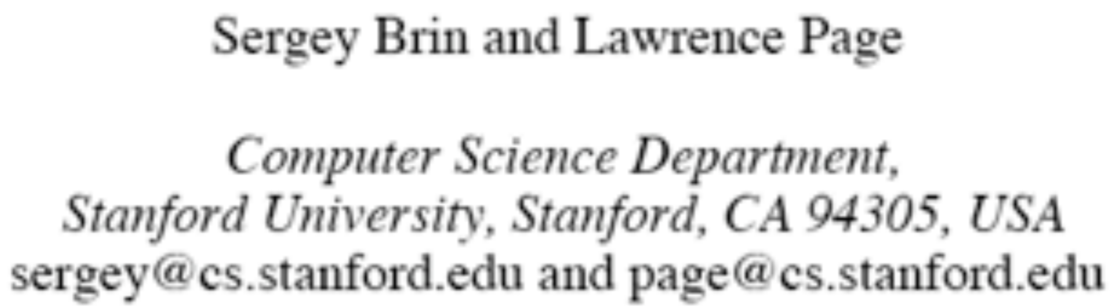
PageRank Centrality

- Eigenvector centrality generalised for directed networks

PageRank

The Anatomy of a Large-Scale Hypertextual Web Search Engine

Brin, S. and Page, L. (1998) The Anatomy of a Large-Scale Hypertextual Web Search Engine. In: Seventh International World-Wide Web Conference (WWW 1998), April 14-18, 1998, Brisbane, Australia.



Sergey Brin and Lawrence Page

Computer Science Department,
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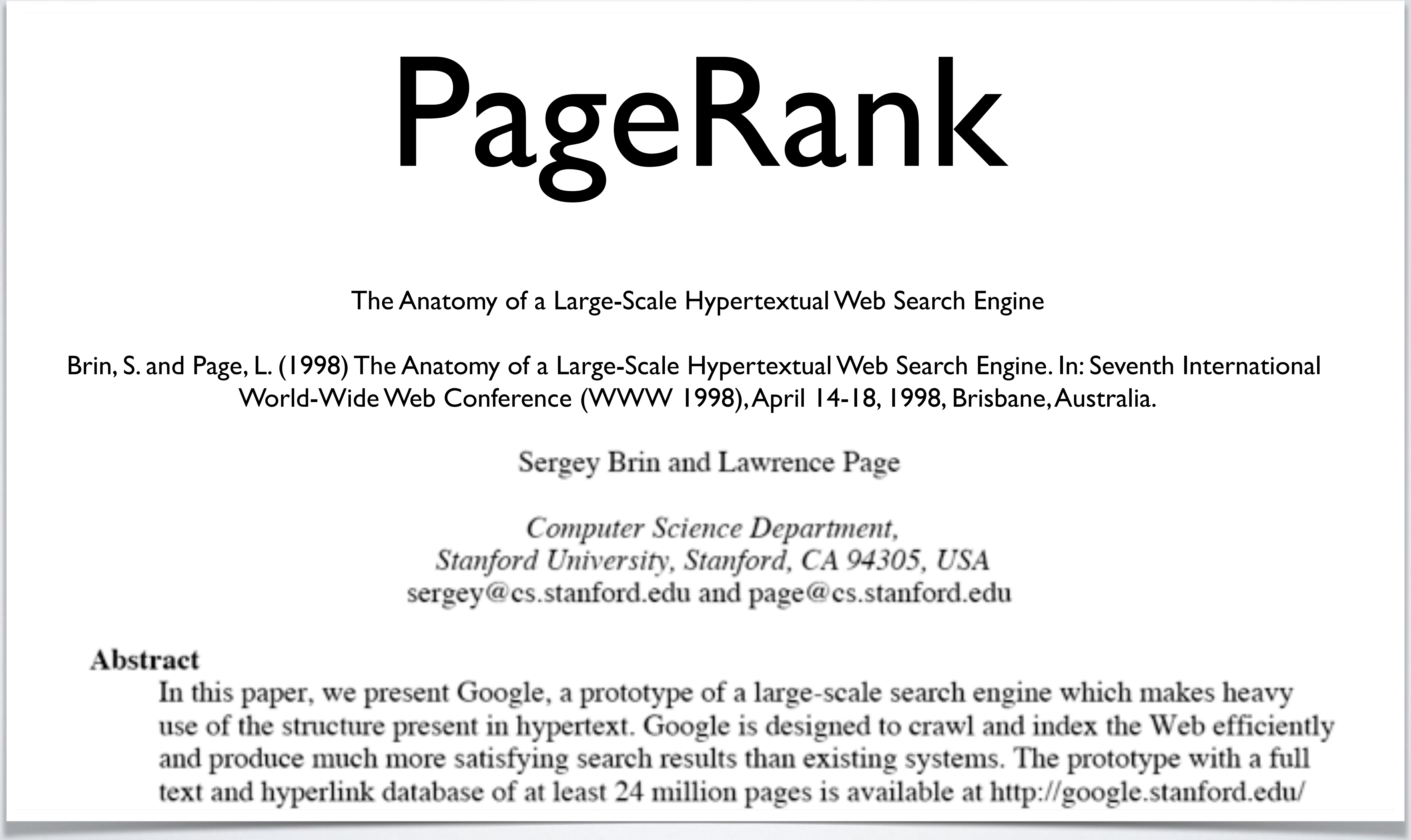
sergey@cs.stanford.edu and page@cs.stanford.edu

PageRank Centrality

- Eigenvector centrality generalised for directed networks



PageRank

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Abstract

In this paper, we present Google, a prototype of a large-scale search engine which makes heavy use of the structure present in hypertext. Google is designed to crawl and index the Web efficiently and produce much more satisfying search results than existing systems. The prototype with a full text and hyperlink database of at least 24 million pages is available at http://google.stanford.edu/

PageRank Centrality

(Side notes)

-“We chose our system name, Google, because it
is a common spelling of googol, or 10^{100} and fits well with our goal of building very large-scale search “

-“[…] at the same time, search engines have migrated from the academic domain to the commercial. **Up until now most search engine development has gone on at companies with little publication of technical details. This causes search engine technology to remain largely a black art and to be advertising oriented (see Appendix A). With Google, we have a strong goal to push more development and understanding into the academic realm.”**

-“[...], we expect that advertising funded search engines will be inherently biased towards the advertisers and away from the needs of the consumers."

PageRank Centrality

(Side notes)

Sergey Brin received his B.S. degree in mathematics and computer science from the University of Maryland at College Park in 1993. Currently, he is a Ph.D. candidate in computer science at Stanford University where he received his M.S. in 1995. He is a recipient of a National Science Foundation Graduate Fellowship. His research interests include search engines, information extraction from unstructured sources, and data mining of large text collections and scientific data.

Lawrence Page was born in East Lansing, Michigan, and received a B.S.E. in Computer Engineering at the University of Michigan Ann Arbor in 1995. He is currently a Ph.D. candidate in Computer Science at Stanford University. Some of his research interests include the link structure of the web, human computer interaction, search engines, scalability of information access interfaces, and personal data mining.

PAGERANK

- 2 main improvements over eigenvector centrality:
  - In directed networks, problem of source nodes
    - => Add a constant centrality gain for every node
  - Nodes with very high centralities give very high centralities to all their neighbors (even if that is their only in-coming link)
    - => What each node "is worth" is divided equally among its neighbors (normalization by the degree)

$$C_u^{t+1} = \frac{1}{\lambda} \sum_{v \in N_u^{in}} C_v^t$$

=>

$$C_u^{t+1} = \alpha \sum_{v \in N_u^{in}} \frac{C_v^t}{k_v^{out}} + \beta$$

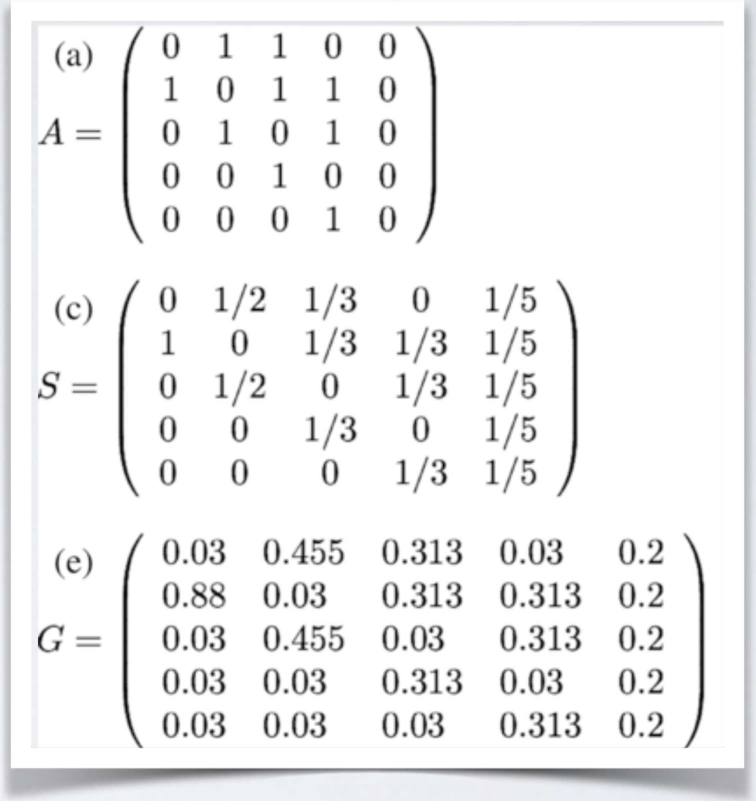
With by convention β=1 and α a parameter (usually 0.85) controlling the relative importance of β

PAGERANK

Matrix interpretation

Principal eigenvector of the “Google Matrix”:
First, define matrix S as:
-Normalization by columns of A
-Columns with only 0 receives 1/n

-Finally, $G_{ij} = \alpha S_{ij} + (1-\alpha)/n$



(a)

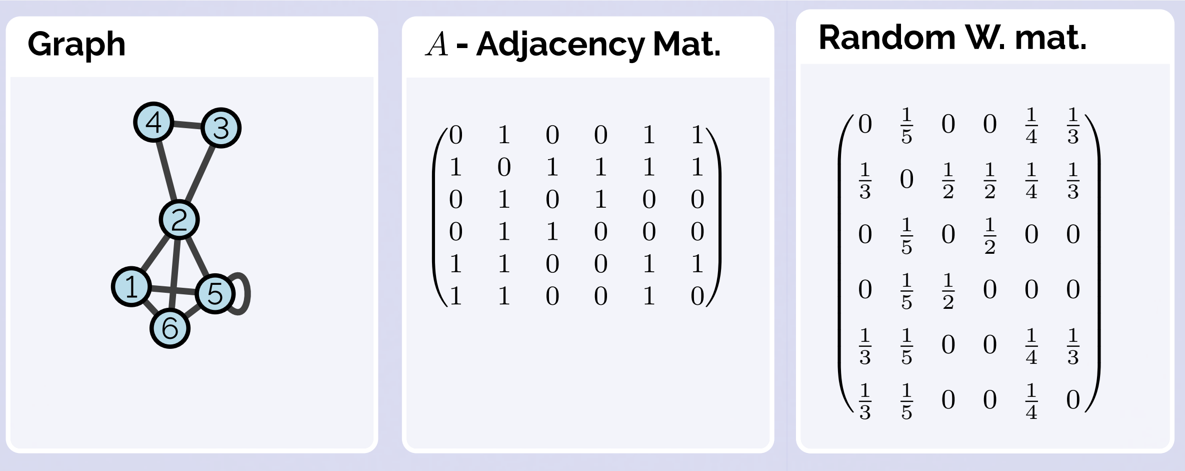
$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(c)

$$S = \begin{pmatrix} 0 & 1/2 & 1/3 & 0 & 1/5 \\ 1 & 0 & 1/3 & 1/3 & 1/5 \\ 0 & 1/2 & 0 & 1/3 & 1/5 \\ 0 & 0 & 1/3 & 0 & 1/5 \\ 0 & 0 & 0 & 1/3 & 1/5 \end{pmatrix}$$

(e)

$$G = \begin{pmatrix} 0.03 & 0.455 & 0.313 & 0.03 & 0.2 \\ 0.88 & 0.03 & 0.313 & 0.313 & 0.2 \\ 0.03 & 0.455 & 0.03 & 0.313 & 0.2 \\ 0.03 & 0.03 & 0.313 & 0.03 & 0.2 \\ 0.03 & 0.03 & 0.03 & 0.313 & 0.2 \end{pmatrix}$$



Graph

A - Adjacency Mat.

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Random W. mat.

$$\begin{pmatrix} 0 & \frac{1}{5} & 0 & 0 & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{3} \\ 0 & \frac{1}{5} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{5} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{5} & 0 & 0 & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{5} & 0 & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

PageRank - as Random Walk

Main idea: The PageRank computation can be interpreted as a Random Walk process with restart

Teleportation probability: the parameter α gives the probability that in the next step of the RW will follow a Markov process or with probability $1\text{-}\alpha$ it will jump to a random node

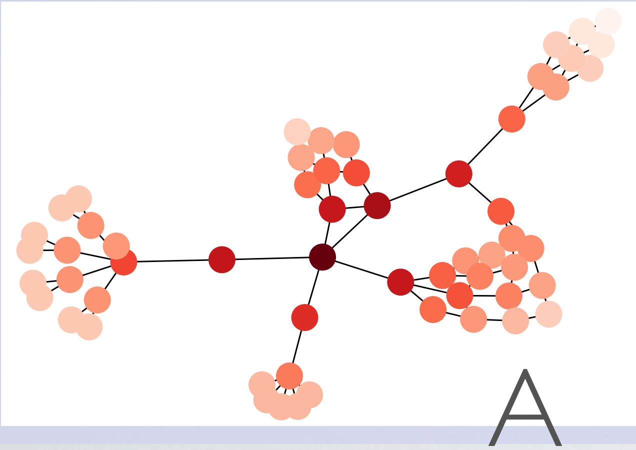
Pagerank score of a node thus corresponds to the probability of this random walker to be on this node after an infinite number of hops.

PAGERANK

- Then how do Google rank when we do a research?
- Compute pagerank (using the power method for scalability)
- Create a subgraph of documents related to our topic
- Of course now it is certainly much more complex, but we don't really know:
  "Most search engine development has gone on at companies with little publication of technical details. This causes search engine technology to remain largely a black art" [Page, Brin, 1997]

OTHERS

- Many other centralities have been proposed
- The problem is how to interpret them ?
- Can be used as supervised tool:
  - Compute many centralities on all nodes
  - Learn how to combine them to find chosen nodes
  - Discover new similar nodes
  - (roles in social networks, key elements in an infrastructure, …)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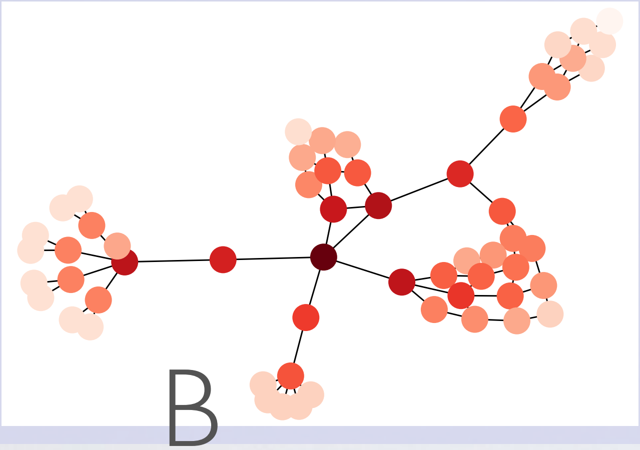
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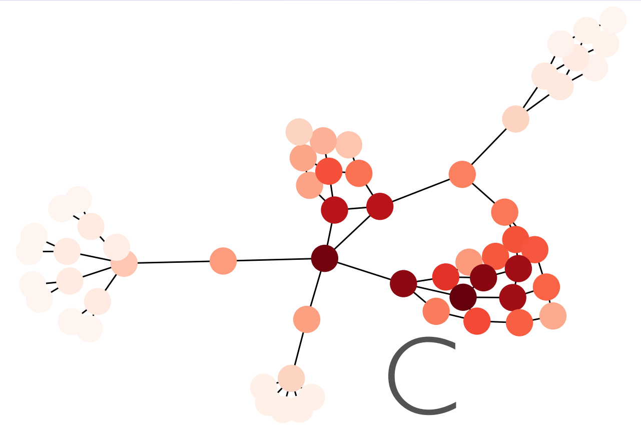
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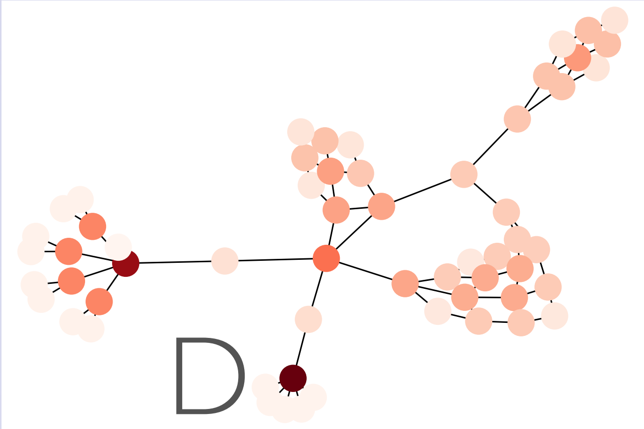
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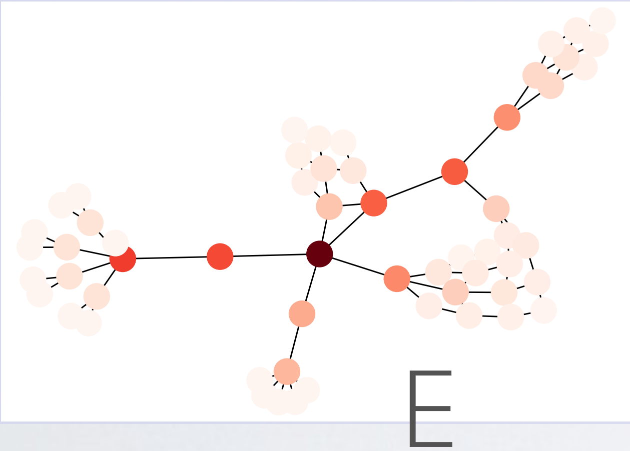
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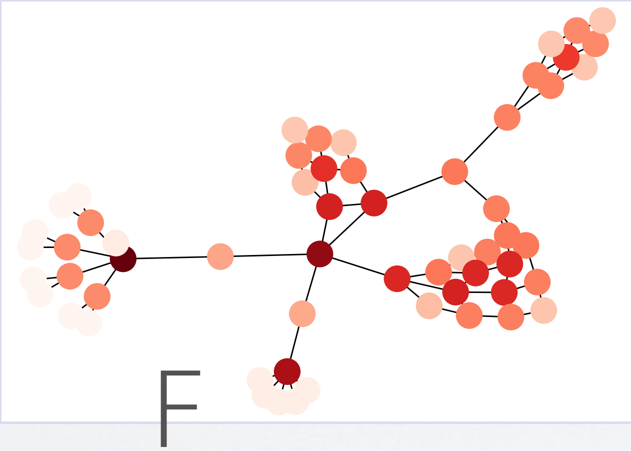
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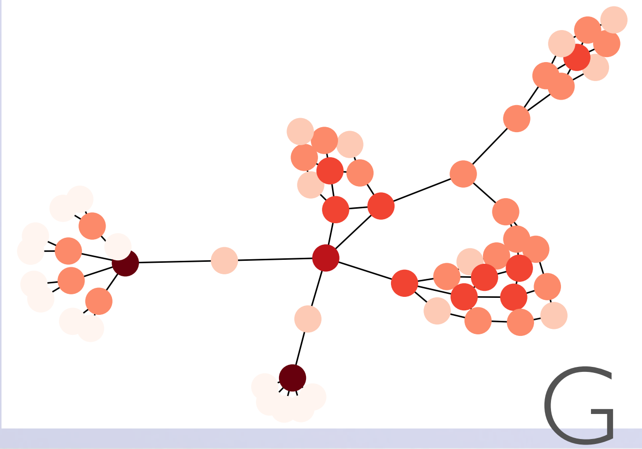
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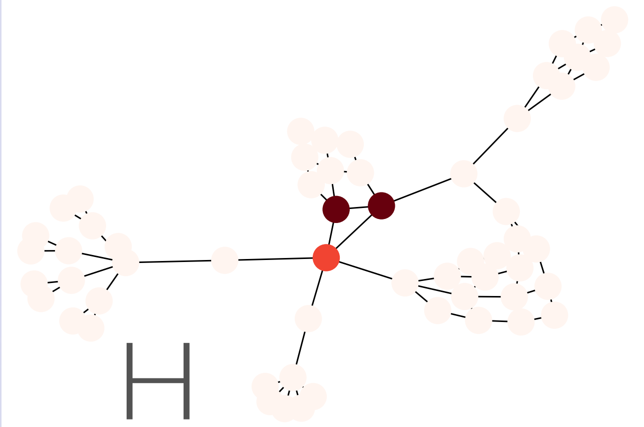
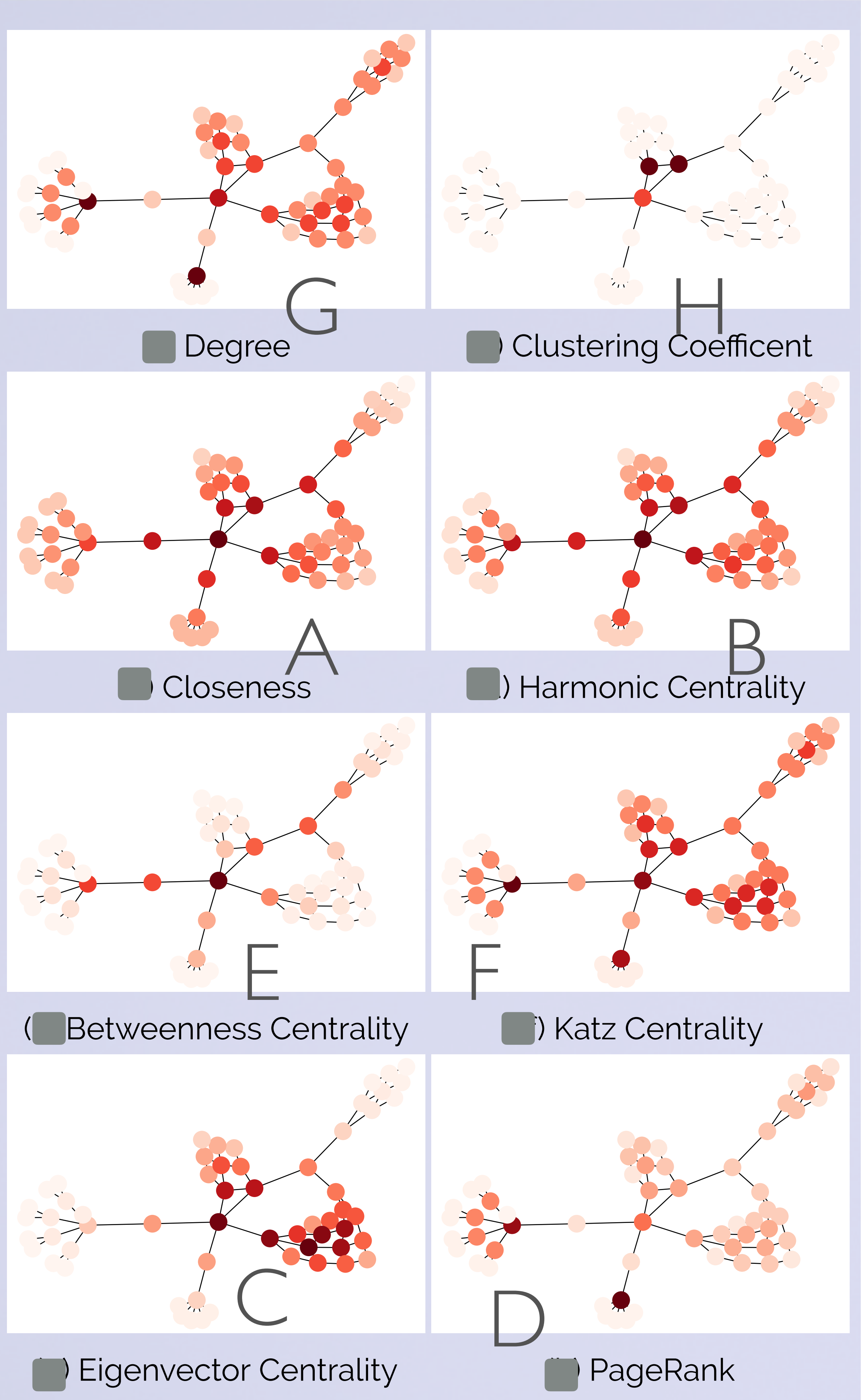

H

Which is which ?

Degree
Clustering coefficient
Closeness
Harmonic Centrality
Betweenness
~~Katz~~
Eigenvector
PageRank



Which is which ?

Degree
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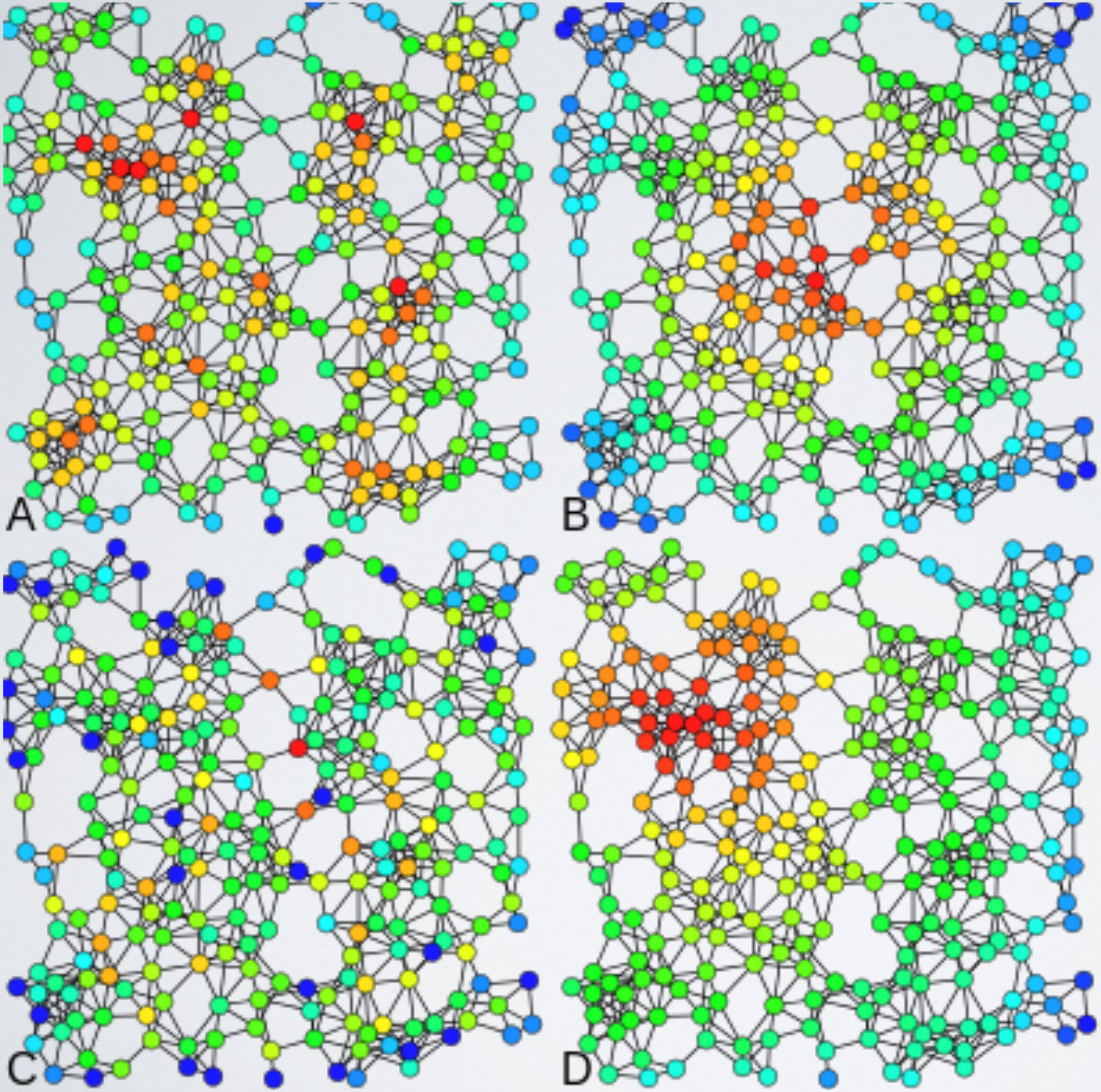
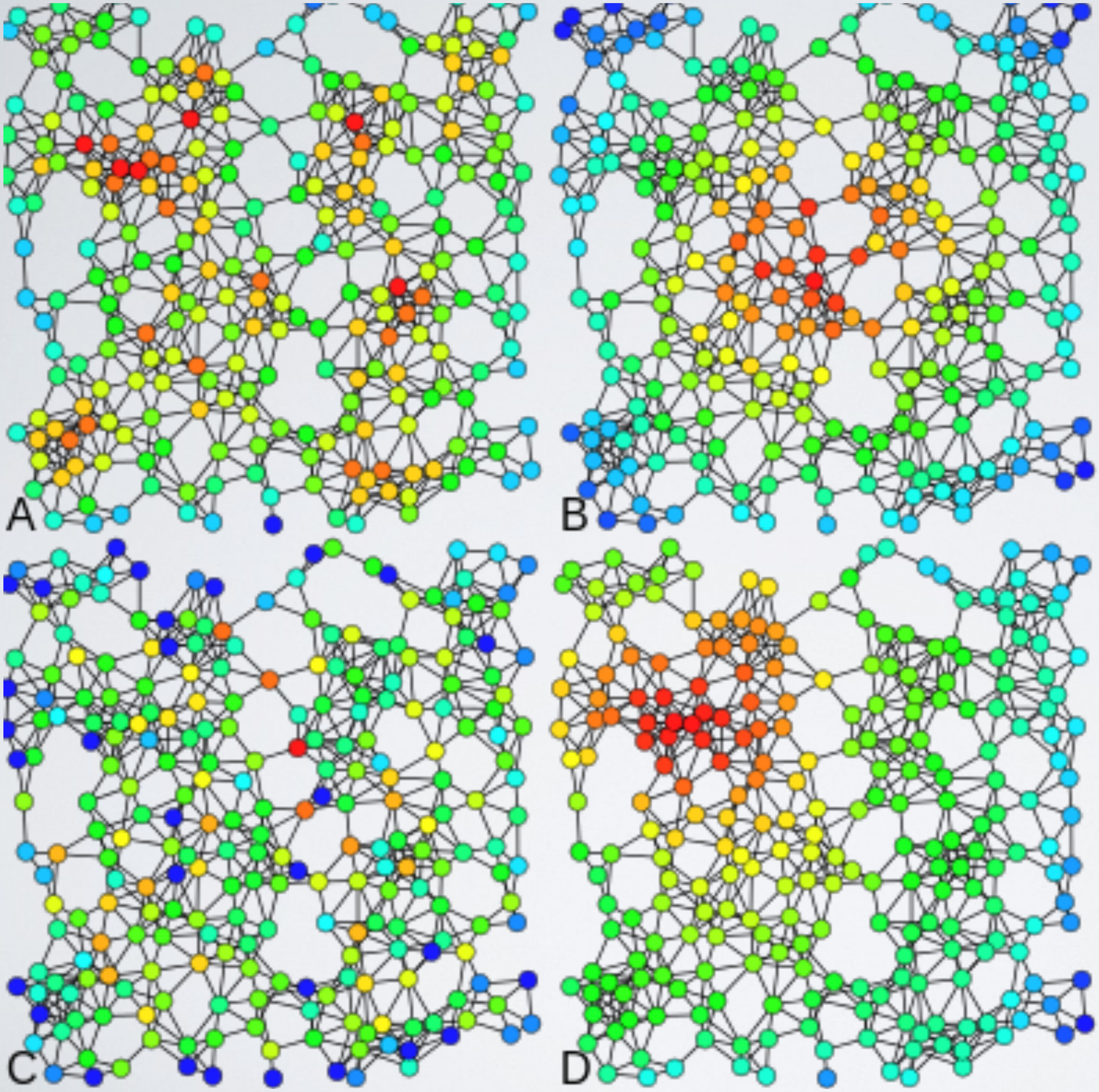


Try again :)

Degree
Betweenness
Closeness
Eigenvector



Try again :)

A: Degree
B:Closeness
C: Betweenness
D: Eigenvector