COMPLEX NETWORKS ANALYSIS INTRODUCTION

Cazabet Rémy

PRESENTATION

- Remy Cazabet (“remi”)
- Associate professor in Computer Science at “Université de Lyon”
- Topics of research:
  - Network Science,
  - Data Mining,
  - Machine Learning,
  - Social Network Analysis,
  - Complex Systems, …

PRESENTATION

Lyon : 2nd city in France

UNIVERSITÉ DE LYON

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PRESENTATION

What about you ?

COURSE ORGANIZATION

- Every day, 2h lectures, 2h practicals.
- We learn a new topic, we apply it on example graphs.
- You can come with your own data. There are many websites with repositories of "interesting" graphs,
  - http://networkrepository.com
  - Marvel, TV series, economics, soccer…

COURSE ORGANIZATION

- Gradation for every week
- End of first week:
  - Send a report on the analysis of a graph you have chosen according to what we have studied (What you think is relevant)
- End of last week:
  - Send a report on the analysis of a DYNAMIC graph according to what we have studied.
- One part of the report should be a Jupyter Notebook

INTRODUCTION

GRAPH OR NETWORKS

- What you have seen last week:
  - Graph theory => Efficient algorithms, complexity analysis, proofs…
- What we will see together:
  - How to make data "speak"
  - Not any kind of data: relational ones, modeled by **networks**

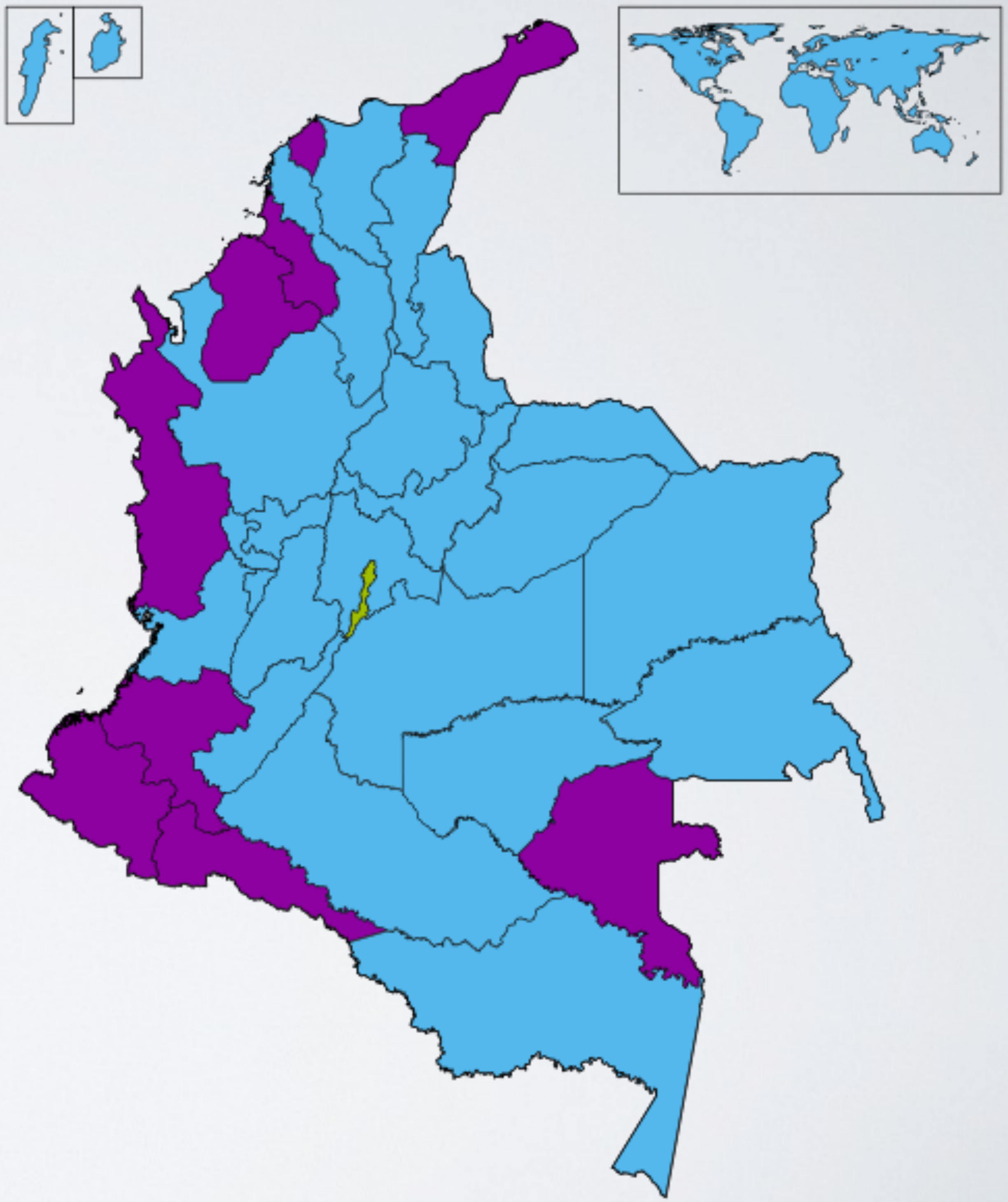
CONTEXT

- Big data, data science, data mining, machine learning, artificial intelligence ….
- Input: Data
- Output:
  - Knowledge
  - Model
  - Prediction

CONTEXT

- Let's take an example: Colombian elections
- Data:
  - Results (by geographical regions)
  - Polls before the vote
  - Surveys: Age, genre, income, marital status, etc.
  - …

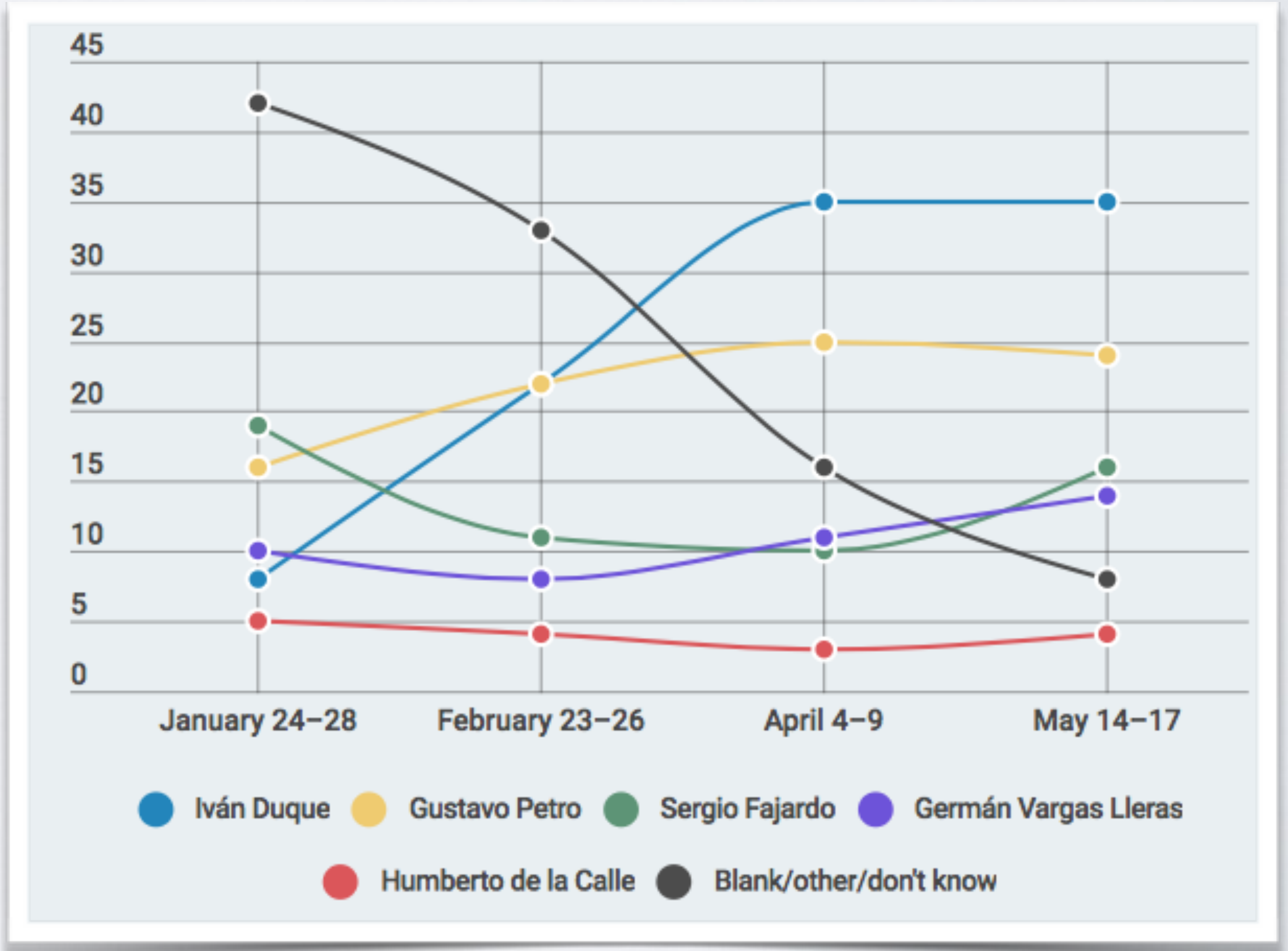
CONTEXT



- Acquiring Knowledge:
  - Geographical disparities
  - Opinions of social classes
  - Long term evolution of the society
  - …

CONTEXT

- Predicting:
  - Time series analysis: predict the futur given trends
  - Predict the vote of a person given its profile
  - Predict how societal evolutions will affect votes
  - …



CONTEXT

- Data oriented decision making/analysis is now ubiquitous:
  - Finance
  - Sport (money game…)
  - Industry (Predictive Maintenance, Supply chain optimisation…)
  - Politics (Cambridge analytica..)

- And Data-Oriented applications continues to expand
  - Self driving cars (data, data, data)
  - Smart cities
  - Physics, Biology, Medicine, …

GRAPHS ?

- Coming back to Colombian elections
  - What information could we add besides features describing each individual ?
  - =>Adding relational data
  - Who is a relative (daughter/sister/grandmother/…) of whom ?
  - Who is a friend of whom ?
  - Who works in the same company ?
  - …
- Tell me who your friends are and I'll tell you who you are
- Knowledge/Opinions propagates and form "social networks"

GRAPHS ?

- “But this information is much harder to obtain than individual ones… right ?”
- On the contrary ! Social Media !
- +, why not, cell phone, emails, WhattsApp, …

GRAPHS ?

- Graphs can also represent any type of data:
  - Step1) Compute correlations between elements
  - Step2) Filter out low values
  - Step3) You have a graph !
- Often used to scale algorithms (DBscan…)
- Or to apply network analysis tools
- (More on that later)

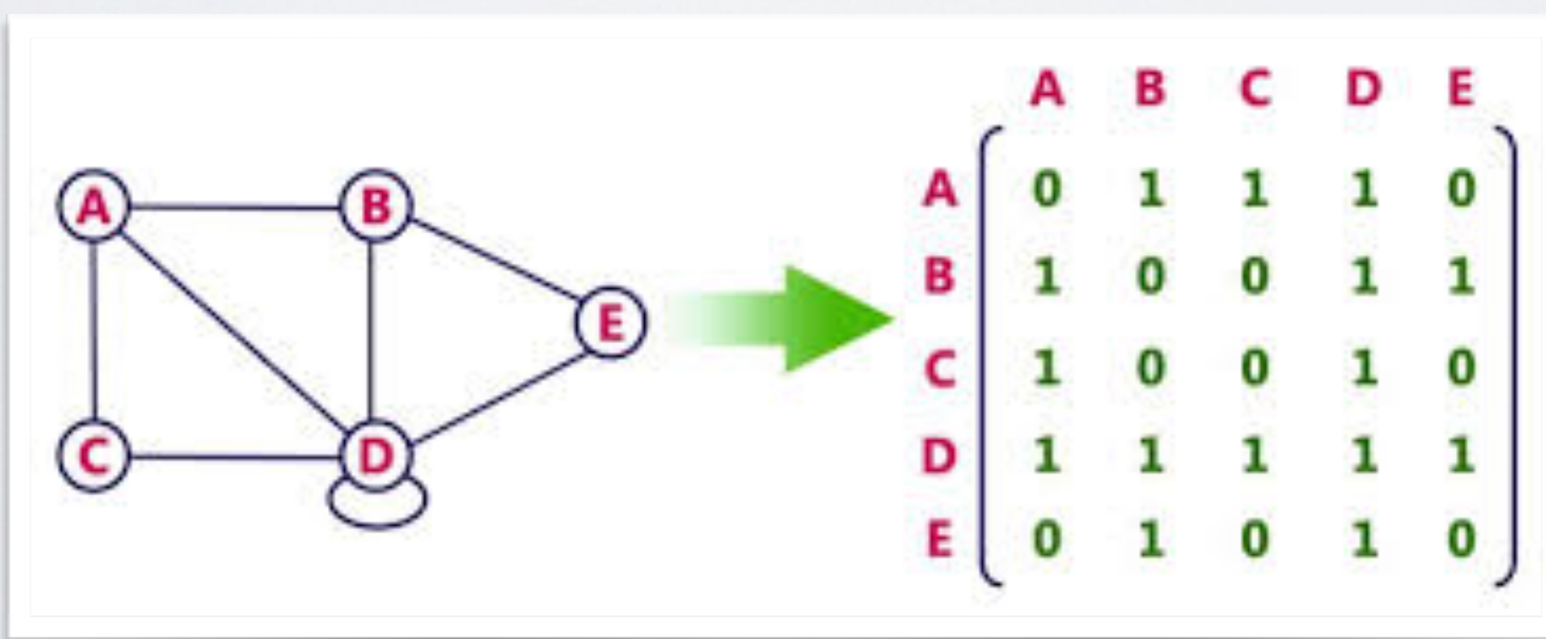
GRAPHS ?

- What is so special about graphs ? Isn't it a feature like any other ?

- Classical data mining/machine learning can be summarized as:
  - An item is described as a VECTOR: [x1,x2,x3,…,xN]
  - We learn sequences of operations on these vectors to predict something
    - IF age>X and income>Y and city in […] THEN Vote=Mr. XXX
  - If your feature is not numeric, you transform it to numbers.
    - For instance: department= NAME
    - Some methods can handle them directly (decision trees, …)
    - Or transformation to vector:
    - 30 departments: Each person has a vector with 29 zeros and a 1

GRAPHS ?

- A graph can be represented as:
  - A list of edges : [{v1,v2}, {v1,v3}, {v5,v7},…]
  - A neighborhood list: {v1:{v2,v3},v2:{v1},v5:{v7},…}
  - An adjacency matrix



GRAPHS ?

- We could use a line of the adjacency matrix as feature vector
- It does not work because:
  - Sparsity: too many 0s
  - Curse of dimensionality
  - Similar features means similar item. Not for adj. matrix:
    - It means connected to the same node
    - What is interesting in graphs is elsewhere: not only direct neighbors

GRAPHS ?

- Field of **Network Science**
- Contributions from physicists, computer scientists/Engineers and mathematicians (beyond traditional scientific fields)
- For me, a "tool" for all scientists, like probabilities, spectral analysis or machine learning
- For computer science: related to ML, DM. Same level as Natural Language Processing, maybe

GRAPHS ?

- Graphs or networks?
- I use both terms interchangeably
- **Graph theory:** older field (env. 70 years), mostly theoretical, studying properties of graphs (usually synthetic) and algorithms on graphs
- **Network Science:** born from graph theory (env. 10 years), interested in real networks, with both theory and applications
- **Social Network Analysis:** Older term than network science (env. 40 years), network science on SN

CHAPTER I
DESCRIBING A NETWORK AT THE GLOBAL SCALE

SIZE

- A network is composed of nodes and edges.
- Size: How many nodes and edges ?

	#nodes	#edges
Wikipedia HL	2M	30M
Twitter 2015	288M	60B
Facebook 2015	1.4B	400B
Brain c. Elegans	280	6393
Roads US	129k	165k
Airport traffic	3k	31k

DENSITY

Defined as:

Directed $D = \frac{|E|}{|V|\,(|V|-1)}$

Undirected $D = \frac{2|E|}{|V|\,(|V|-1)}$

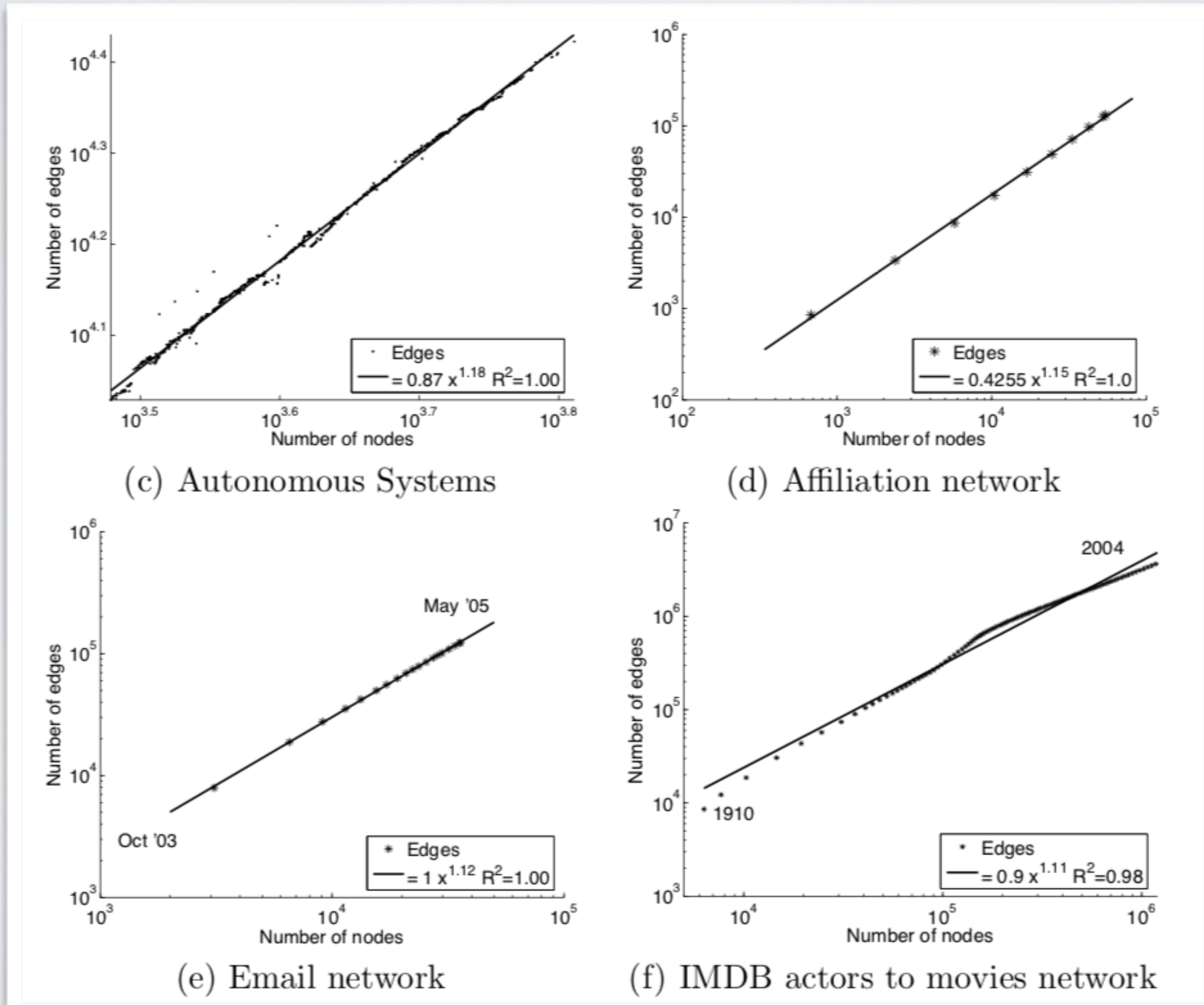
Often more relevant: average degree (2|E| / |V|)

	#nodes	#edges	Density	avg. deg
Wikipedia	2M	30M	1.5×10^{-5}	30
Twitter 2015	288M	60B	1.4×10^{-6}	416
Facebook	1.4B	400B	4×10^{-9}	570
Brain c.	280	6393	0.16	46
Roads Calif.	2M	2.7M	6×10^{-7}	2.7
Airport	3k	31k	0.007	21

DENSITY

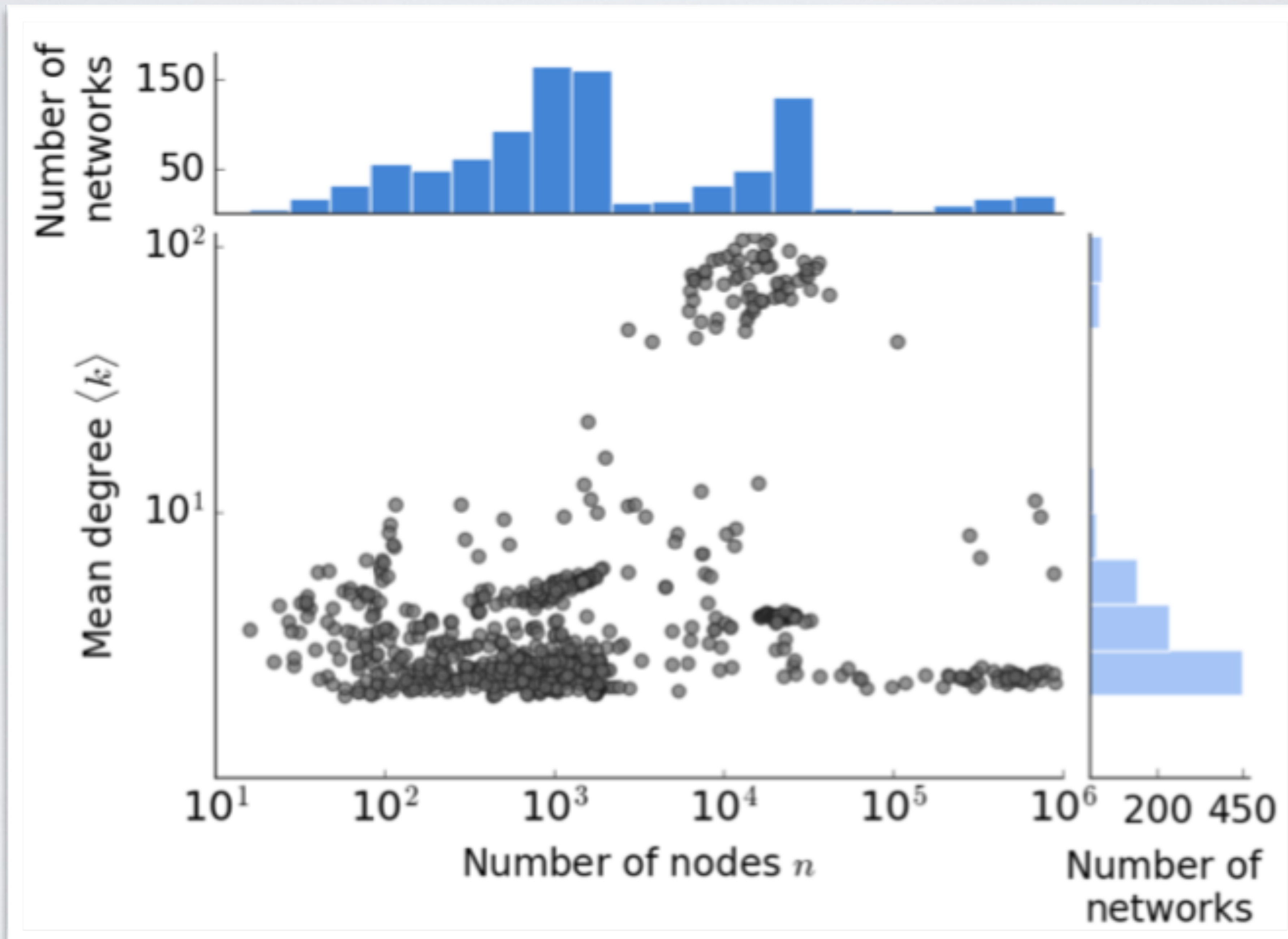
- It has been observed that: [Leskovec. 2006]
  - When graphs increase in size, the average degree increases
  - This increase is very slow
- Think of friends in a social network

DENSITY



Number of edges
Number of nodes
Edges
= 0.87 x^1.18 R^2=1.00
(c) Autonomous Systems
Edges
= 0.4255 x^1.15 R^2=1.0
(d) Affiliation network
May '05
Oct '03
Edges
= 1 x^1.12 R^2=1.00
(e) Email network
2004
1910
Edges
= 0.9 x^1.11 R^2=0.98
(f) IMDB actors to movies network

DENSITY

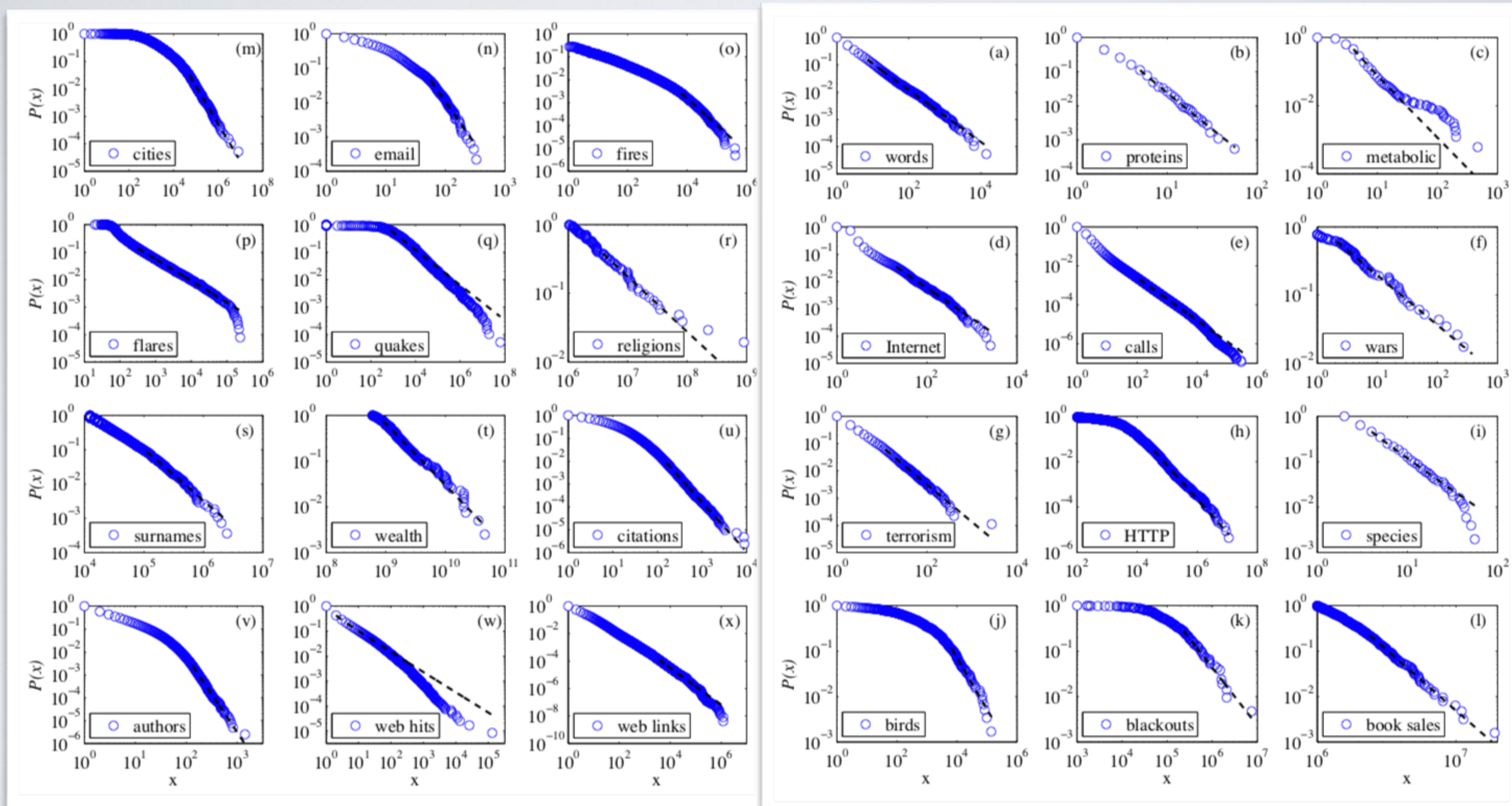


[Broido, Clauset 2018]

DEGREE DISTRIBUTION

- In a fully random graph (Erdos-Renyi), degree distribution is a normal distribution centered on the average degree
- In real graphs, in general, it is not the case:
    - A high majority of small degree nodes
    - A small minority of nodes with very high degree (Hubs)
- Often modeled by a **power law**

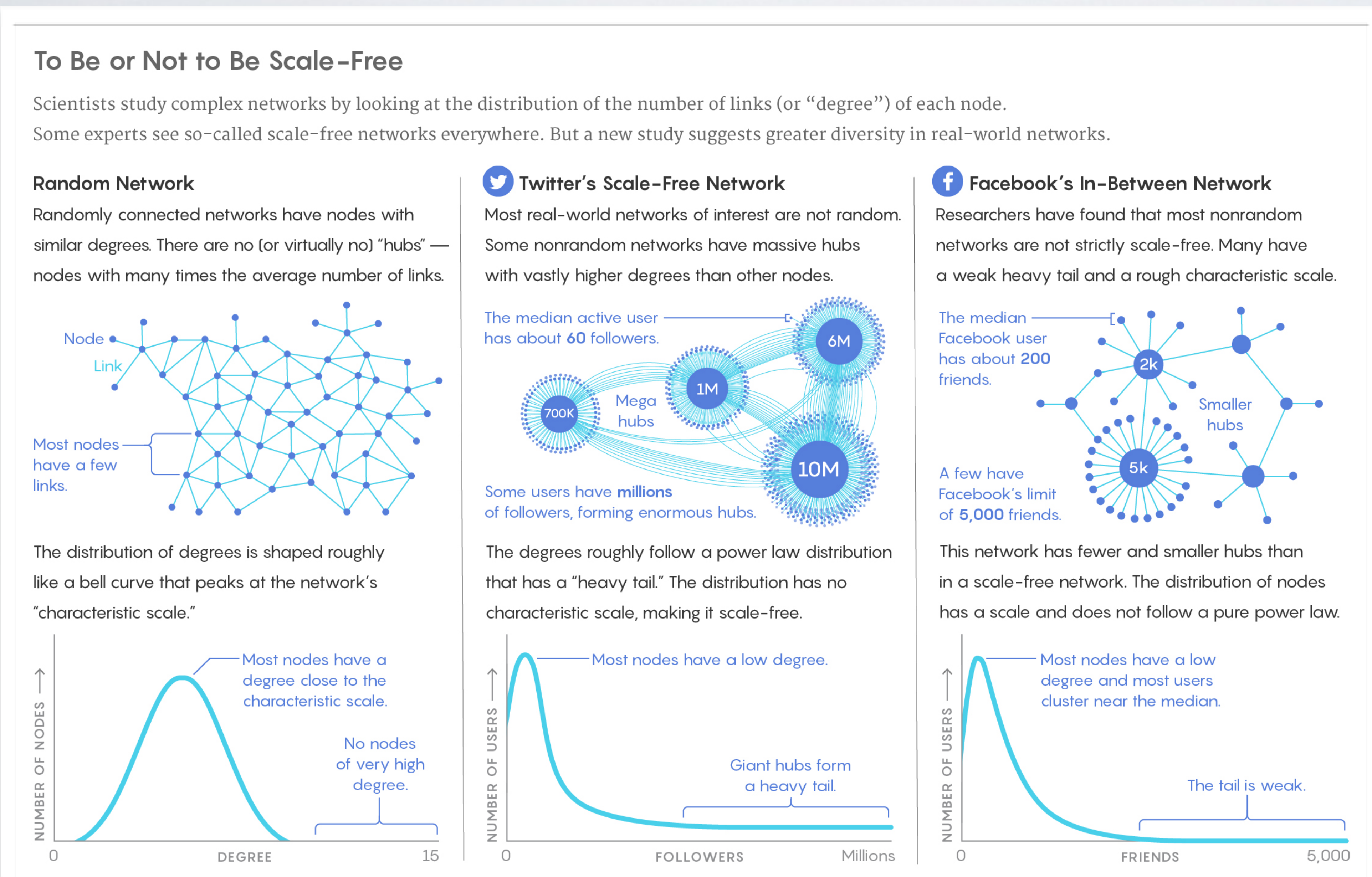
DEGREE DISTRIBUTION



[Clauset 2009]

DEGREE DISTRIBUTION

Power law/Scale free distribution: $f(x) = ax^{-k}$



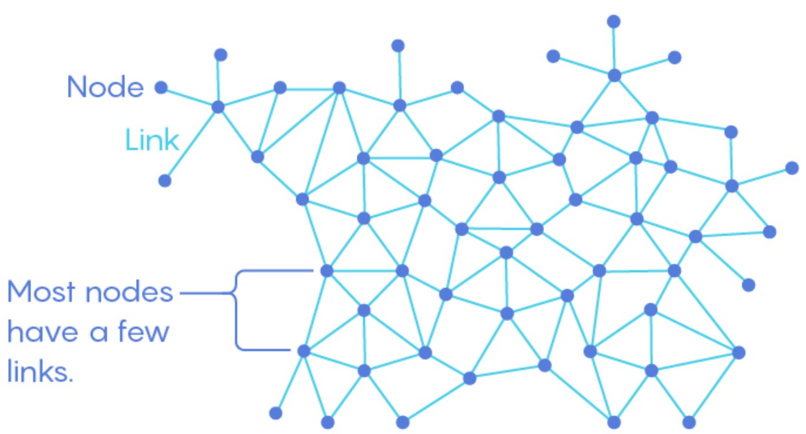
[Quanta magazine 2018]

To Be or Not to Be Scale-Free

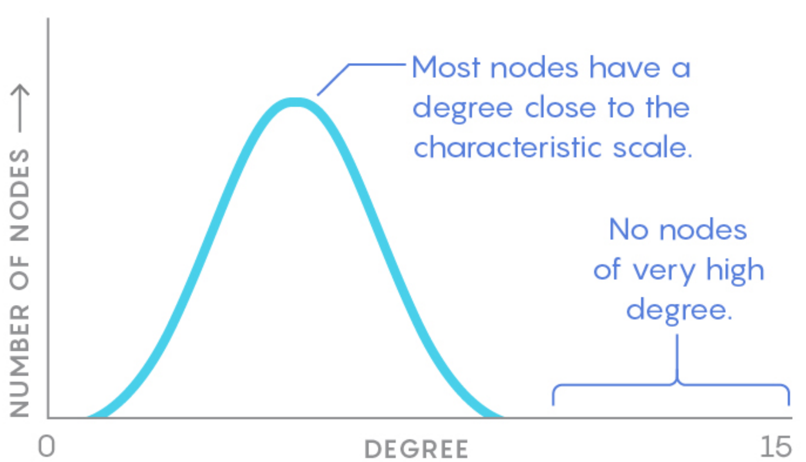
Scientists study complex networks by looking at the distribution of the number of links (or "degree") of each node. Some experts see so-called scale-free networks everywhere. But a new study suggests greater diversity in real-world networks.

Random Network

Randomly connected networks have nodes with similar degrees. There are no (or virtually no) "hubs" — nodes with many times the average number of links.

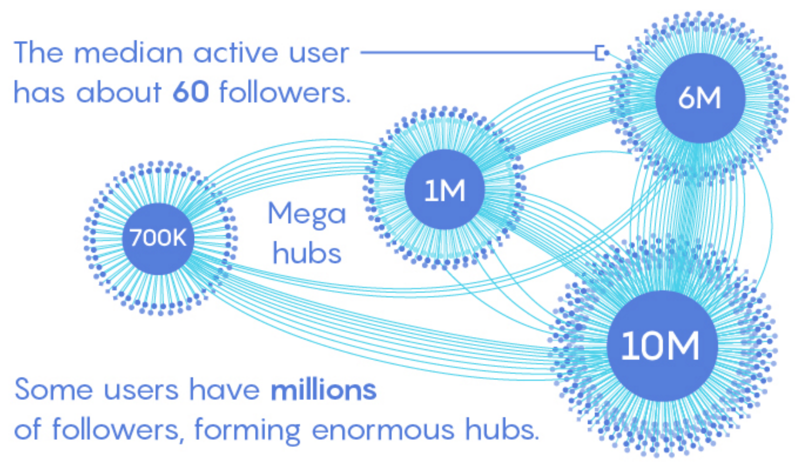


The distribution of degrees is shaped roughly like a bell curve that peaks at the network's "characteristic scale."

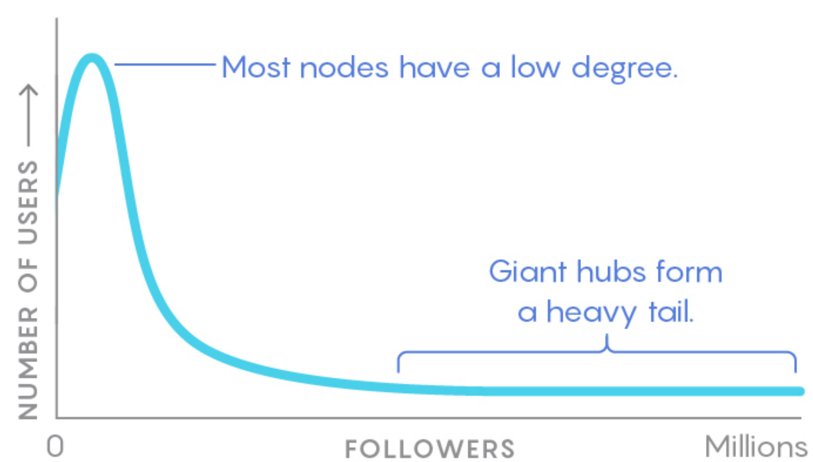


Twitter's Scale-Free Network

Most real-world networks of interest are not random. Some nonrandom networks have massive hubs with vastly higher degrees than other nodes.

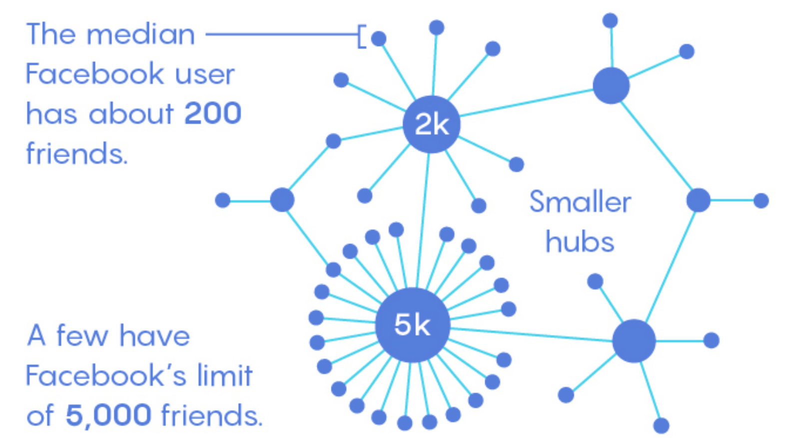


The degrees roughly follow a power law distribution that has a "heavy tail." The distribution has no characteristic scale, making it scale-free.

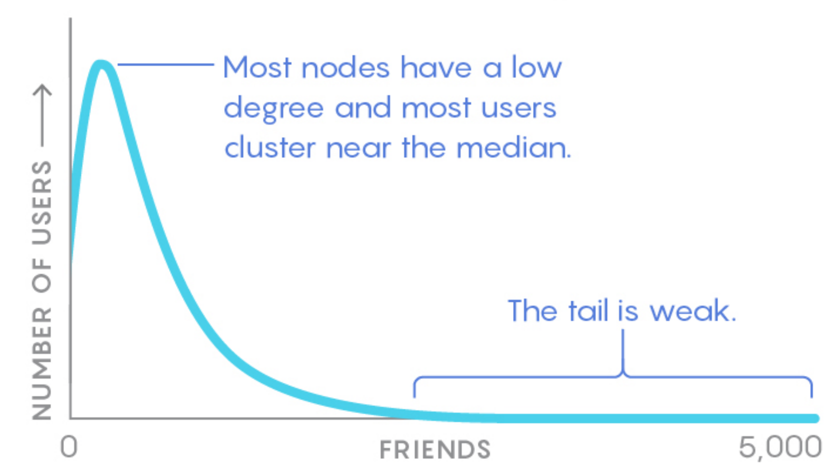


Facebook's In-Between Network

Researchers have found that most nonrandom networks are not strictly scale-free. Many have a weak heavy tail and a rough characteristic scale.



This network has fewer and smaller hubs than in a scale-free network. The distribution of nodes has a scale and does not follow a pure power law.



DEGREE DISTRIBUTION

- This has important implications:
  - There is no "scale" in the degree: the average degree is not representative
  - It is not realistic to use "random graphs" (ER) for evaluating algorithms performance
- If the degree distribution is not a power law, some algorithms might not behave as expected (spatial networks…)

CLUSTERING COEFFICIENT

Global clustering coefficient

$$C = \frac{\text{number of closed triplets}}{\text{number of all triplets (open and closed)}}.$$

Triplet: set of 3 nodes connected by 2 or 3 edges

Average Clustering Coefficient

Clustering coefficient of a node: $$C_i = \frac{2|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}$$

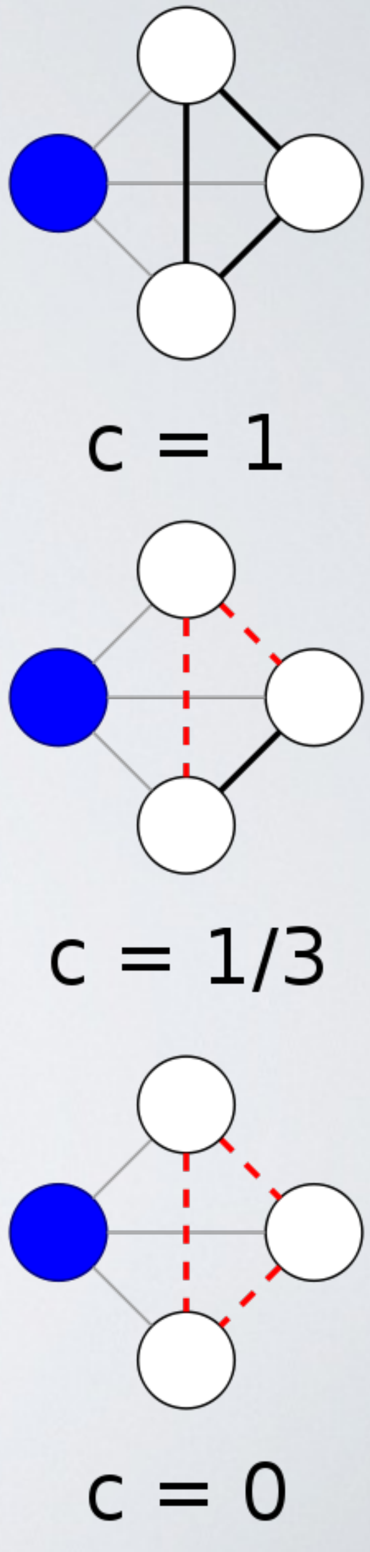
Average CC: $$\bar{C} = \frac{1}{n}\sum_{i=1}^{n} C_i$$

CLUSTERING COEFFICIENT

The higher the value,
the more **locally dense** is the network.

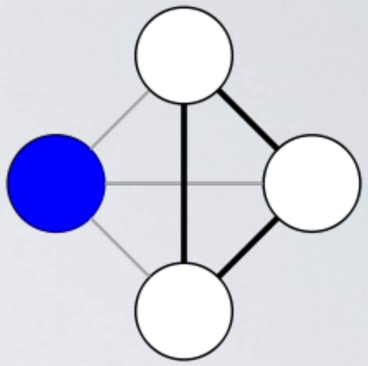
"Friends of my friends are my friends"

Higher in real networks than random

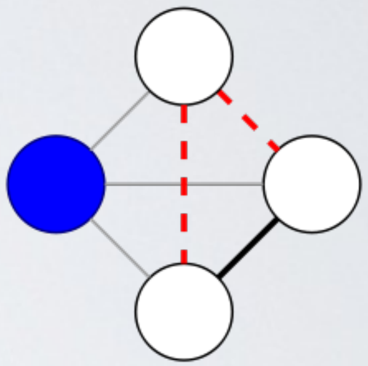


CLUSTERING COEFFICIENT

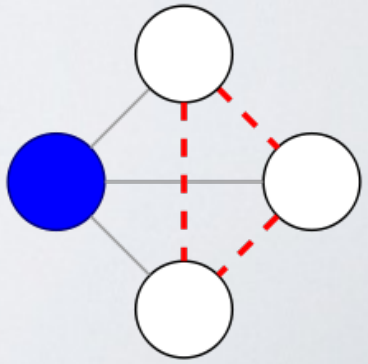
- Facebook ego-networks: 0.6
- Twitter lists: 0.56
- California Road networks: 0.04
- Random (ER): =density: very small for large graphs



c = 1



c = 1/3



c = 0

CONNECTED COMPONENTS

- A connected component: a group of nodes all mutually reachable
- Most real networks:
  - A "Giant connected component" including >99% nodes
  - A few small connected components
- E.g.: Facebook 2011: 99.91%

DIAMETER

- Shortest path between nodes *u* and *v*: minimal number of hops between them.
- Diameter: the longest shortest path in the network
- Very sensible to outliers, not reliable

AVERAGE PATH LENGTH

- Average shortest path between all pairs of nodes
- The famous 6 degrees of separation (Milgram experiment)
  - In fact 6 hops
  - (More on that next slide)
- Not too sensible to noise
- Tells your if the network is "stretched" or "hairball" like

SIDE-STORY: MILGRAM EXPERIMENT

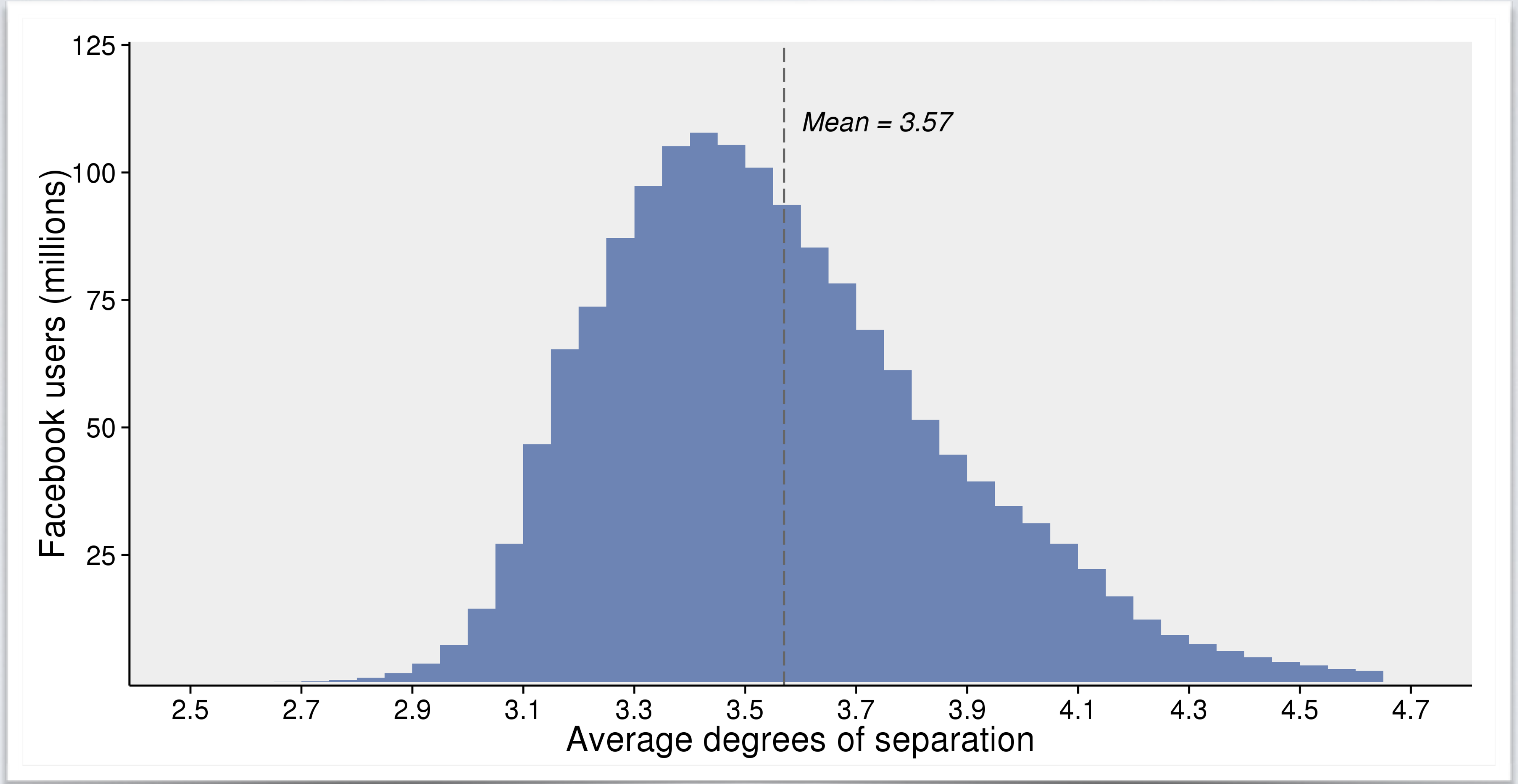
- Small world experiment (60's)
  - Give a (physical) mail to random people
  - Ask them to send to someone they don't know
    - They know his city, job
  - They send to their most relevant contact
- Results: In average, 6 hops to arrive

SIDE-STORY: MILGRAM EXPERIMENT

- Many criticism on the experiment itself:
  - Some mails did not arrive
  - Small sample
  - …

- Checked on “real” complete graphs (giant component):
  - MSN messenger
  - Facebook
  - The world wide web
  - …

SIDE-STORY: MILGRAM EXPERIMENT



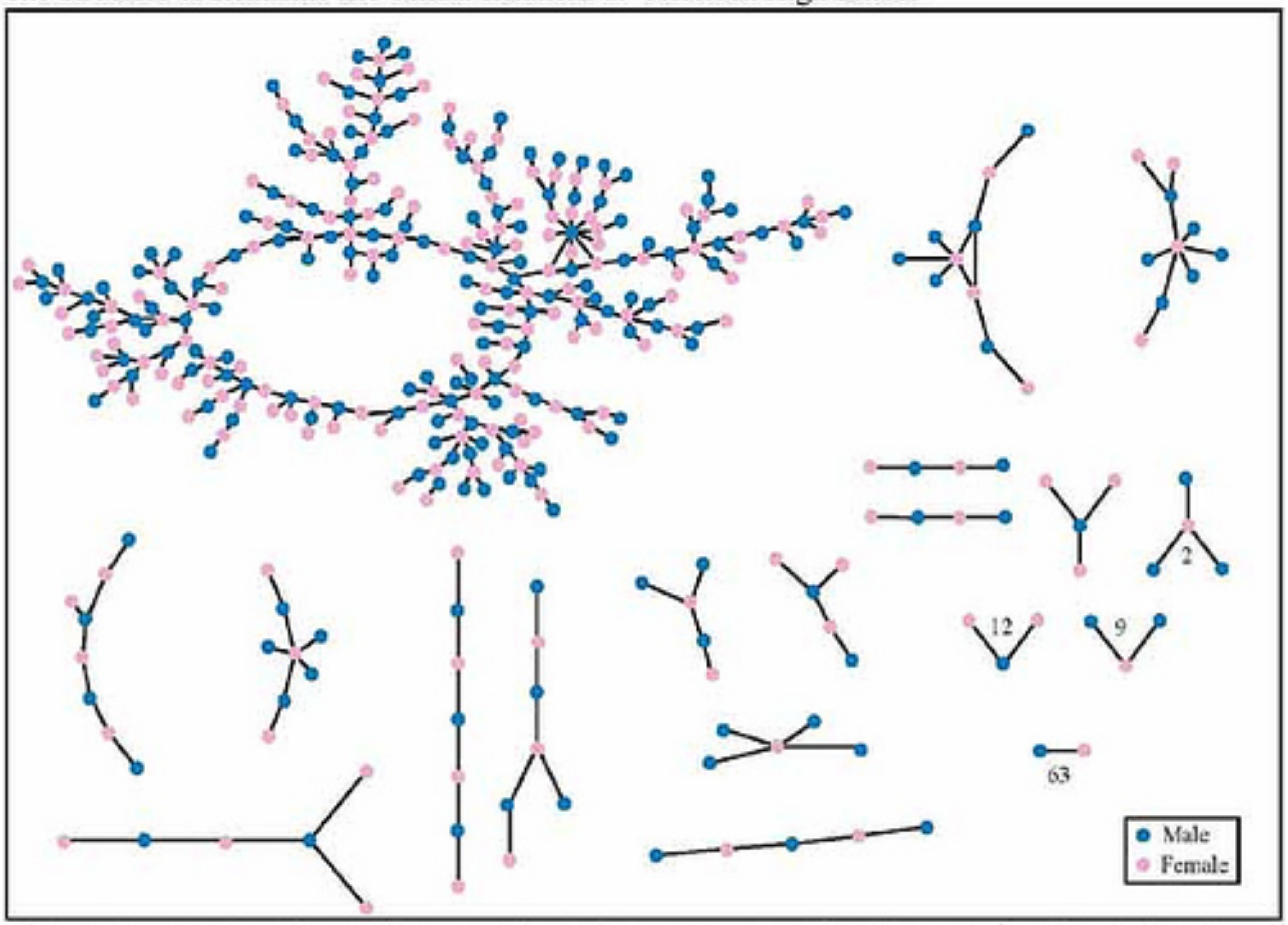
Facebook

HOMOPHILY/ASSORTATIVITY

- Nodes might have a preference for some other nodes
  - Similar nodes (age in social networks)
  - Different nodes (genre in sentimental networks (yes, it has been done!))
  - Nodes with a particular property
- "Assortativity" alone often used to mean "degree assortativity"
  - Large nodes are preferentially connected to large nodes
- All this implies: "compared with a random network"

HOMOPHILY/ASSORTATIVITY

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

HOMOPHILY/ASSORTATIVITY

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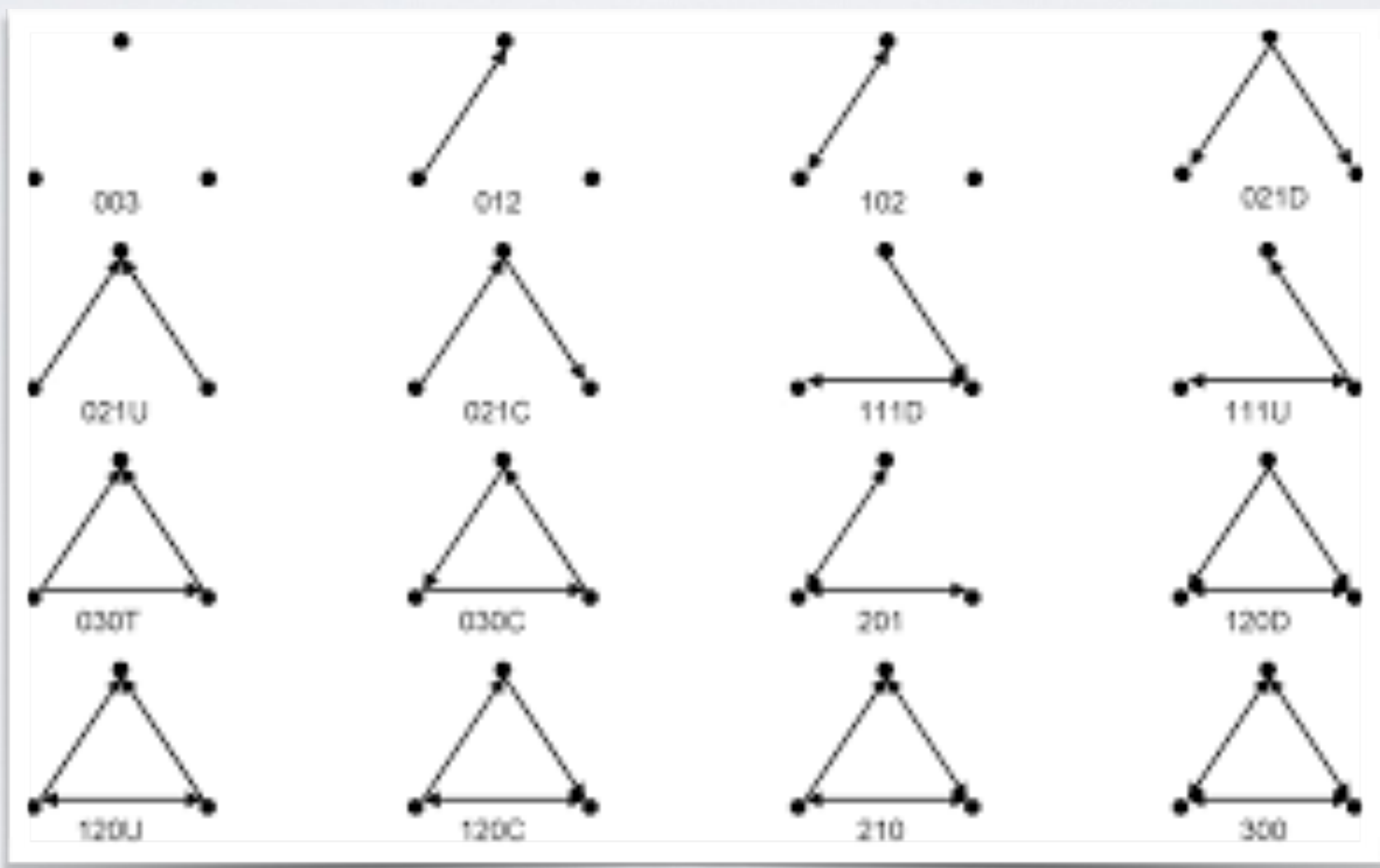
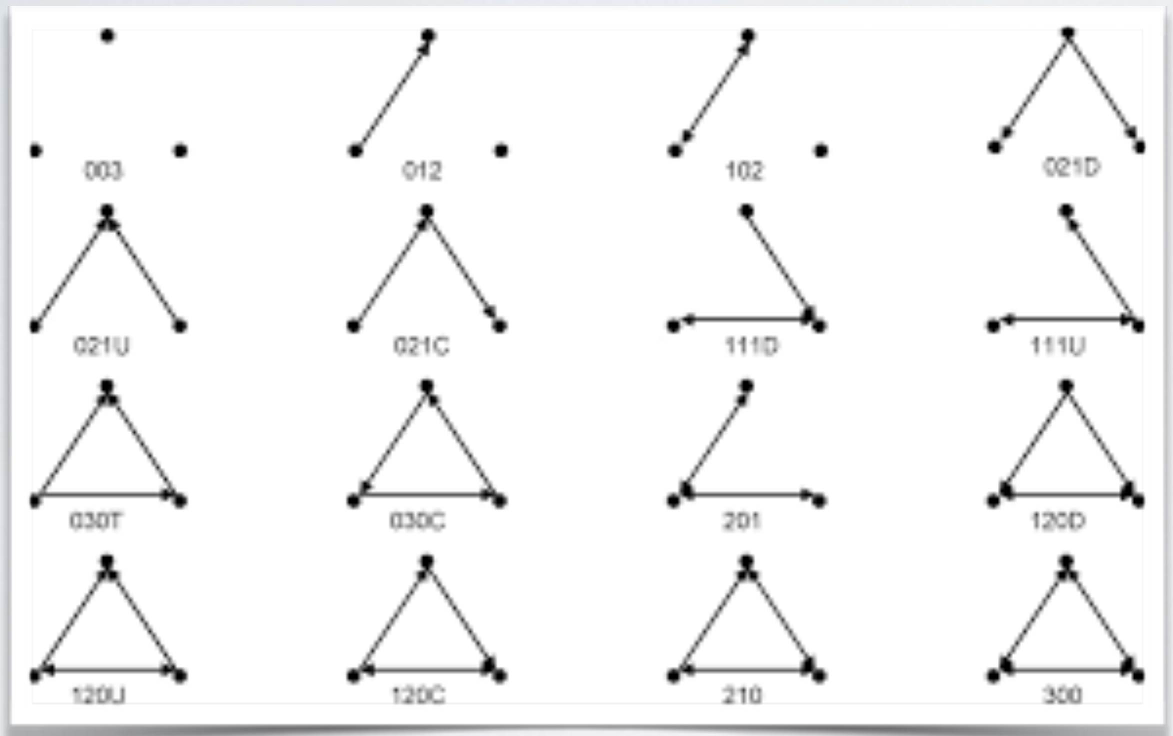
OTHER (A FEW EXAMPLES)

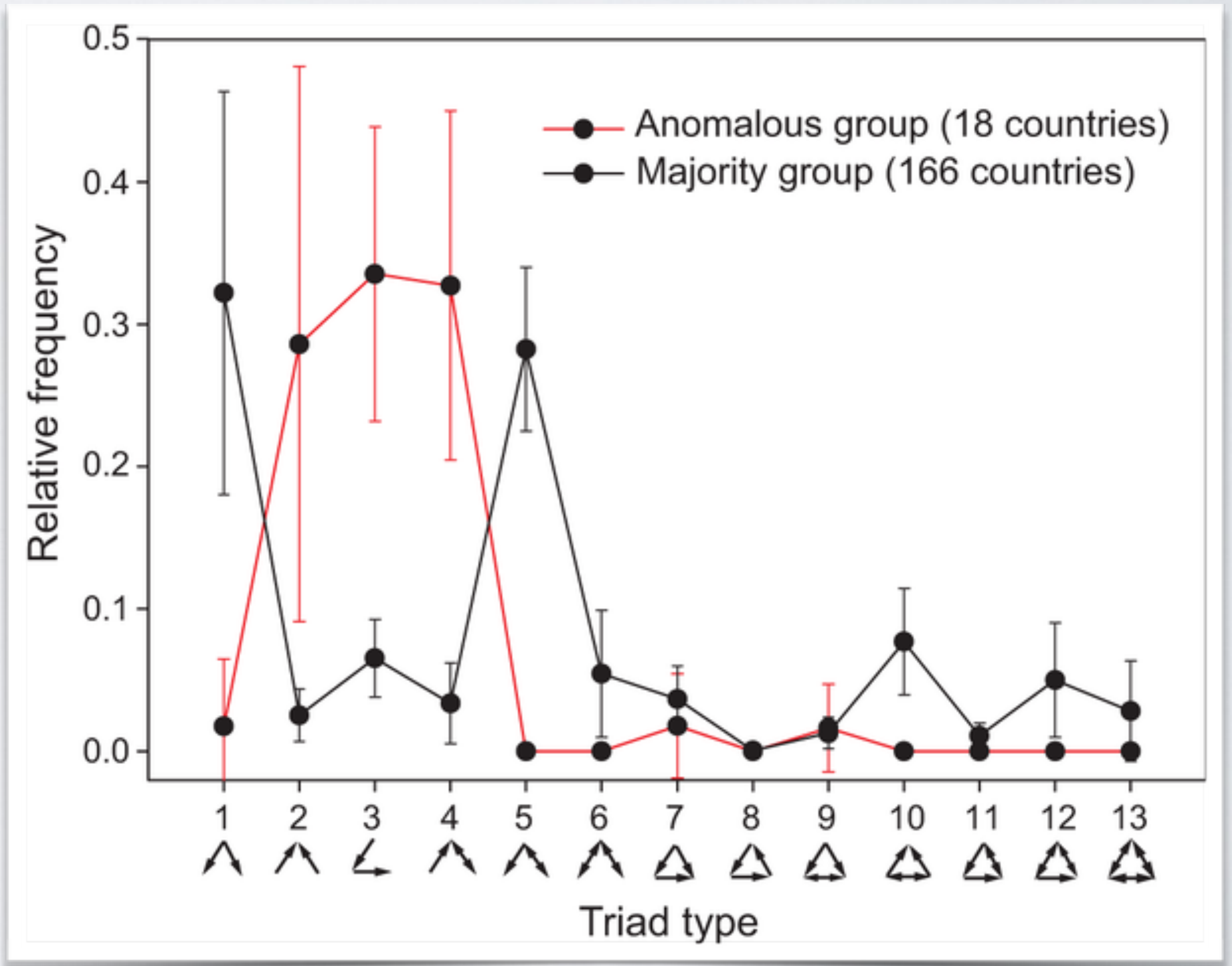
Triads counting



OTHER

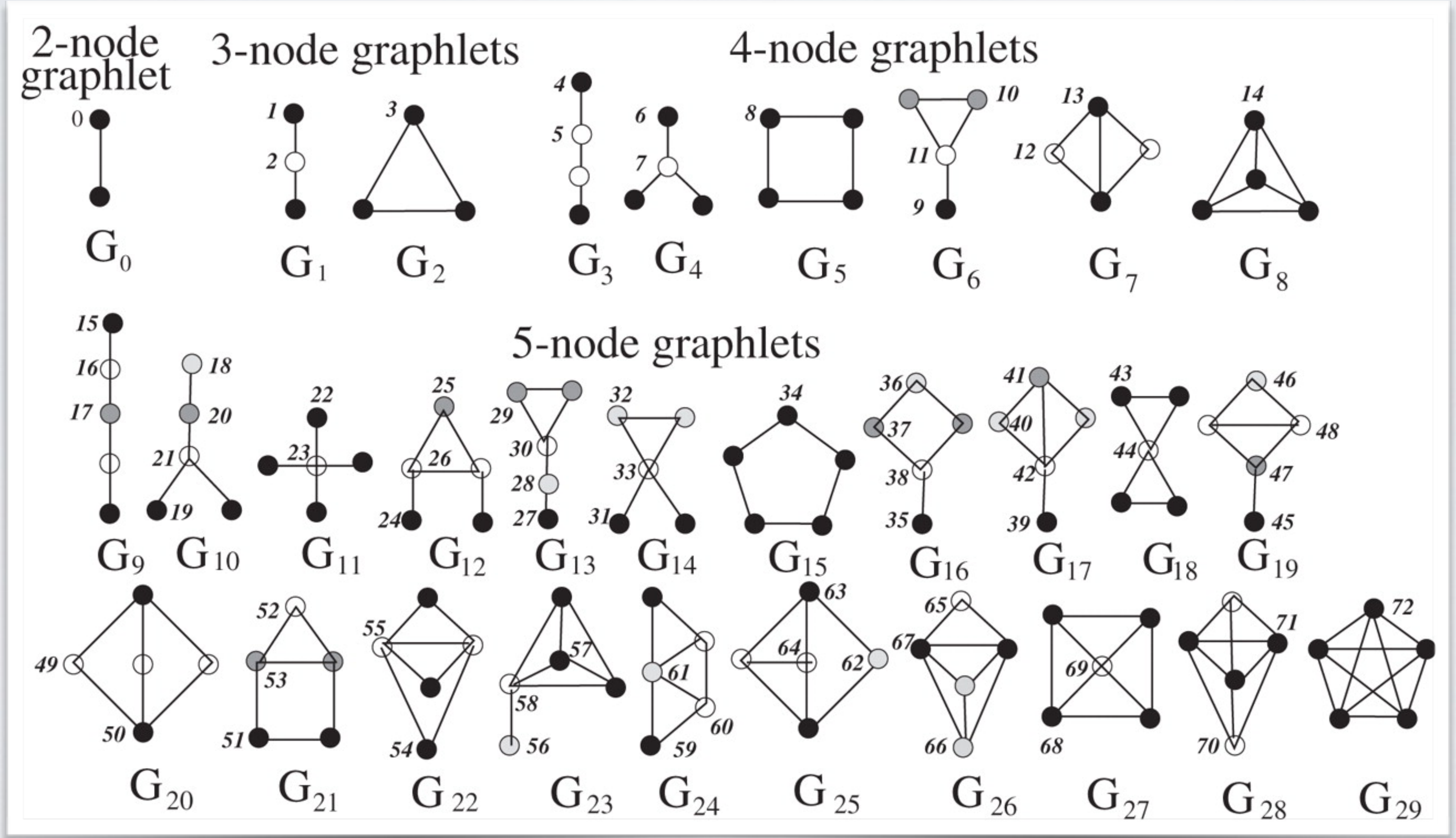
Triads counting



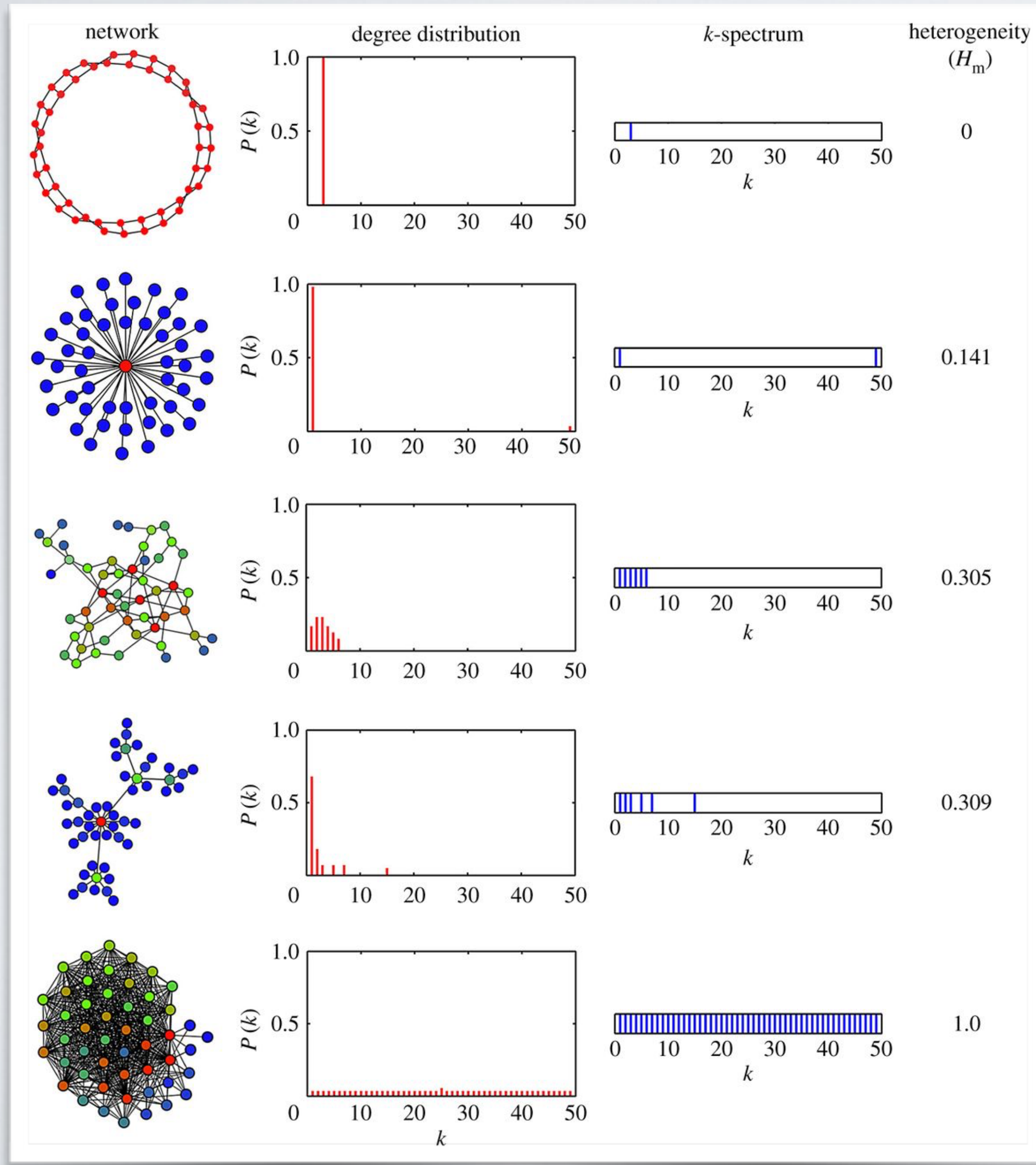


OTHER

Graphlets



OTHER



Spectral properties

Look for
Spectral graph theory

PROPERTIES OF REAL NETWORKS

SMALL WORLD NETWORK

- Not formally defined.
  - **Small** average distance (< log(N) ?)
  - **High** Clustering (>0.1 ?)

- Random networks (ER) have small avg. distance but low clustering

- Spatial networks have high clustering but high avg. distance

SMALL WORLD NETWORK

- Not formally defined.
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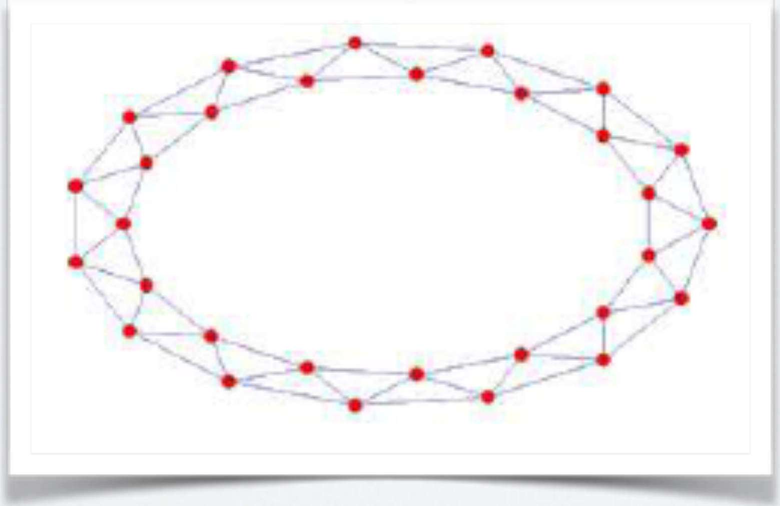


CLASSIFYING NETWORKS

TABLE I. DISTRIBUTION OF NETWORKS OVER DOMAINS

Domain	Number of Networks
Social	25
Citation	20
Communication	28
Ecology	20
Biomolecular	32
Computer	21
Transportation	5

[Kantarci et al. 2013]

CLASSIFYING NETWORKS

TABLE II. OVERVIEW OF TOPOLOGICAL MEASURES RELATIVELY TO DOMAINS

	n	δ	$\langle k \rangle$	C	$\langle d \rangle$	D	R	Q
Social	[11, 1882] μ:143.88 σ: 448.52	[0.0004, 0.38] μ: 0,29 σ: 0,25	[1.85, 66.69] μ: 11.39 σ: 14.54	[0.01, 0.87] μ: 0.38 σ:0.25	[1.26, 9.33] μ: 2.80 σ: 1.68	[2, 305124] μ: 12212.12 σ: 61023.31	[2, 16] μ: 3.2 σ:4.07	[-0,03, 0.89] μ: 0.31 σ: 0.29
Citation	[35, 27779] μ:3424.53 σ: 7547.97	[0.0004, 0.26] μ: 0.07 σ: 0.09	[3.24, 516.80] μ: 39.81 σ: 104.77	[0.03, 0.69] μ: 0.23 σ: 0.17	[1.76, 8.46] μ: 3.88 σ: 1.55	[3, 37] μ: 13.93 σ: 0.26	[2, 49] μ: 8.29 σ: 13.67	[0.14, 0.93] μ: 0.41 σ: 0.20
Communication	[12, 3861] μ: 427.93 σ:103.822	[0.0004, 0.36] μ: 0.12 σ: 0.11	[1.83, 27.70] μ: 7.50 σ: 5.66	[0.01, 0.88] μ: 0.25 σ: 0.22	[1.21, 6.53] μ: 2.98 σ: 1.50	[3, 33] μ: 10.35 σ: 8.42	[2, 22] μ: 5.25 σ: 6.64	[0.01, 0.79] μ: 0.42 σ: 0.24
Ecological	[24, 128] μ: 65.38 σ: 35.00	[0.0816, 0.23] μ: 0.15 σ: 0.03	[5.13, 33.39] μ: 18.15 σ: 10.11	[0.25, 0.49] μ: 0.38 σ: 0.08	[1.81, 3.36] μ: 2.31 σ: 0.35	[8, 947493] μ: 133126.5 σ: 302590.7	[2, 11] μ: 3 σ: 2.16	[0.01, 0.53] μ: 0.04 σ: 0.12
Biomolecular	[23, 3839] μ 1099.44 σ:889.27	[0.0012, 0.34] μ: 0.02 σ: 0.06	[2.15, 15.88] μ: 5.34 σ: 2.37	[0.02, 0.57] μ: 0.07 σ: 0.14	[1.80, 7.65] μ: 4.66 σ: 1.16	[3, 35] μ: 13.03 σ: 5.33	[2, 63] μ: 9.79 σ: 15.90	[0.01, 0.78] μ: 0.52 σ: 0.17
Computer	[18, 10680] μ: 158.28 σ:2973.78	[0.0002, 0.50] μ: 0.05 σ: 0.11	[2.54, 39.1] μ: 6.95 σ: 8.67	[0.01, 0.50] μ: 0.12 σ: 0.14	[1.49, 18.98] μ: 4.31 σ: 3.48	[2, 46] μ: 11.65 σ: 8.71	[2, 352] μ: 38.13 σ: 86.11	[0.01, 0.88] μ: 0.43 σ: 0.26
Transportation	[75, 332] μ:174.40 σ: 107.60	[0.0327, 0.24] μ: 0.22 σ: 0.26	[4.23, 194,64] μ: 37.90 σ: 69.61	[0.01, 0.84] μ: 0.32 σ: 0.26	[1.21, 3.48] μ: 2.37 σ: 0.70	[3, 19] μ: 6.94 σ: 6.27	[2, 16] μ: 4.28 σ: 5.67	[0.01, 0.44] μ: 0.15 σ: 0.16

Density **avg. degree** **avg. CC** **avg. distance** **Diameter** Radius Modularity

[Kantarci et al. 2013]

CLASSIFYING NETWORKS

TABLE III. CORRELATION BETWEEN GLOBAL MEASURES

	$\boldsymbol{\delta}$	$\langle \boldsymbol{k} \rangle$	$\boldsymbol{C}$	$\langle \boldsymbol{d} \rangle$	$\boldsymbol{D}$	$\boldsymbol{R}$	$\boldsymbol{Q}$
$\boldsymbol{\delta}$	-	0.16	0.76	-0.45	0.02	-0.14	-0.71
$\langle \boldsymbol{k} \rangle$	-	-	0.12	-0.16	-0.01	0.00	-0.13
$\boldsymbol{C}$	-	-	-	-0.43	0.04	-0.09	-0.51
$\langle \boldsymbol{d} \rangle$	-	-	-	-	-0.09	0.59	0.60
$\boldsymbol{D}$	-	-	-	-	-	-0.03	-0.12
$\boldsymbol{R}$	-	-	-	-	-	-	0.16
$\boldsymbol{Q}$	-	-	-	-	-	-	-

TABLE VII. DISTRIBUTION OF DOMAINS OVER CLUSTERS

	Cluster 1	**Cluster 2**
Biomolecular	29	3
Citation	16	4
Computer	19	2
Ecology	1	19
Transportation	0	5
Social	5	20
Communication	5	23

[Kantarci et al. 2013]

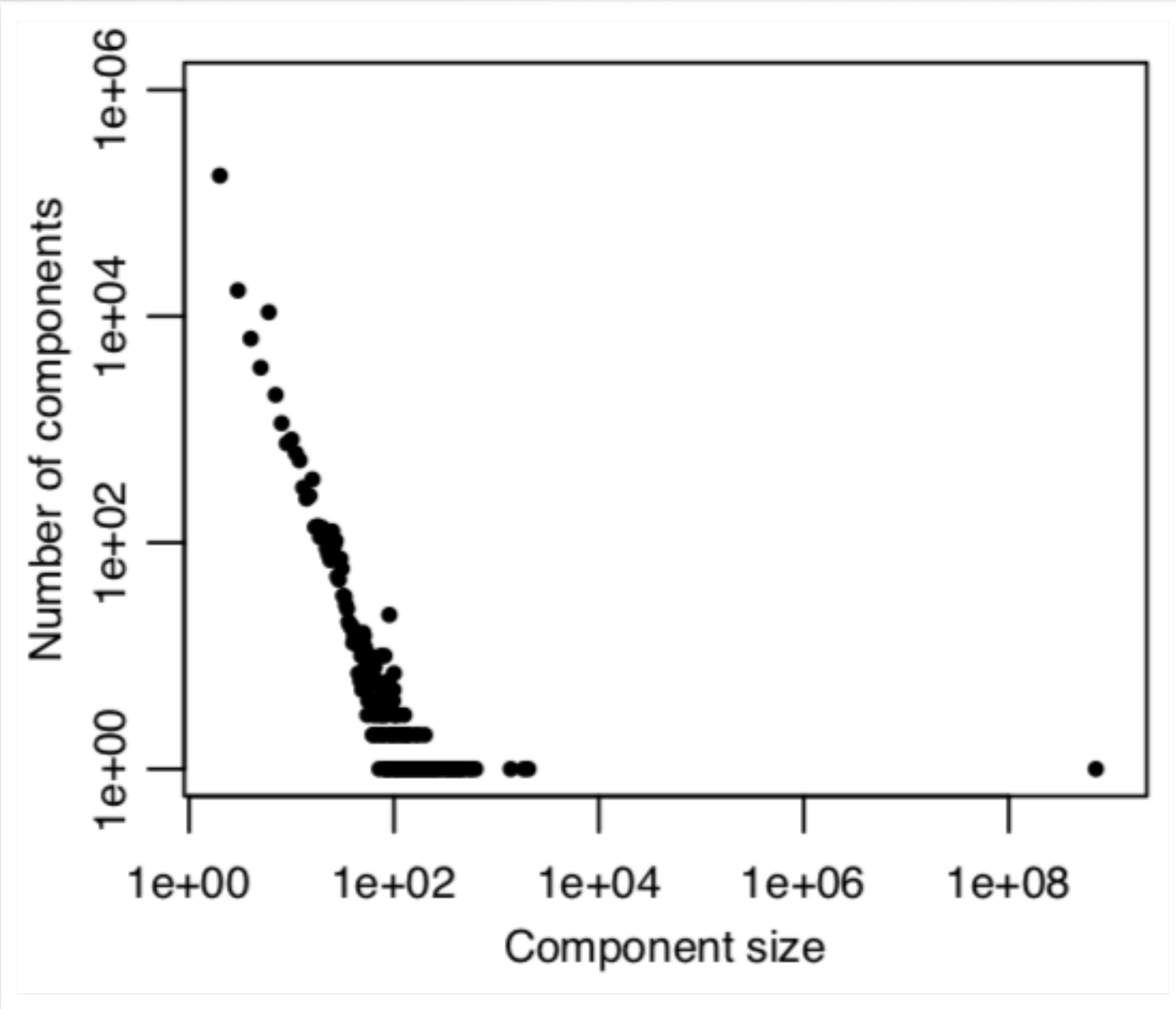
EXEMPLE OF GRAPH ANALYSIS

- Source: [The Anatomy of the Facebook Social Graph, Ugander et al. 2011]
- The Facebook friendship network in 2011

EXEMPLE OF GRAPH ANALYSIS

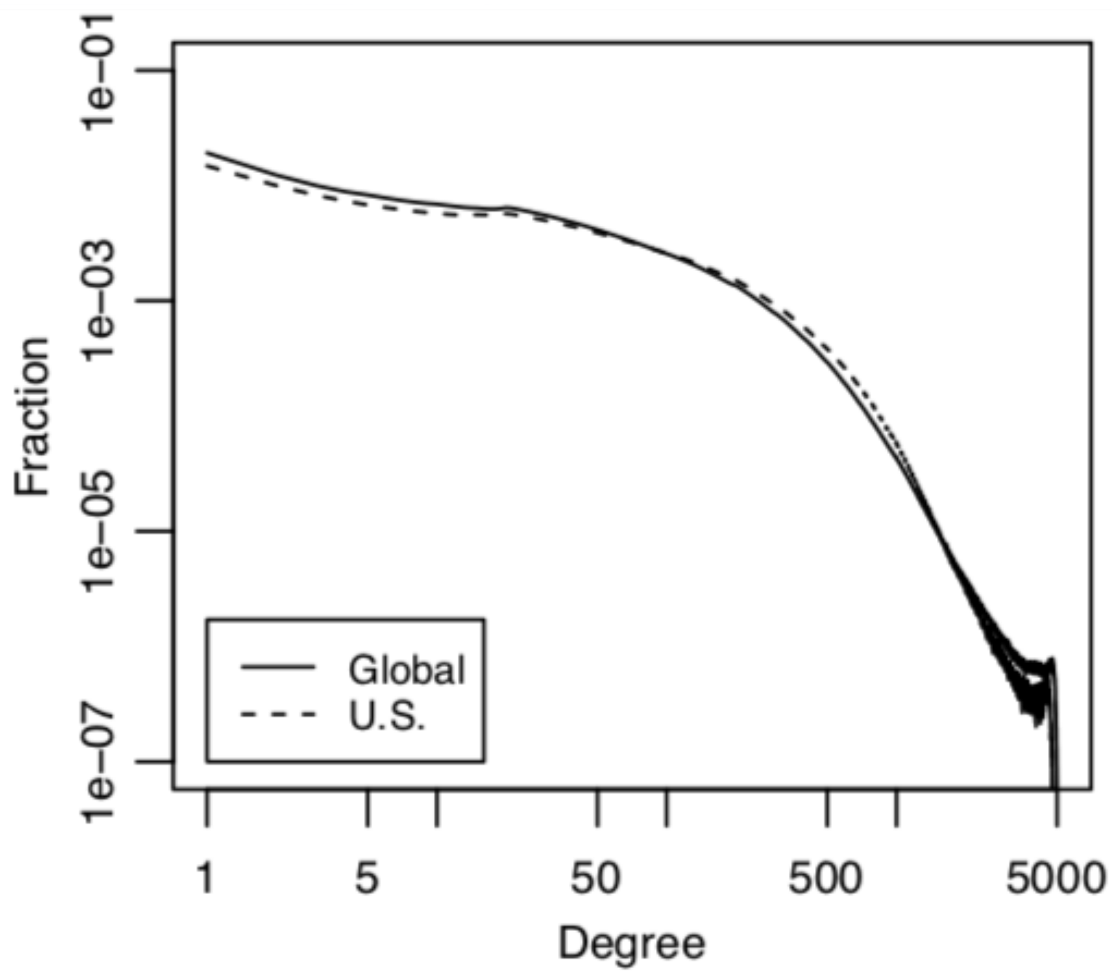
- 721M users (nodes) (active in the last 28 days)
- 68B edges
- Average degree: 190 (average # friends)
- Median degree: 99
- Connected component: 99.91%

EXEMPLE OF GRAPH ANALYSIS

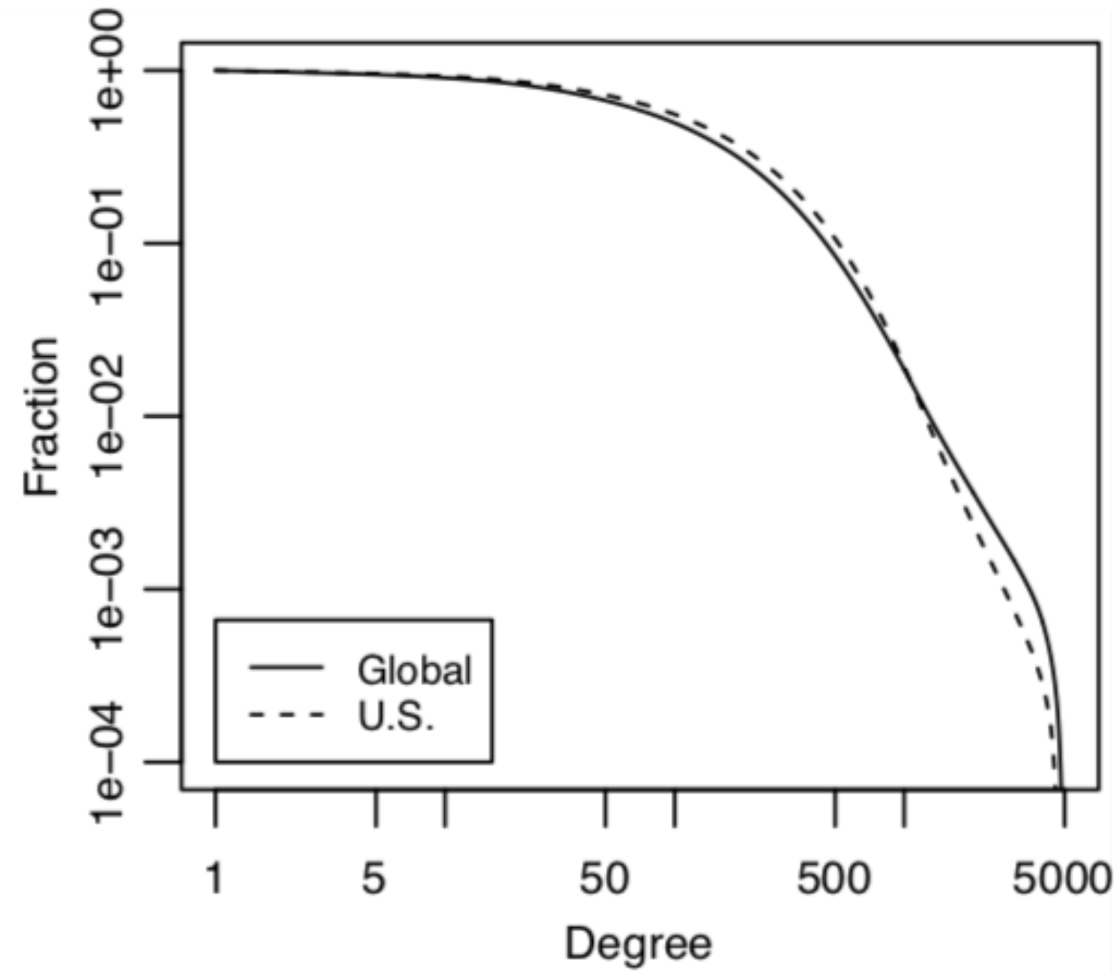


Component size
Distribution

EXEMPLE OF GRAPH ANALYSIS



(a)

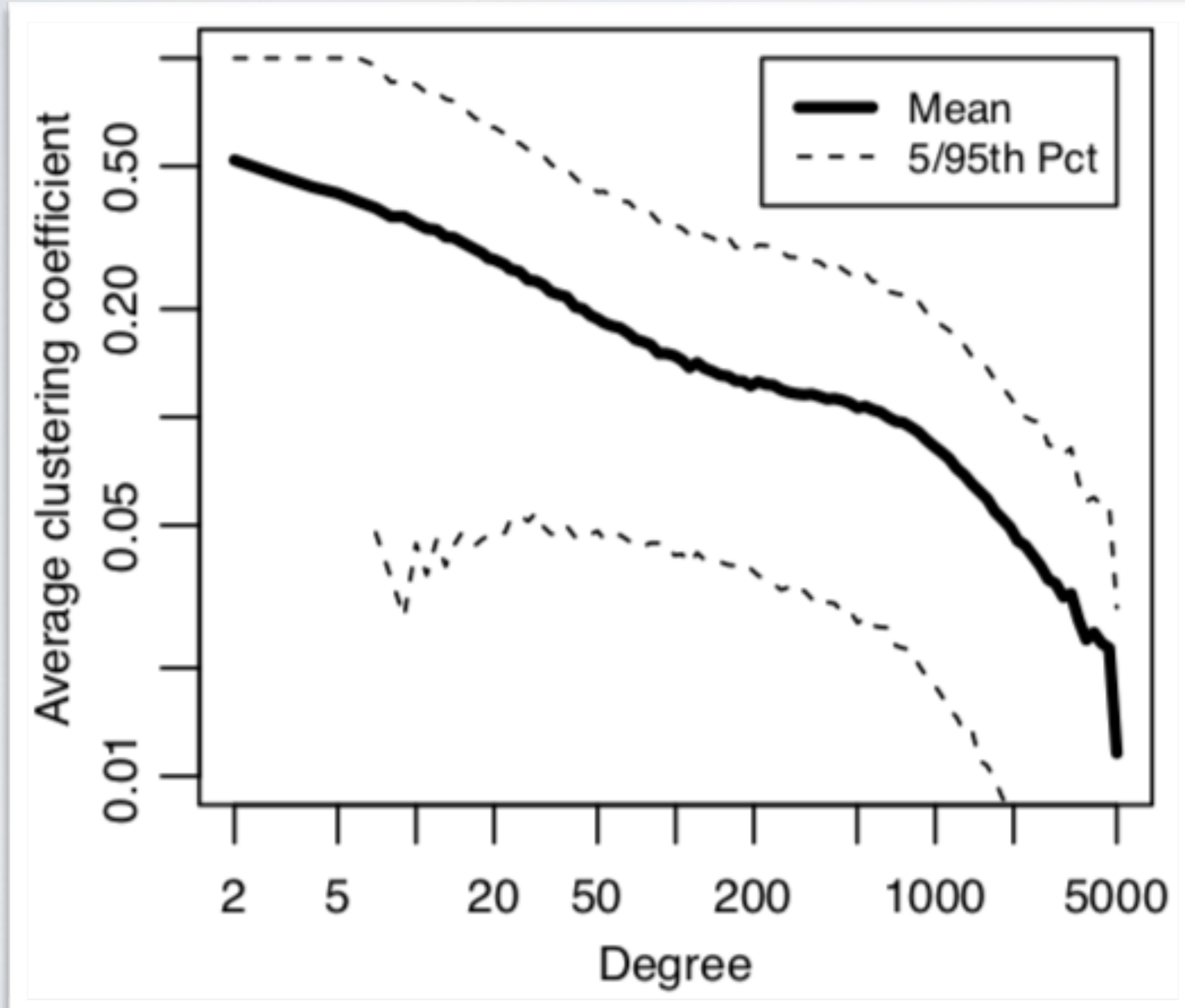


(b)

Cumulative

Degree distribution

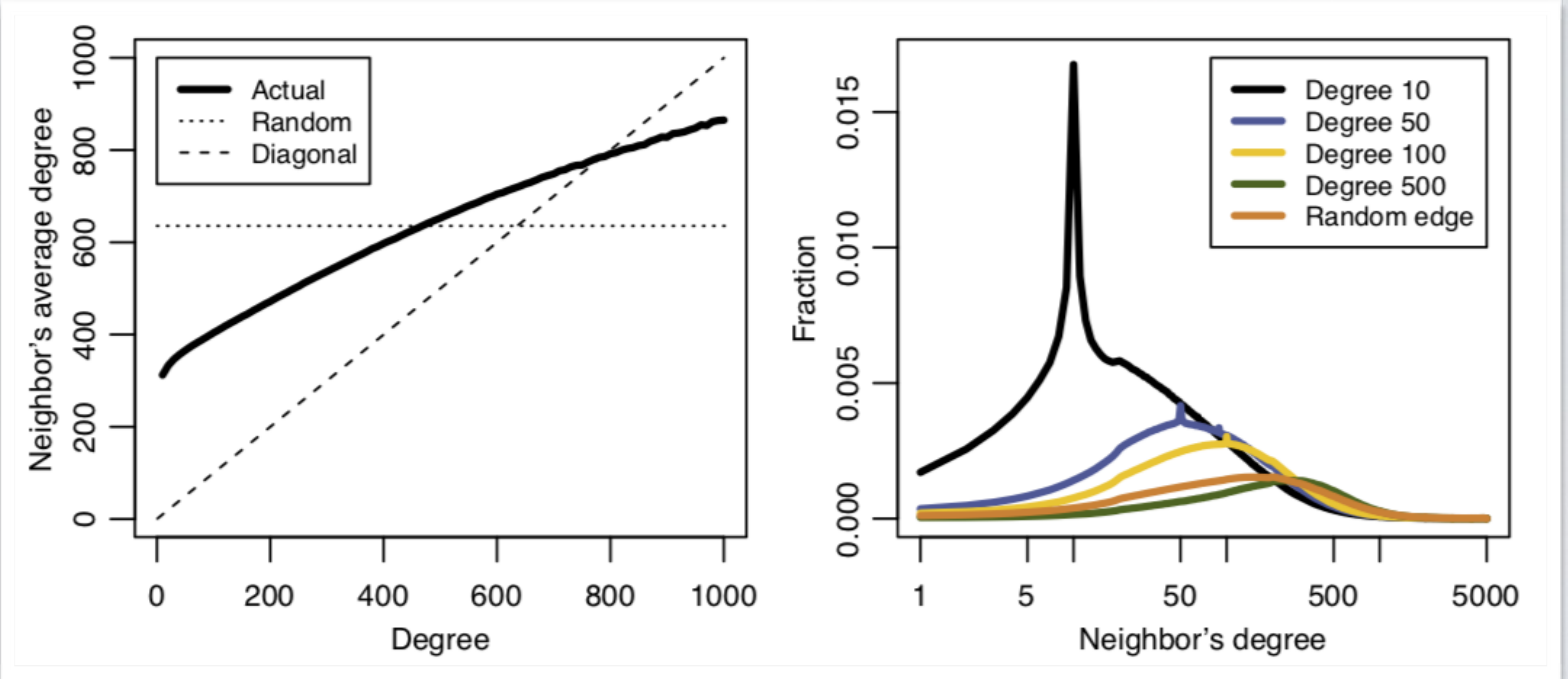
EXEMPLE OF GRAPH ANALYSIS



Clustering coefficient
By degree

Median user: 0.14:
14% of pair friends
Are actually friends

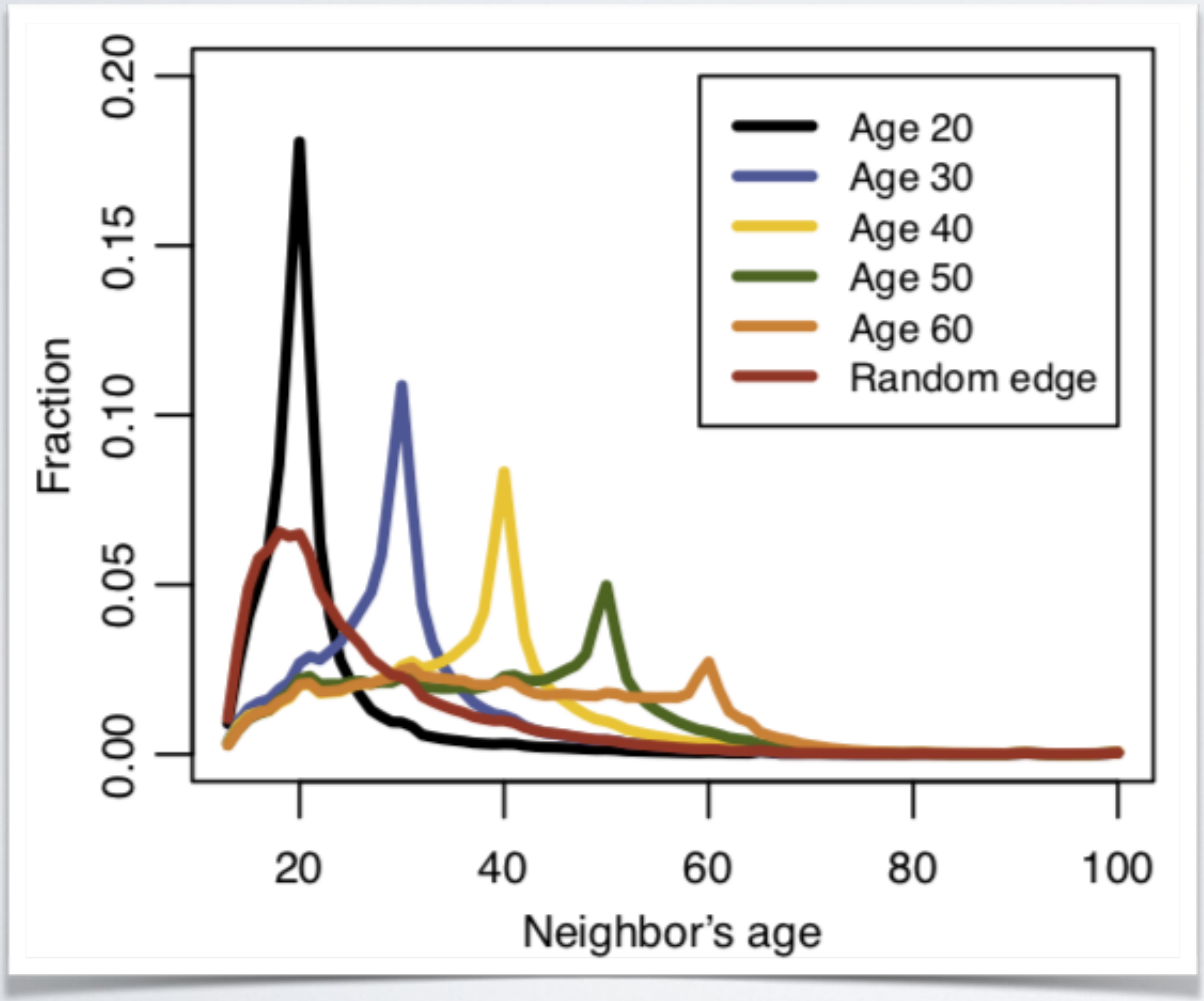
EXEMPLE OF GRAPH ANALYSIS



My friends have more
Friends than me!

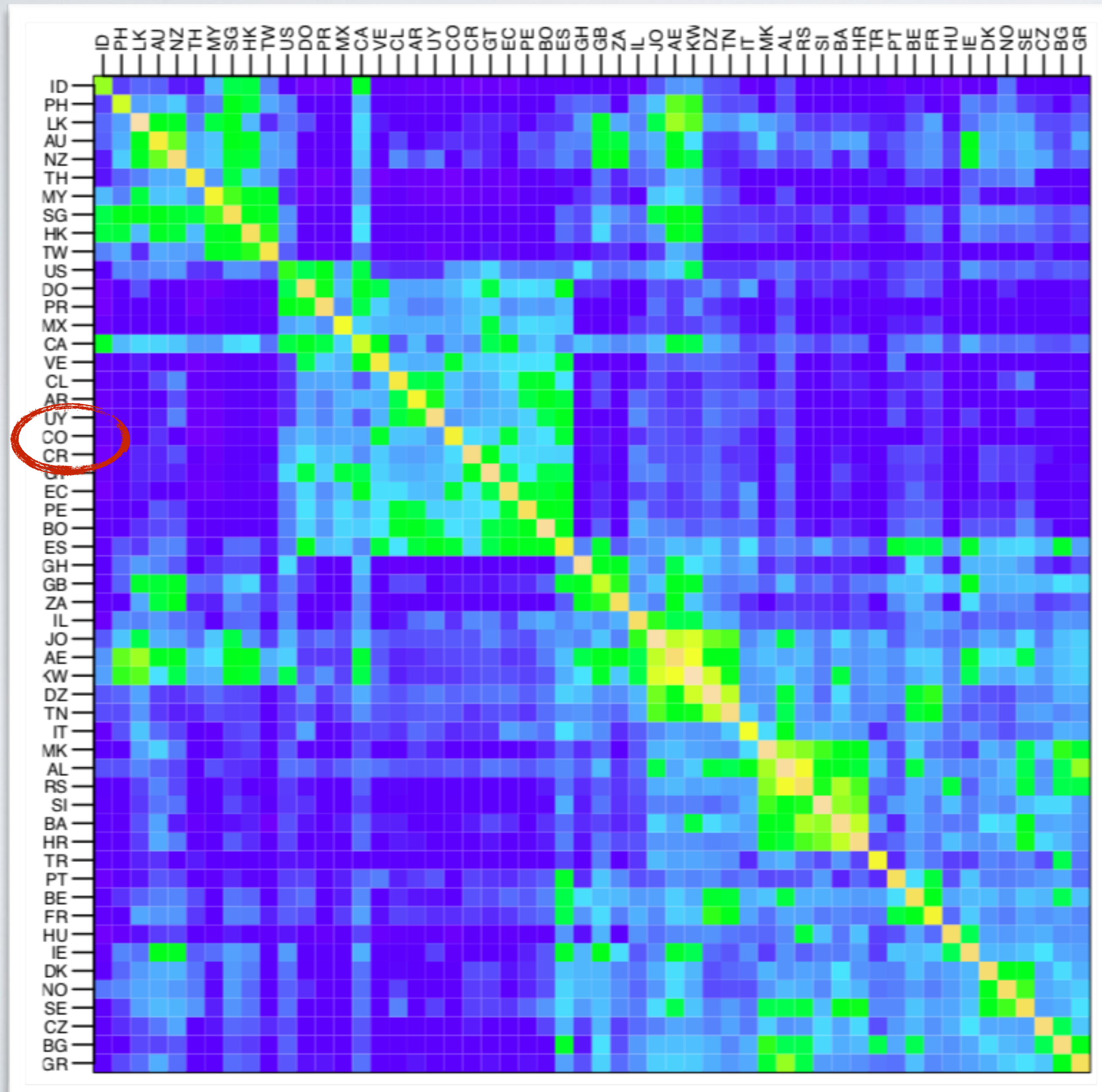
Many of my friends have the
Same # of friends than me!

EXEMPLE OF GRAPH ANALYSIS



Age homophily

EXEMPLE OF GRAPH ANALYSIS



Country similarity

84.2% percent of edges are within countries

(More in the community detection class)

MANIPULATING AND VISUALIZING GRAPHS

Using Gephi (Demo)

PRACTICAL

- Choose a network (I recommend to start with the soccer one ;))
  - http://cazabetremy.fr/Teaching/catedra.html
- Use Gephi to visualize it
  - Layout, node size and colors, edge size and colors, name…
- Choose a larger graph and try to visualize it
- Use filtering tools to clarify
- Export and interpret